NON-UNIFORM LINEAR ARRAY CONFIGURATION FOR MIMO RADAR

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Abstract: Array configuration of multiple-input multiple-output (MIMO) radar with non-uniform linear array (NLA) is proposed. Unlike a standard phased-array radar where NLA is used to generate thinner beam patterns, in MIMO radar the property of NLA is exploited to get more distinct virtual array elements so as to improve parameter identifiability, which means the maximum number of targets that can be uniquely identified by the radar. A class of NLA called minimum redundancy linear array (MRLA) is employed and a new method to construct large MRLAs is described. The numerical results verify that compared to uniform linear array (ULA) MIMO radars, NLA MIMO radars can retain the same parameter identifiability with fewer physical antennas and achieve larger aperture length and lower Cramer-Rao bound with the same number of the physical antennas.

Key words: MIMO radar; parameter identifiability; non-uniform linear array; virtual array element; array configuration

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INTRODUCTION

Multiple-input multiple-output (MIMO) radar has drawn considerable attention recently because of a number of advantages[1-5], including high sensitivity of detecting slow moving targets, excellent interference rejection capability, good parameter identifiability, and enhanced flexibility for transmitted beam pattern design. Unlike a standard phased-array radar which transmits scaled versions of a single waveform, a MIMO radar system emits orthogonal waveforms in each of the transmit antennas and utilizes a bank of matched filters to extract the waveforms at the receiver. As a new radar system, MIMO radar has many problems to be dealt with among which non-uniform spacing can achieve higher performance for parameter identifiability, that is, the maximum number of targets that can be uniquely identified by the radar[1]. It is known that the non-uniform linear array (NLA) of phased-array radar can be used to develop thinner beam patterns to improve the system performance. In this paper NLA is extended to the case of MIMO radars to obtain more distinct virtual array elements with fewer physical antennas, which is important to improve the parameter identifiability and also reduce the design cost and complexity of the radar system. A class of NLA called minimum redundancy linear array is employed for MIMO radar array configuration. A new method to generate large low redundancy arrays from small ones is also described.

1 SIGNAL MODEL OF MIMO RADAR

Assume a MIMO radar system that utilizes an array of M_t for transmit antennas and M_r for receive antennas, and many far field independent scattering point targets. Let $\boldsymbol{x}_m(n)$ denote the dis-

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crete-time baseband signal transmitted by the mth transmit antenna, and $\boldsymbol{y}_m(n)$ the signal received by the mth receive antenna.

$$\boldsymbol{x}(n) = [\boldsymbol{x}_1(n)\ \boldsymbol{x}_2(n)\ \cdots\ \boldsymbol{x}_{M_t}(n)]^{\mathrm{T}} \tag{1}$$

$$\boldsymbol{y}(n) = [\boldsymbol{y}_1(n)\ \boldsymbol{y}_2(n)\ \cdots\ \boldsymbol{y}_{M_r}(n)]^{\mathrm{T}}$$

$$n = 1,2,\cdots,N \tag{2}$$

where N is the number of samples of each signal pulses.

Let θ denote the direction-of-arrival (DOA) of a generic target. Then, under the assumption that the transmitted probing signals are narrowband and the propagation is non-dispersive, the transmitted and received steering vectors can be described by the following expression respectively

$$\boldsymbol{a}(\theta) = [\mathrm{e}^{-\mathrm{j}\omega_0\tau_1(\theta)}\ \mathrm{e}^{-\mathrm{j}\omega_0\tau_2(\theta)}\cdots\ \mathrm{e}^{-\mathrm{j}\omega_0\tau_{M_t}(\theta)}]^{\mathrm{T}} \tag{3}$$

$$\boldsymbol{b}(\theta) = [\mathrm{e}^{-\mathrm{j}\omega_0\tau'_1(\theta)}\quad \mathrm{e}^{-\mathrm{j}\omega_0\tau'_2(\theta)}\cdots\ \mathrm{e}^{-\mathrm{j}\omega_0\tau'_{M_r}(\theta)}]^{\mathrm{T}} \tag{4}$$

where $\tau_m(\theta)$ is the time delay via the mth transmit antenna to the target located at θ and $\tau'_m(\theta)$ via the target to the mth receive antenna, and ω_0 the carrier frequency. Then assume the number of the far field point targets is Q, the received signal $\boldsymbol{y}(n)$ is[1]

$$\boldsymbol{y}(n) = \sum_{q=1}^{Q} \alpha_q \boldsymbol{b}^{\mathrm{c}}(\theta_q)\boldsymbol{a}^{\mathrm{H}}(\theta_q)\boldsymbol{x}(n) + \varepsilon(n)$$

$$n = 1,2,\cdots,N \tag{5}$$

where $\varepsilon(n)$ denotes the interference noise uncorrelated with $\boldsymbol{x}(n)$, α_q the complex amplitudes proportional to the radar cross-sections (RCSs) of those targets, $(\cdot)^{\mathrm{H}}$ the conjugate transpose, and $(\cdot)^{\mathrm{c}}$ the complex conjugate. Then a new virtual array steering vector $\boldsymbol{G}$ is formed, and it is the Kronecker product of the transmitted and the received array steering vectors of MIMO radar

$$\boldsymbol{G} = \boldsymbol{a}(\theta) \otimes \boldsymbol{b}(\theta) \tag{6}$$

The concerned problem is the maximum number of targets that can be distinguished by a certain MIMO radar system. Consider the case that the transmitting array is also the receiving array (Fig. 1), for most radar systems are active ones and the array is used for both transmitting and receiving. Assume that the array is a uniform linear array, i. e. $M_t = M_r = M$, so its steering vector is

$$\boldsymbol{a}(\theta) = \boldsymbol{b}(\theta) = [1\ \mathrm{e}^{-\mathrm{j}2\pi\sin\theta d/\lambda}\ \cdots\ \mathrm{e}^{-\mathrm{j}2\pi\sin\theta(M-1)d/\lambda}]^{\mathrm{T}} \tag{7}$$

where d denotes the distance of the adjacent antennas and λ the carrier wavelength. So $\boldsymbol{G}$ is

$$\boldsymbol{G} = \boldsymbol{a}(\theta) \otimes \boldsymbol{b}(\theta) = \boldsymbol{a}(\theta) \otimes \boldsymbol{a}(\theta) \tag{8}$$

where $\boldsymbol{G}$ is supposed to have $M \times M$ distinct elements which represent the $M \times M$ distinct signal channels formed at the receiver for each of the transmitted waveforms. As there are overlaps in the results of the convolution products, only $2M-1$ distinct elements can be obtained when the array is uniform linearly designed, however, for a NLA the number may reach $M(M+1)/2$[1,6].

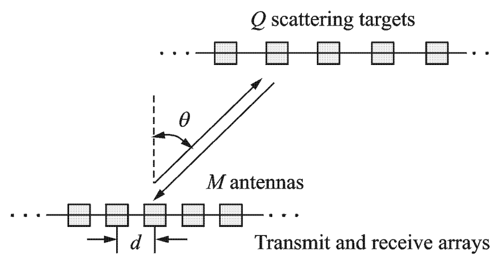

Fig. 1 ULA MIMO radar scenario

2 MINIMUM REDUNDANCY LINEAR ARRAY

The non-uniform linear array applied in this paper is the minimum redundancy linear array (MRLA)[7-8], which is to minimize the number of the antennas by reducing the redundancy of the spacing. The nomenclature used to denote MRLA of M antennas is a bracketed list of M numbers $\{u_k\}$ indicating the normalized antennas locations. For example, it is $\{0,1,4,6\}$ as shown in Fig. 2. This is a 4-antenna array whose aperture is equal to a 7-antenna ULA. Its steering vector is $\boldsymbol{a}(\theta) = [1\ \mathrm{e}^{-\mathrm{j}\omega}\ \mathrm{e}^{-\mathrm{j}4\omega}\ \mathrm{e}^{-\mathrm{j}6\omega}]^{\mathrm{T}}$, where $\omega = 2\pi d\sin\theta/\lambda$. Then its distinct elements in $\boldsymbol{G}=\boldsymbol{a}(\theta)\otimes\boldsymbol{a}(\theta)$ are $(1,\mathrm{e}^{-\mathrm{j}\omega},\mathrm{e}^{-\mathrm{j}2\omega},\mathrm{e}^{-\mathrm{j}4\omega},\mathrm{e}^{-\mathrm{j}5\omega},\mathrm{e}^{-\mathrm{j}6\omega},\mathrm{e}^{-\mathrm{j}7\omega},\mathrm{e}^{-\mathrm{j}8\omega},\mathrm{e}^{-\mathrm{j}10\omega},\mathrm{e}^{-\mathrm{j}12\omega})$.

However, for a 4-antenna ULA, $\boldsymbol{a}'(\theta) = [1\quad \mathrm{e}^{-\mathrm{j}\omega}\quad \mathrm{e}^{-\mathrm{j}2\omega}\quad \mathrm{e}^{-\mathrm{j}3\omega}]^{\mathrm{T}}$, its distinct elements in $\boldsymbol{G}'$ are $(1,\mathrm{e}^{-\mathrm{j}\omega},\mathrm{e}^{-\mathrm{j}2\omega},\mathrm{e}^{-\mathrm{j}3\omega},\mathrm{e}^{-\mathrm{j}4\omega},\mathrm{e}^{-\mathrm{j}5\omega},\mathrm{e}^{-\mathrm{j}6\omega})$.

Note that the number of the distinct virtual array elements obtained by the 4-antenna NLA is up to $M(M+1)/2=10$, while it is only $2M-1=7$ by ULA[1,6,7]. As the distinct elements represent the effective signal channels, it can be inferred that the MIMO radar parameter identifiability mostly depends on the number of the distinct elements in the Kronecker product $\boldsymbol{G}$[1,9]. More distinct elements lead to a higher identification performance under the same experimental conditions.

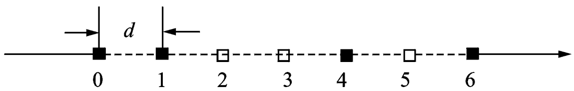

Fig. 2 MRLA of 4 antennas

The idea of MRLA was first proposed by Moffet[6]. It suggests that one should minimize the number of the physical antennas as long as the spacings between pairs of array antennas include all the integers between 1 and L, where L is the desired normalized aperture of NLA. The spacing are defined as $\{u_k - u_{k'}\}$. The optimization solution is

$$\begin{aligned} &\min_{\{u_k\}} \quad M \\ &\text{subject to } |\{u_k\}| = M \\ &\qquad \{u_k - u_{k'}\} \supset \{1,2,\cdots,L\} \end{aligned} \tag{9}$$

where $|\cdot|$ denotes the cardinality of the set. For a smaller M, the optimization solution can be found by an exhaustive search algorithm. However, when M becomes larger, it requires an extremely long time for the exhaustive searching, which is a problem not easy to solve.

In order to avoid the exhaustive search for a larger M, a new method is proposed to grow small MRLAs into large ones by inserting a seed repeatedly. Redundancy R is quantitatively defined by the ratio of the number of pairs of antennas to the desired aperture length L[7]

$$R = \frac{1}{2L}M(M-1) \qquad R \geqslant 1 \tag{10}$$

MRLAs are designed to make the redundancy R as small as possible. A bracketed list of $M-1$ numbers $(m_1, m_2, \cdots, m_{M-1})$ indicating the spacing between adjacent antennas is used to denote MRLA. Firstly, split the parent array $(m_1, m_2, \cdots, m_{M-1})$ into two parts. When M is odd, the number of elements in the bracketed list is even and it is split at the midpoint. When M is even, the list can be split at either the $(M-1)/2$ or the $M/2$ position. Then the new array is constructed by inserting a number repeatedly at the position where the list has been split. This number is equal to the number of antennas (M) in the parent configuration. The number inserted at the midpoint of the list must appear at least twice in order to ensure that the array is restricted. For example, when $M=13$, the array configuration can be generated by the following sequence with the redundancy $R=1.34$

$$\underset{M=9,L=22}{(1,4,3,4,5,1,2,2)} \rightarrow \cdots \rightarrow$$

$$\underset{M=13,L=58}{(1,4,3,4,9,9,9,9,5,1,2,2)}$$

In this way, the redundancy R of the large MRLAs can be constrained within $R \leqslant 1.60$ when $M \leqslant 37$[6-8], which is an acceptable redundancy.

3 NUMERICAL RESULTS

Several numerical examples are presented to compare the parameter identifiability of the NLA MIMO radar with its ULA counterpart. The applied transmitted waveforms are quadrature phase shift keyed (QPSK) sequences which are orthogonal to each other[1].

Firstly, consider a scenario where Q targets are located with $\Delta\theta = 10°$ to adjacent ones. The number of the snapshot is $N=256$. Assume the received signal is mixed with a Gaussian noise with mean zero and variance 0.01. An MIMO radar system with $M=7$ antennas is used for testing. Let the array as MRLA$\{0,1,4,6,13,21,31\}$ and all the distances between antennas are times of half-wavelength. Fig. 3 shows the simple least-squares (LS) spatial spectrum $\eta_{\mathrm{LS}}(\theta)$, as a function of θ, when $Q=12$ and the targets are located from $-50°$ to $+60°$. Note that all the 12 targets can be identified by the peak of the LS spatial spectrum. However, compared with an ULA MIMO radar[1], at least 10 antennas are needed to get 12 targets separated under the same simulation conditions.

Then consider a NLA MIMO radar system with $M=6$. Its array configuration is $\{0,1,4,5,11,13\}$. The corresponding 6-antenna ULA MIMO radar system is also tested where the adjacent

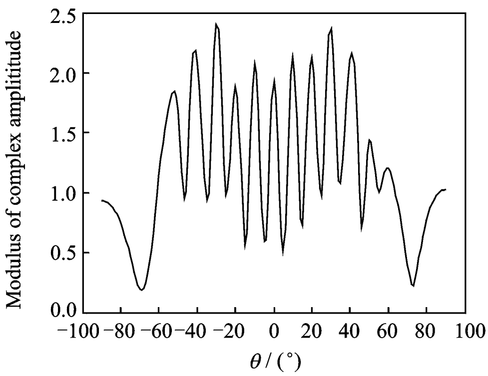

Fig. 3 LS spatial spectrum ($M=7, Q=12$)

antennas are half-wavelength spaced. All the simulation parameters are the same as those in the above example except that $Q=10$. It can be observed from Fig. 4 that the NLA system can distinguish the 10 targets clearly, while it is very hard for the ULA system to gain the similar performance with the same number of antennas.

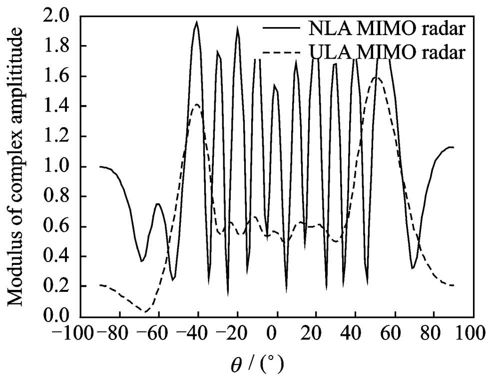

Fig. 4 Comparison of LS spatial spectrum between NLA and ULA MIMO radars ($M=6, Q=10$)

The numerical results are also provided on the Cramer-Rao bound (CRB) of θ, which is probably the best known lower bound on the MSE of unbiased estimators[5,10]. CRB has the following form in the simulation

$$\mathrm{CRB}(\theta) = \left[\| \boldsymbol{a}^{c}(\theta) \|^{2} - \frac{\| [\boldsymbol{a}^{\mathrm{H}}(\theta)]^{c} \boldsymbol{a}(\theta) \|^{2}}{M} \right]^{-1} \times \frac{M}{2(M-1)NSNR} \times \left[1 + \frac{M}{(M-2)NSNR} \right] \quad (11)$$

Fig. 5 shows CRB of θ as a function of Q and a comparison of the two systems. Note that CRB of NLA MIMO radar is always lower than its ULA counterpart as Q increases from 1 to 10.

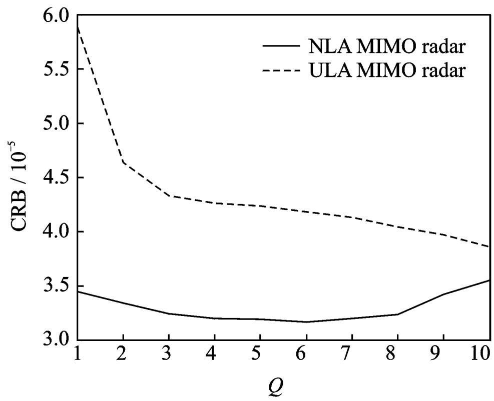

Fig. 5 CRB of θ for NLA and ULA MIMO radars

4 CONCLUSION

In this paper, a non-uniform linear array configuration method is presented for MIMO radars. As demonstration of the potential advantages that a NLA MIMO radar can offer, the LS and the Cramer-Rao bound are evaluated for parameter estimation. The numerical results show that compared with ULA MIMO radars, NLA MIMO radars can achieve the same parameter identifiability with fewer physical antennas and obtain more distinct virtual array elements and lower Cramer-Rao bound with the same number of the antennas. The NLA configuration method can reduce the cost and complexity of the array design in MIMO radars. How to design the optimum non-uniform linear arrays for a MIMO radar system is valuable in deep research.

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多输入多输出雷达的非均匀线性阵列配置

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摘要:提出了利用非均匀线性阵列(Non-uniform linear array,NLA)对多输入多输出(Multiple-input multiple-output,MIMO)雷达系统进行阵列配置优化的方法。在传统的相控阵雷达中,非均匀线性阵列配置被用来形成较窄的波束方向图,而在MIMO雷达中,利用非均匀线性阵列来获得更多的互不相同的虚拟阵元,以此来提高雷达的参数可辨识性能。文中所采用的一种非均匀线性阵列是最小冗余线性阵列,并给出了一种在物理阵元数量较大时最小冗余线性阵列的生成方法。实验结果表明:与均匀线性阵列(Uniform linear array,ULA)配置的MIMO雷达相比,非均匀线性阵列MIMO雷达能够利用较少的物理天线阵元获得相同的参数可辨识性能;而在两种配置的雷达系统的物理阵元个数相同的情况下,非均匀线性阵列MIMO雷达可以获得更大的阵列孔径长度和更低的克拉美·罗界。

关键词:多输入多输出雷达;参数可辨识性能;非均匀线性阵列;虚拟阵元;阵列配置

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