MODAL FREQUENCY CHARACTERISTICS OF AXIALLY MOVING BEAM WITH SUPERSONIC/HYPERSONIC SPEED

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Abstract: The vibration characteristics of transverse oscillation of an axially moving beam with high velocity is investigated. The vibration equation and boundary conditions of the free-free axially moving beam are derived using Hamilton's principle. Furthermore, the linearized equations are set up based on Galerkin's method for the approximation solution. Finally, three influencing factors on the vibration frequency of the beam are considered: (1) The axially moving speed. The first order natural frequency decreases as the axial velocity increases, so there is a critical velocity of the axially moving beam. (2) The mass loss. The changing of the mass density of some part of the beam increases the beam natural frequencies. (3) The thermal effect. The temperature increase will decrease the beam elastic modulus and induce the vibration frequencies descending.

Key words: axially moving beam; vibration; thermal effect; supersonic/hypersonic

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INTRODUCTION

Axial motion has obvious effect on the stability of the high-speed slender members, which even can lead to the structure instability[1]. Zajaczkowski and Lipinski[2], Zajaczkowski and Yamada[3] analyzed the dynamic stability of an Euler-Bernoulli beam under periodically sliding motion. Vu-Quoc et al[4] investigated dynamics of sliding geometrically-exact beams. Zhu and Ni[5] derived the energy and stability characteristics of a class of translating beams and strings with arbitrarily varying length.

Axial moving beams with high-speed, such as rockets, missiles, space shuttle and so on, draw more and more attentions of researchers, which can be modeled as axially moving free-free beams. However, the dynamics has been analyzed without considering the axially moving effect by now. Williams et al[6] computed the natural frequencies and mode shapes of the hypersonic vehicle, where the structural dynamics model was based on the free-free, Euler-Bernoulli beam. Culler et al[7] modeled the hypersonic vehicle by a beam whose mass and stiffness characteristics captured the expected flexible modes. They analyzed the modal frequencies and mode shapes under the combined effects of mass change and temperature change.

Recently, more and more people have realized that the thermal effect is important for the design of supersonic or hypersonic aircraft. One of the hypersonic candidates is the X-51a hypersonic cruise missile, which is designed to hit *Ma* 5. The goal, according to the U. S. Strategic Command's deputy commander Lt Gen C Robert Kehler, is "to strike virtually anywhere on the face of the Earth within 60 min"[8]. Aerodynamic heating problem is one of the key issues of supersonic or hypersonic flying problems. The aircraft nose cone, wing windward surface of the front edge might suffer thousands of degrees Celsius temperature under hypersonic speed. And this aerodynamic heating has severe influence on the vibration characteristics of the structure. Emil and Pedro[9] studied geometrically nonlinear vibrations of

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a Timoshenko beam under the combined action of mechanical and thermal loads. It is found that the thermal loading duration and thermal loading amplitude had serious influence on the response of the structures. Jurij and Maks[10] investigated the effect of the temperature field on the vibrations of the beams. It was obtained that small change of temperature could cause obvious changes of the natural frequencies of the beams. The aerodynamic heating, modeling and dynamic characteristics of supersonic aircraft were investigated by Séror et al[11]. They derived a new preferential vibration-dissociation-exchange reaction coupling model resulting from an extension of the well-known Treanor and Marrone coupled vibration dissociation vibration (CVDV) model, which had been derived to take into account the coupling between the vibrational excitation of the N_2 and O_2 molecules and the two Zeldovich exchange reactions. Dinkelmann et al[12] used optimal trajectory to reduce heat input for the range cruise of a hypersonic flight system propelled by a turbo/ram jet engines combination. Culler et al[7] focused on effects of aerodynamic heating on the rigid-body and structural dynamics of an air-breathing hypersonic vehicle. It was shown that the effects of the aerodynamic heating on the aircraft rigid-body poles and zeros were negligible.

Up to now, there is no study on the vibration characteristics of the supersonic or hypersonic flying aircraft considering heating and axially moving effects. In this paper, the authors focus on the transverse vibration characteristics of an axial moving beam together with aerodynamic heating and mass changing influence.

1 BEAM MODEL

1.1 Equation of motion

Fig. 1 shows an Euler-Bernoulli beam which is a free-free elastic beam and moves along the axial direction. The length of the beam is l, the flexural rigidity is $EI(x)$, and the cross section is arbitrary with mass per unit length $\rho(x)$ which is similar to the X-51a. The beam is assumed to be inextensible with an arbitrarily prescribed axially velocity $v(t)$. Small transverse vibration of the beam about its trivial equilibrium is considered here. According to the fixed coordinate system shown in Fig. 1, the transverse displacement of the beam particle instantaneously located at spatial position x, where $0<x<l$, is $y(x,t)$.

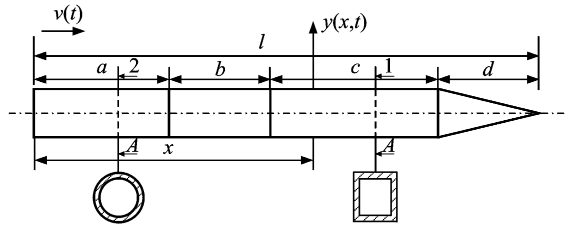

Fig. 1 Axially moving beam with arbitrary cross section

The total kinetic energy of the beam is

$$TE = \frac{1}{2}\int_0^l \rho v^2 \mathrm{d}x + \frac{1}{2}\left[\int_0^l \rho (y_t + v y_x)^2 \mathrm{d}x\right] \tag{1}$$

where the first term in Eq. (1) represents the kinetic energy associated with the longitudinal translational motion of the beam, and the second term the kinetic energy associated with the transverse vibration.

The potential energy of the system, including the potential energies associated with the axial force and the flexure rigidity, is

$$VE = \frac{1}{2}\int_0^l [P(x) y_x^{\,2} + EI y_{xx}^{\,2}]\mathrm{d}x \tag{2}$$

where $P(x)$ is the axial force. The axial force in the horizontally translating beam in Fig. 1 is

$$P(x) = -\int_x^l \rho \mathrm{d}x \dot{v} \tag{3}$$

Substitute Eq. (1) and Eq. (2) into Hamilton's principle

$$\delta\left[\int_{t_1}^{t_2} (TE - VE)\mathrm{d}t\right] = 0 \tag{4}$$

Setting the coefficients of δy to zero yields the following equation

$$\begin{aligned}\rho(y_{tt} + 2v y_{xt} + \dot{v} y_x + v^2 y_{xx}) + \rho_x v (y_t + v y_x) - \\ (P_x y_x + P y_{xx}) + (E_{xx} I + 2E_x I_x + \\ E I_{xx}) y_{xx} + 2(E_x I + E I_x) y_{xxx} + \\ E I y_{xxxx} = 0\end{aligned} \tag{5}$$

where only the temperature effect on the elastic modulus is considered. The elastic modulus of metal can be expressed as the following formula by thermo-elastic theory, where the second order item is neglected[13]

$$E = E_0 + E_1\Delta T = E_0\left(1 + \frac{E_1}{E_0}\Delta T\right) = E_0(1 + \alpha_E \Delta T) \quad (6)$$

where E_0, E_1, E_2 are the material constants, $\alpha_E = E_1/E_0$ is the thermoelastic coefficient.

The resulting boundary conditions for Eq. (5) are

$$y_{xx}(l,t) = y_{xx}(0,t) = 0$$
$$y_{xxx}(l,t) = y_{xxx}(0,t) = 0 \quad (7)$$

1.2 Spatial discretization

The Galerkin's method is employed to truncate the above governing partial differential equation to a set of time-dependent ordinary differential equations. The solution of the displacement can be expressed as a superposition of every mode as

$$y = \sum_{j=1}^{n} q_j(t)\phi_j(x) \quad j = 1,2,3,\cdots \quad (8)$$

where n is the number of kept modes, $q_j(t)$ are the generalized coordinates, and $\phi_j(x,t)$ ($j=1,2,3,\cdots,n$) the instantaneous orthonormal eigenfunctions and have forms as

$$\phi_j(x,t) = \frac{1}{2}[\mathrm{ch}(\lambda_j x) + \cos(\lambda_j x)] - \frac{1}{2}\frac{\mathrm{sh}(\lambda_j l) + \sin(\lambda_j l)}{\mathrm{ch}(\lambda_j l) - \cos(\lambda_j l)}[\mathrm{sh}(\lambda_j x) + \sin(\lambda_j x)]$$

where $\mathrm{ch}(\lambda_j l)\cos(\lambda_j x) = 1$.

Substituting Eq. (8) into Eq. (5) with Eq. (7), multiplying the equation by $\phi_i(x,t)$, integrating it over the domains $[0,l]$, yields

$$\boldsymbol{M}(t)\ddot{\boldsymbol{q}}(t) + \boldsymbol{C}(t)\dot{\boldsymbol{q}}(t) + \boldsymbol{K}(t)\boldsymbol{q}(t) = 0 \quad (9)$$

where $\boldsymbol{q} = [q_1, q_2, \cdots, q_n]^{\mathrm{T}}$ is the vector of generalized coordinates, $\boldsymbol{M}$ the mass matrix, $\boldsymbol{C}$ the damping matrix, and $\boldsymbol{K}$ the stiffness matrix. Entries of these matrices are given by

$$m_{ij} = \int_0^l \rho\phi_i\phi_j \mathrm{d}x \quad (10)$$

$$c_{ij} = \int_0^l [2\rho v\phi_i(\phi_j)_x + v\rho_x\phi_i\phi_j]\mathrm{d}x \quad (11)$$

$$k_{ij} = \int_0^l [\dot{v}\rho\phi_i(\phi_j)_x + v^2\rho\phi_i(\phi_j)_{xx} + v^2\rho_x\phi_i(\phi_j)_x]\mathrm{d}x - \int_0^l [P_x\phi_i(\phi_j)_x + P\phi_i(\phi_j)_{xx}]\mathrm{d}x + \int_0^l EI(\phi_i)_{xx}(\phi_j)_{xx}\mathrm{d}x \quad (12)$$

2 PARAMETER EFFECT ON DYNAMIC CHARACTERISTICS

According to X-51a, it is set that $l=8$ m, $a=2$ m, $b=1.5$ m, $c=2.5$ m, $d=2$ m. The cross sections of the beam are put forward as follow. Segment a is a cylinder with the thickness 0.08 m and the external diameter 0.8 m. Segment c is a cuboid with the side length and thickness 0.8 m and 0.05 m, respectively. And segment b is a transition segment for a and c. Segment d of the beam is a wedge whose cross section is a rectangle with thickness 0.08 m. The Young's modulus with room temperature and mass density are $E_0 = 100\times10^9$ N/m and $\rho^* = 0.6\times10^3$ kg/m^3. There are an extra mass per unit length 250.8 kg/m added to a and b which simulate the fuel. The thermoelastic coefficient[13-14] is $\alpha_E = -4\times10^{-4}$.

Here, the first two order modes are studied. Firstly, there include a symmetrical mode and a antisymmetrical mode, so it is representative. Secondly, the first two order modes are important for navigation and control.

2.1 Effect of axial motion

On this occasion, it is assumed that the beam is moving axially with no acceleration. Figs. 2, 3 show the relationships of stiffness and the first order frequency with the axial velocity, respectively.

It can be seen that the stiffness $K(1,1)$ and $K(2,2)$ and the first order natural frequency decrease as the axial velocity increases, so there is a critical velocity of instability. In this example,

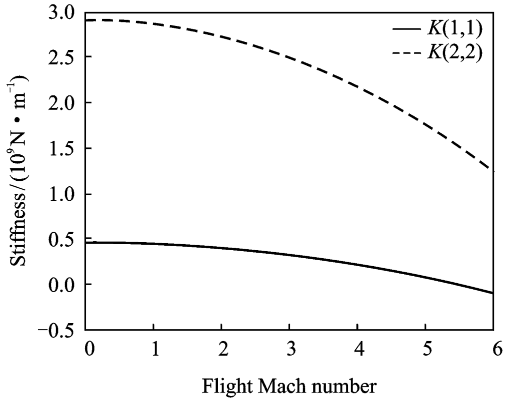

Fig. 2 Relationship between stiffness and axial velocity

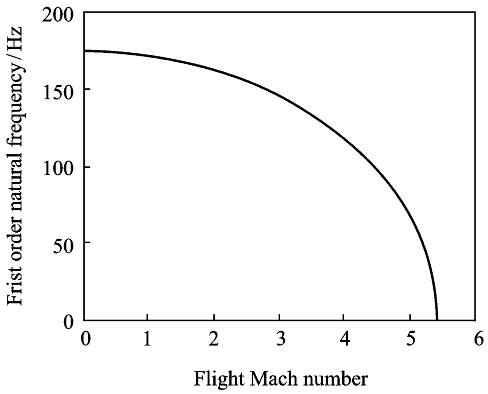

Fig. 3 Relationship between first order frequency and velocity

the critical velocity is about *Ma* 5.4.

2.2 Effect of mass density change

Here, we assume that the axial velocity and acceleration of the beam are 0 m/s and 45 m/s^2, respectively. The following two cases are dissussed.

Case 1 The unit length mass density keeps invariant.

Case 2 The mass of the sections *a* and *b* decrease as a slope of 8.4 kg/(m · s).

The change of the first order frequency with time is shown in Fig. 4. Fig. 4 shows that the decreasing of the mass induces significant increasing of the natural frequency.

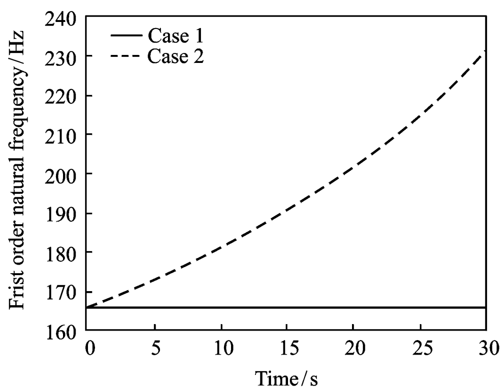

Fig. 4 Change of first order frequency with time

2.3 Thermal effect

When the axial speeds are *Ma* 2 and *Ma* 4, the vertical temperature distribution of leeward and windward sides are shown in Figs. 5, 6. Here the temperature data are mainly referred to Ref. [15]. In Figs. 5, 6, the abscissa coordinates are normalized by the length of the beam. The temperatures are normalized by the maximum temperature value of 880 and 1 031 K, respectively.

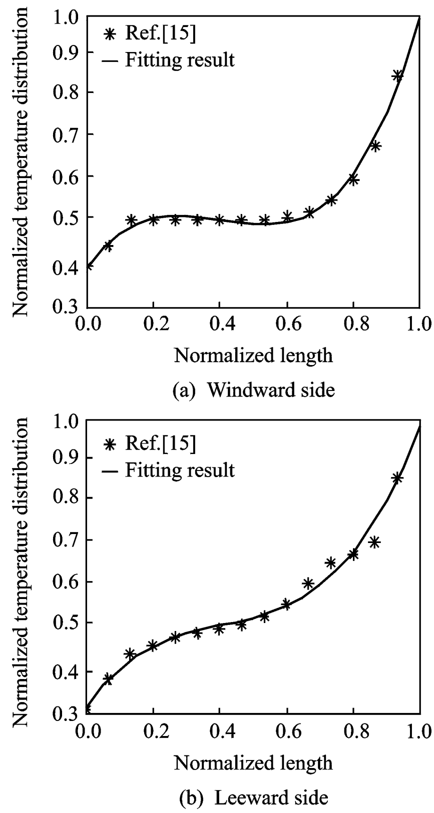

Fig. 5 Temperature distribution of beam at *Ma* 2

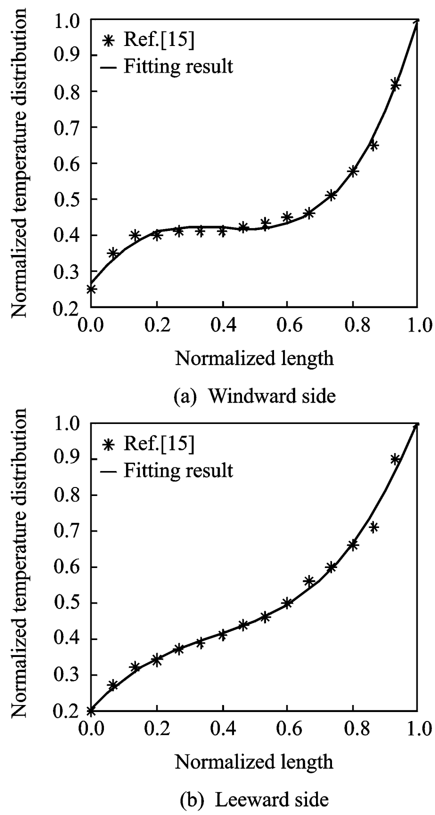

Fig. 6 Temperature distribution of beam at *Ma* 4

Figs. 7, 8 show the thermal effects on the stiffness and the first order natural frequency where the temperature averages of leeward and windward sides at the same position are applied to the structure.

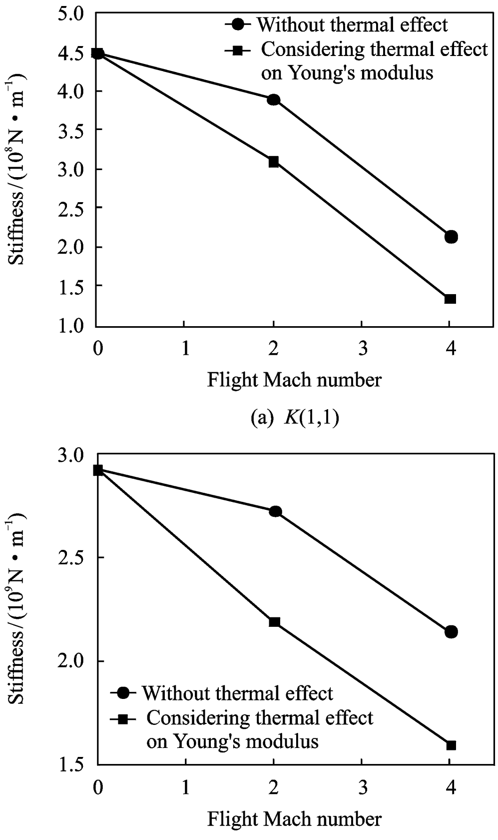

Fig. 7 Effect of thermal effect on stiffness

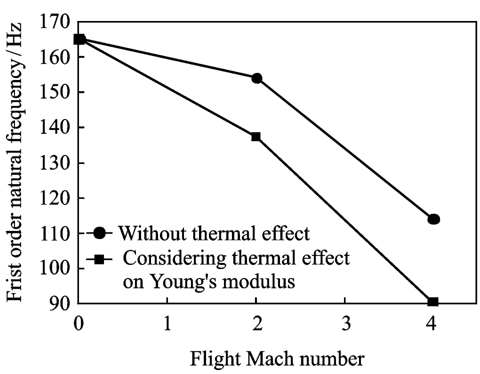

Fig. 8 Effect of thermal effect on first order frequency

From the results we can see that the diagonal elements of the stiffness matrix and the first-order natural frequency will decrease if the thermal effect is considered.

2.4 Effects of axially moving speed, mass changing and thermal

In this section we study the dynamic characteristics of the beam with axially moving speed together with mass changing and thermal effects. We also assume that the initial axial velocity and a constant acceleration of the beam are 0 m/s and 200 m/s^2, respectively. The results are shown in Fig. 9.

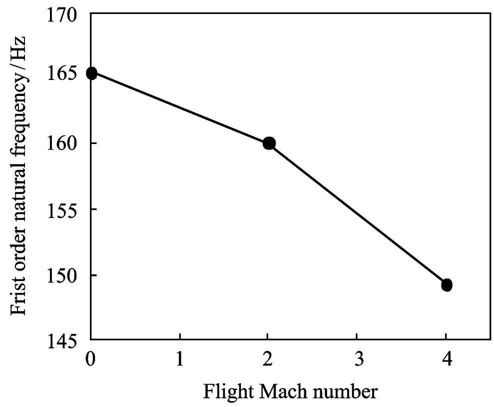

Fig. 9 First order natural frequency vs velocity

It can be seen that the first order frequency is decreased significantly when the axial speed of the beam approaches a supersonic/hypersonic value. It can be concluded that there must be a critical axial velocity of the beam when it flies hypersonically.

3 CONCLUSIONS

In the paper, the dynamic characteristics of the transverse oscillation of a free-free axially moving beam with supersonic velocity is firstly discussed. The change of unit length mass density and thermal effects are discussed. The following conclusions are obtained.

(1) The first order natural frequency decreases as the axial velocity increases, so there is a critical velocity of instability, and this velocity is a relative to the structure. The critical velocity in the paper is about *Ma* 5.4.

(2) The mass loss of some part of the beam introduces the increasing of natural frequency, and the frequency increases about 65 Hz in the paper.

(3) The thermal effect on the elastic modulus decreases stiffness diagonal elements and first-order natural frequency significantly.

(4) The two effects of mass loss and temper-

ature on the first order natural frequency of the beam are cancelled by each other. So, the synthetic result (whether the first order natural frequency is increased or decreased) is determined by both of them.

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超高/超声速轴向运动梁的模态频率特性

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摘要:研究了高速轴向运动梁的横向振动特性。首先采用Hamilton 原理得到了两端自由轴向运动梁的振动方程和边界条件。再运用伽辽金法得到了求解系统响应的近似方程。最后研究了对梁的振动频率影响较大的3个方面:(1)轴向速度,第一阶固有频率随轴向速度增加而减小,存在一个失稳的临界速度;(2)质量亏损,梁的某部分质量亏损可以增加固有频率;(3)热效应,温度上升会减小梁的弹性模量,使得固有频率减小。

关键词:轴向运动梁;振动;热效应;超高/超声速

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