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# DAMAGE TOLERANCE AND FATIGUE LIFE ESTIMATION FOR PIPE/ROD BAR STRUCTURE

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**Abstract**: The damage tolerance for pipe/rod bar in the auxiliary power units (APUs) support system is studied. The main objective is to study whether planes can safely land when the fatigue crack appears on the bar. Firstly, the stress intensity factors ($K_\mathrm{I}$) of two bar structures are computed, and the iso-stress intensity factor (iso-$K_\mathrm{I}$) model is presented. The model uses the finite element model (FEM) instead of the parameters of material to compute the crack propagation. Then, the general relations between $K_\mathrm{I}$ vs crack size are obtained for different materials, inner to outer diameter ratios ($D_\mathrm{in}/D_\mathrm{out}$) and external crack propagation angles ($\theta$). Finally, the FEM analysis results are input to the ANSYS parametric design language(ANSYS-APDL) software. And the fatigue life of damaged bar is estimated under the condition of loading spectrum.

**Key words**: fatigue crack propagation; circumferential crack; stress intensity factor; pipe/rod bar

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## INTRODUCTION

Pipe structures are widely used in many engineering applications. Tension pipes are commonly found in engineering applications, such as in aircraft auxiliary power units (APUs) support system, reactor, rotorcraft, machine and machine components, etc. The structures are applied to the random loading spectrum. The cracks greatly reduce the load bearing capacity of pipes. Structure fracture leads airplane in danger. The primary techniques used in fracture mechanics are the finite element method and the line-spring element technique. As far as the crack propagation phase is concerned, the most dominant parameter is the near-crack-tip elastic stress intensity factor. The solutions for the external and internal circumferential cracks are of the same order. In engineering sense they show no significant difference[1]. There are relatively few solutions for pipes containing partly elliptical circumferential surface flaws. Similar studies about the circumferential crack propagation in a pipe have been done in Refs. [1-4].

The crack propagation phase in a pipe is studied in three phases shown in Fig. 1.

(1)Phase Ⅰ: The shape of the crack front is nearly elliptical and the crack front only crosses the outer surface of pipe.

(2)Phase Ⅱ: Crack growth shape is near straight line at the beginning and more complex at later phase. Crack front crosses both the inner and the outer surfaces of pipe.

(3)Phase Ⅲ: Crack growth shape is nearly elliptical and the crack front has just broken through in the opposite inner surface. Practically it cannot sustain any loading. It is very unstable.

To estimate the crack propagation life under fatigue loading, the following information must be known:

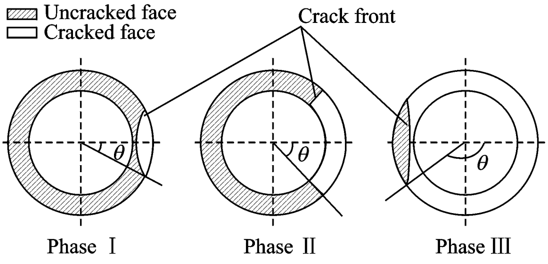

Fig. 1 Crack propagation phases in pipe

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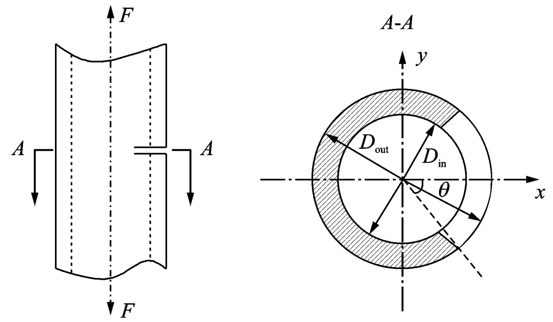
(1)Material properties such as d$a$/d$N$ curve;

(2)Fatigue loading spectrum;

(3)Relations between $K_{\mathrm{I}}$ vs crack size under specific loading. The finite element model (FEM) is used to find the relations.

# 1 FINITE ELEMENT MODELING

EFM is the most dominant technique for investigating these structures due to its flexibility in complex structure modeling[1,2,4,5]. The crack configuration shown in Fig. 2 is described by some non-dimensional parameters, i. e. , the inner to outer diameter ratio of the pipe ($D_{\mathrm{in}}/D_{\mathrm{out}}$) and the external crack propagation angle($\theta$).



(a) Pipe containing circumferential crack (b) Crack plane

Fig. 2 Circumferential cracked pipe under tension loading

## 1.1 Crack tip element

A significant advancement in the use of FEM for linear elastic fracture mechanic (LEFM) problems was simultaneous and independent development of "quarter-point" element[6-7]. The quarter-point element achieves more accurate result. The singular elements were utilized around the crack front in order to induce a square-root singularity of stress/strain field in the vicinity of crack front[8]. The twenty-node iso-parametric brick elements (Solid 95[9]) were regarded as crack tip (Fig. 3) and the other parts of the model were used with eight-node brick element (Solid 45[9]) for the higher computational efficiency. The half-elliptical crack front consists of 20—60 crack tip elements depending on the crack propagation phase. The crack front uses the focused type of mesh with typically 5—10 elements to enclose each crack front element in radial direction as shown in Fig. 4. In order to avoid the large number of required analyses and save time, the code in ANSYS Parametric Design Language (ANSYS-APDL) software is developed.

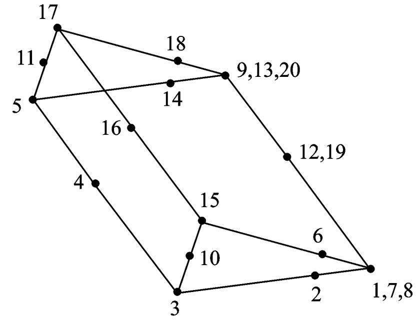


Fig. 3 Twenty-node crack tip element

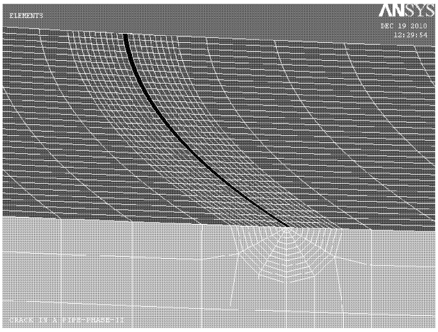


Fig. 4 Focused type of mesh

## 1.2 Boundary condition and loading

The symmetry conditions in the longitudinal and lateral directions are exploited to reduce the computation and FEM efforts. FEM for a pipe containing the circumferential crack is shown in Fig. 5. The loading condition includes the uniform pressure of $p=1$ MPa on the lower extreme surface of cylinder. Nodes $A$, $B$ and $C$ of the tip element in Fig. 6 are constrained in $z$-direction to achieve the singularity in the strain[8]. Fig. 6 shows the stress distribution on the crack tip, i. e. , $\sigma_z = C_\sigma / \sqrt{r}$ , and the displacement curve of the distorted element, i. e. , $u_z = C_u \sqrt{r}$ , where $C_\sigma$ and $C_u$ are constants[8]. Moreover, Fig. 6 shows the crack tip elements 1—6, where $u_{z1/4}$ and $u_z$ are the crack tip opening displacements of the quarter chord node and the corner node, respectively.

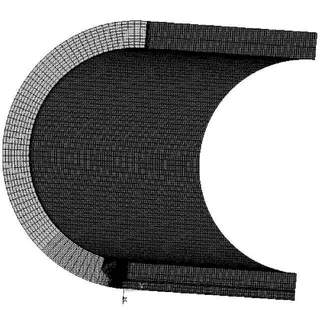


Fig. 5 Finite element model

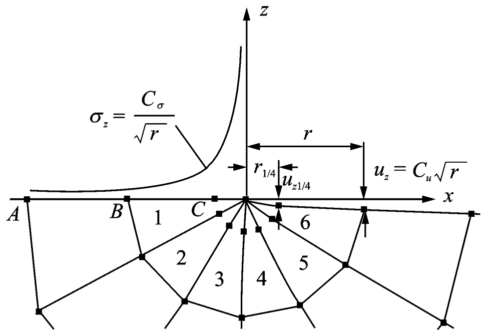


Fig. 6 Stress distribution on crack tip and element singularities

## 2 FATIGUE CRACK PROPAGATION

### 2.1 Iterative crack front propagation

Most commonly used fatigue crack propagation model is the iterative crack front propagation[3-4]. And it is also called the two-parameter theoretical model[4]. The model uses the Paris-Erdogan law (Eq. (1)) to assume the crack propagation.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{\mathrm{I}})^m \tag{1}$$

where $\mathrm{d}a/\mathrm{d}N$ is the crack propagation rate expressed in m/cycle, $\Delta K_{\mathrm{I}}$ is expressed in Pa · $\mathrm{m}^{1/2}$, $C$ and $m$ are constants. The parameters influencing the crack shape change are[3]:

(1) The relative crack size $a/R$ (crack depth to radius) and $a/L$ (crack aspect ratio).

(2) The exponent $m$ in the Paris-Erdogan law.

(3) Type of loading.

### 2.2 Iso-$K_{\mathrm{I}}$ crack front propagation

In FEM calculation of $K_{\mathrm{I}}$ for pipe or rod bar, the crack front form is adjusted so that the parameters $K_{\mathrm{I}}$ of nodes in the front are equal. Carpinteri[4] noted that the distribution of $K_{\mathrm{I}}$ along the crack front is approximately constant for this particular value of the crack aspect ratio and the iso-$K_{\mathrm{I}}$ criterion can be successfully applied only when the front of the initial surface defect is nearly circular-arc-shaped.

According to the iso-$K_{\mathrm{I}}$ criterion[4], the surface flaw grows by redistributing $K_{\mathrm{I}}$ along the defect front in order to obtain a constant distribution of $K_{\mathrm{I}}$, i. e. , the initial flaw tends to a particular configuration during propagation to satisfy this assumption (constant $K_{\mathrm{I}}$ along the crack front).

For a given angle($\theta$), the crack tip nodes and their respective stress intensity factors are numbered by $i = 1, 2, 3, \cdots$ and $K_{\mathrm{I}1}, K_{\mathrm{I}2}, K_{\mathrm{I}3}, \cdots$, respectively. The acceptable value of $K_{\mathrm{I}}$ is

$$K_{\mathrm{I}} = \frac{[K_{\mathrm{I}i}]_{\max} - [K_{\mathrm{I}i}]_{\min}}{[K_{\mathrm{I}i}]_{\max}} \times 100\% < 10\% \tag{2}$$

In order to obtain $K_{\mathrm{I}}$ within the acceptable value, an APDL program is created with geometry variables "$a$" and "$b$" as shown in Fig. 7. Two "DO" loops are used to change the geometry of crack front and calculate $K_{\mathrm{I}}$ within the acceptable limit.

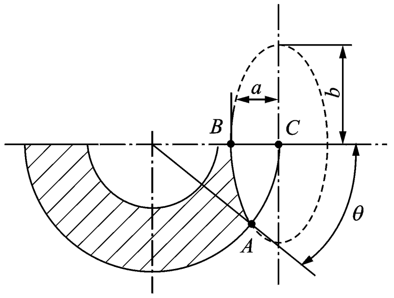


Fig. 7 Crack front parameters

Unlike the iterative crack front propagation geometry, the iso-$K_{\mathrm{I}}$ crack front propagation is independent of the initial crack geometry. The iso-$K_{\mathrm{I}}$ assumption avoids using the exponent $m$ of Paris-Erdogan law in the calculation of $K_{\mathrm{I}}$, and the relation between $K_{\mathrm{I}}$ vs crack size is generic and may be used in any material.

# 3 FINITE ELEMENT RESULTS

## 3.1 Crack propagation profile

Once the crack propagates up to a certain relative depth, the subsequent stage is independent of the initial crack aspect ratio[2]. FEM results show that the crack profile in the Phase I of any thickness pipe is nearly elliptical. When $D_{in}/D_{out}$ is 0.6—0.9 (thin pipe), the crack profile in the early Phase Ⅱ is straight line, and with $\theta$ increasing the profile is more curved. As $D_{in}/D_{out}$ goes on decreasing to 0.5—0.6 (thick pipe), the crack propagation profile is near straight line in the early phase and has more curved in the later phase, so it is extremely difficult to determine.

The crack propagation profiles in a pipe and a rod are simulated by the iso-$K_I$ criterion, and their distinct difference is shown in Fig. 8. Early and later phases of the Phase Ⅱ in pipe and rod have distinct crack propagation fringes under fatigue.

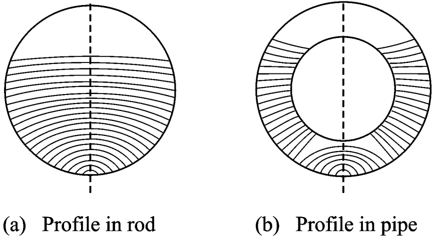

(a) Profile in rod (b) Profile in pipe

Fig. 8 Iso-$K_I$ crack propagation profiles

## 3.2 Stress intensity factor

Ref. [10] proposed that the distorted elements are more accurate than the undistorted ones. Thus for obtaining the accurate result crack tip opening displacement (CTOD) is calculated with respect to the distorted element (element 6 in Fig. 6). With distorted element different authors have used different nodes to calculate CTOD, the stress intensity factor and hence the fracture life.

Ref. [7] used the quarter-point node displacement ($u_{z1/4}$) and quarter-point node distance ($r_{1/4}$) to calculate $K_I$.

$$K_I = \frac{E}{4(1-\mu^2)}\sqrt{\frac{2\pi}{r_{1/4}}}u_{z1/4} \qquad (3)$$

Ref. [8] used the corner-node displacement ($u_z$) and the corner-node distance ($r$) to calculate the stress intensity.

$$K_I = \frac{E}{1-\mu^2}\sqrt{\frac{\pi}{2r}} \cdot \frac{u_z}{2} \qquad (4)$$

Under specific loading, $K_I$ increases with the crack growth. For the iso-$K_I$ model, only one parameter is enough to describe the crack size. Here, the external crack propagation angle $\theta$ (Fig. 2) is used. Fig. 9 shows FEM results in pipe and rod bars under specific loading for any material.

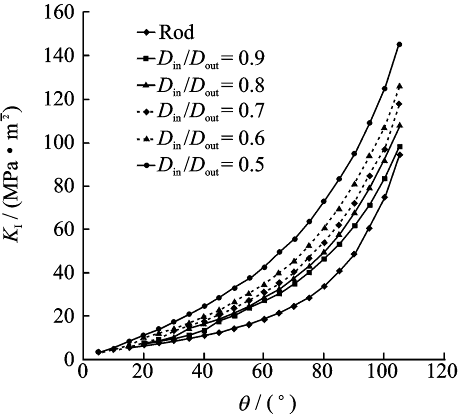


Fig. 9 Relation between $K_I$ vs $\theta$ for different $D_{in}/D_{out}$

The stress intensity factor curve is as expected. For the given pipe with fixed $D_{out}$ and different thickness, the thicker pipes are more resistant to the fracture. Fig. 9 shows the transition from Phase Ⅰ to Phase Ⅱ where the increase of $K_I$ is significant, and also indicates that in the early phase of the Phase Ⅱ $K_I$ slowly increases, but in the later phase of Phase Ⅱ $K_I$ exponentially increases.

# 4 CRACK PROPAGATION LIFE

The fatigue crack growth analysis of a component subjected to a constant amplitude loading is rather simple because loading history can be ignored. Numerous fatigue crack growth models have existed which are capable of representing the fatigue rate data. Paris model, Walker model and Forman model etc are some of the famous fatigue propagation models.

### 4.1 Forman model

Forman model improves the Walker model by considering the instability of crack growth when the stress intensity factor approaches its critical value[11]. Moreover, it is capable of describing all the region of fatigue crack (i. e., early development of fatigue crack, intermediate crack propagation zone and high growth rate of fatigue crack) and the effect of stress ratio[11].

Forman model is expressed as follows

$$\frac{\mathrm{d}a}{\mathrm{d}N}=\frac{C(\Delta K_{\mathrm{I}})^{m}}{[K_{\mathrm{IC}}(1-R)-\Delta K_{\mathrm{I}}]} \tag{5}$$

where $\Delta K_{\mathrm{I}}=K_{\mathrm{Imax}}-K_{\mathrm{Imin}}$, $C$ and $m$ are the material properties, $K_{\mathrm{IC}}$ is the critical stress intensity factor depending on the material, $N$ the cycle of applied loading, $R=\frac{\sigma_{\min}}{\sigma_{\max}}$, $\sigma_{\min}$ and $\sigma_{\max}$ are the minimum and the maximum stress applied to the tensile pipe during the certain period. Stress in a pipe can be obtained in the experiment and the real loading situations.

### 4.2 Cumulative damage law

Two main approaches for cumulative damage are considered: One is the direct postulation of lifetime damage (such as the Miner rule[12]), the other is the residual strength. Miner rule is also called the Palmgren-Miner linear damage hypothesis and expressed as follows

$$\sum\frac{n_j}{N_j}=1 \tag{6}$$

where $n_j$ is the number of cycles under the loading corresponding to the lifetime $N_j$.

The linear cumulative damage (LCD) accumulates damage in a linearly additive manner independent of the sequence of the loading applications. Then, the total damage is used to predict the failure. So, the Miner equation (Eq. (6)) is very useful and safer to use. However, it is well known that the fatigue life is dependent on the loading sequence. That is the non-linear cumulative damage. Since the loading is a random spectrum in the structures such as APU in airplanes, the loading sequence cannot be uniquely determined. In this case, Miner rule gives a conservative fatigue life[12], and can enforce the safety in airplanes.

### 4.3 Visual C++

The visual C++ code has been developed by using the finite element analysis (FEA) results. Miner rule is used to determine the cumulative damage life of the pipe. The input parameters in the program are $D_{\mathrm{in}}$, $D_{\mathrm{out}}$, $\theta$, $C$, loading data and $m$. Fig. 10 shows the flow chart of the program. Forman model is used to calculate the crack growth life.

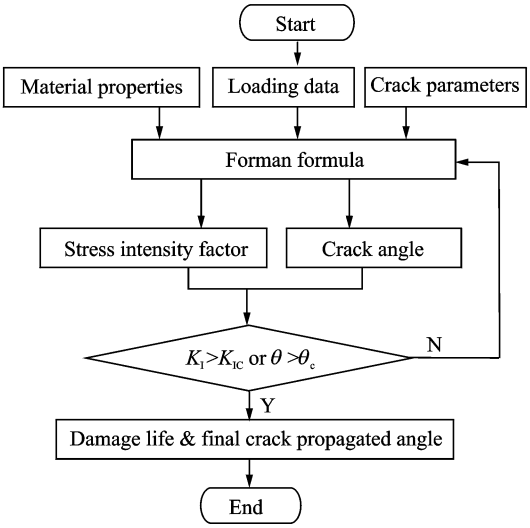
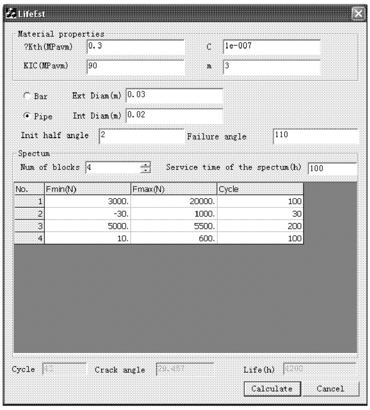


Fig. 10 Flow chart of program

$K_{\mathrm{IC}}$ is used for the crack propagation criterion. For the given crack geometry and loading condition, if $K_{\mathrm{I}}>K_{\mathrm{IC}}$, the crack propagates rapidly and fails.

The geometric criterion is used for the failure of pipe. In the LifeEst software program—GUI, the critical angle for pipe is set to 110°. For the rod, the equivalent crack depth to 110° circumferential crack angle is taken as the critical crack depth. Normally, in the aircraft structures the pipes are replaced once the crack is visible. From the view point of static design, the pipe with 110° circumferential crack cannot sustain any designed static loading. Finally, GUI is created in VC++ for the more convenience. Fig. 11 shows the GUI window of LifeEst software. The LifeEst program is useful to design the fracture tolerant for the pipe/rod bar structure under tension-tension or tension-compression spectrum.



LifeEst

Material properties

?Kth(MPavm) 0.3

C 1e-007

KIC(MPavm) 90

m 3

○ Bar Ext Diam(m) 0.03

● Pipe Int Diam(m) 0.02

Init half angle 2

Failure angle 110

Spectum

Num of blocks 4

Service time of the spectum(h) 100

| No. | Fmin(N) | Fmax(N) | Cycle |
|---|---|---|---|
| 1 | 3000. | 20000. | 100 |
| 2 | -30. | 1000. | 30 |
| 3 | 5000. | 5500. | 200 |
| 4 | 10. | 600. | 100 |

Cycle 45

Crack angle 29.457

Life(h) 4200

Fig. 11 GUI window of LifeEst software

During the operation, APU pipes suffer random loading in which the peak sequence randomly occurs. Based on this kind of loading sequence, a block spectrum can be statistically formed by eliminating small peaks without considering the sequence. For each loading block in the spectrum, there are the maximum force $F_{\max}$ and the minimum force $F_{\min}$ in certain service time of the spectrum (h). Each loading block has $n_i$ cycles. The material properties are known from the experiments or the material handbooks and can be input through GUI. Users can also define the crack initial size and the failure size depending on applications. The software can give the estimated lifetime.

A distribution example of the loading spectrum is given in Table 1.

**Table 1 Loading spectrum distribution**

| No. | $F_{\min}$/N | $F_{\max}$/N | Cycle ($n_i$) |
|---|---|---|---|
| 1 | 3 000 | 20 000 | 100 |
| 2 | −30 | 1 000 | 80 |
| 3 | 5 000 | 5 500 | 200 |
| 4 | 10 | 600 | 100 |
| Service time of spectrum/h=100 | | | |

Fig. 12 shows the relation between life vs circumferential angle ($\theta$) for the thick pipe with $D_{\text{in}}=20$ mm, $D_{\text{out}}=30$ mm and length (10 times of the outer diameter) under the tensile-compression loading spectrum in Table 1. It also shows that the crack propagation life of pipe exponentially decreases.

Fig. 13 shows the influence of the critical angle on the fracture life of pipe structures containing circumferential crack angle 1°. It also shows that the assumed 110° as a critical crack geometry angle is quite safe. For the given pipe with circumferential crack angle 1° under the loading spectrum in Table 1, the total life beyond the critical crack angle 30° is constant and is safe to calculate the fracture life.

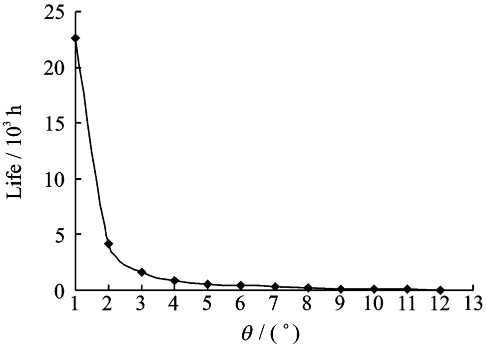


Fig. 12 Life estimation for thick pipe

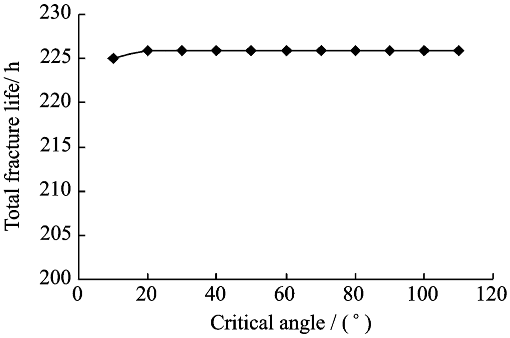


Fig. 13 Total life estimation

# 5 CONCLUSIONS

(1) The iso-$K_{\text{I}}$ criterion is independent of the material property and is useful for the study of crack propagation in a pipe or a rod under simple loading. Using the iso-$K_{\text{I}}$ criterion, the profiles can present the fatigue crack fringe.

(2) As the crack propagates from the Phase Ⅰ to the Phase Ⅱ, the stress intensity factor significantly increases.

(3) The crack propagation profiles on a pipe and a rod are distinctly different in the early and later phases of the Phase Ⅱ.

(4) The developed software is useful if the loading spectrum is known. It is also easy to be modified for other types of crack and structure

since it is implemented by object-oriented programming (OOP) language.

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# 实心/空心杆结构的损伤容限与疲劳寿命估算

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**摘要:**对飞机辅助动力装置的支撑杆(实心和空心)的损伤容限进行了研究。该研究探索一旦支撑杆损伤出现微小裂纹,飞机是否还能够安全着陆。首先,计算了两种可能用作支杆结构的裂纹应力强度因子$K_{\mathrm{I}}$,并提出了等应力强度因子 的计算模型iso-$K_{\mathrm{I}}$,该模型避免使用材料参数,采用有限元法计算裂纹。然后,将问题进行了参数化,对于不同的材料、内外径比($D_{\mathrm{in}}/D_{\mathrm{out}}$)及外部裂纹扩展角($\theta$),得到了通用的应力强度因子$K_{\mathrm{I}}$与裂纹尺寸的关系。最后,用面向对象编程方法,将计算结果集成到程序中,在有统计载荷谱的情况下,可以估算出支撑杆的损伤容限寿命。

**关键词:**疲劳裂纹扩展;周向裂纹;应力强度因子;实心/空心管

**中图分类号:**O346.2

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