

# NOVEL APPROACH TO LOCATOR LAYOUT OPTIMIZATION BASED ON GENETIC ALGORITHM

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**Abstract:** Proper fixture design is crucial to obtain the better product quality according to the design specification during the workpiece fabrication. Locator layout planning is one of the most important tasks in the fixture design process. However, the design of a fixture relies heavily on the designer's expertise and experience up to now. Therefore, a new approach to locator layout determination for workpieces with arbitrary complex surfaces is proposed for the first time. Firstly, based on the fuzzy judgment method, the proper locating reference and locator numbers are determined with consideration of surface type, surface area and position tolerance. Secondly, the locator positions are optimized by genetic algorithm(GA). Finally, a typical example shows that the approach is superior to the experiential method and can improve positioning accuracy effectively.

**Key words:** locator layout; locating error; fuzzy judgment; genetic algorithm(GA)

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## INTRODUCTION

Fixtures are used to locate and constrain a workpiece during a machining operation. Proper fixture design is crucial to guarantee the machining requirements of the workpiece. Functions of such devices aim at locating and ensuring the desired positions and orientations of workpieces during the manufacturing process. It is therefore important to configure the locator layout. In practice, the design of a fixture relies heavily on the designer's expertise and experience due to some uncertain factors. So it is difficult to evaluate the performance of a fixture and in turn, determine the optimal one.

A design method of locating layout for workpiece with arbitrary surface is presented based on the fuzzy judgment. Firstly, the feasible locating reference and locator numbers are automatically selected based on the fuzzy evaluation matrix. And then the locator positions are optimized on

the selected locating reference by genetic algorithm(GA).

Many efforts have been done into fixture locator layout determination. Gulesin<sup>[1]</sup> established a system of automatic selection of the locating surfaces for prismatic workpiece based on 3-2-1 guideline. Wayne et al<sup>[2]</sup> proposed a method to conduct a robust fixture design to minimize workpiece positional errors as a result of workpiece surface and fixture setup errors. Li and Melkote<sup>[3]</sup> used a nonlinear programming method to solve the layout optimization problem. The method minimized workpiece location errors due to the localized elastic deformation of the workpiece. Choudhuri et al<sup>[4]</sup> presented a model and analyzed the influence of the accuracy of locators on geometrical tolerance of workpiece. Asada et al<sup>[5]</sup> used a Jacobian matrix to formulate the relationship between the fixture displacements and workpart displacements. Rong et al<sup>[6]</sup> established three

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perpendicular locating reference planes based on locator types and positions. Locator displacements were mapped into the deviations of locating reference planes. The machining surface deviation was then calculated based on the locating reference plane deviations. Marin et al<sup>[7]</sup> presented a method to determine the tolerance of the error in the contact zone between fixture and components so that a given geometric tolerance was satisfied. Qin et al<sup>[8]</sup> presented a mathematical model based on locating principle, and then analyzed the correctness of the locating scheme according to the solution construction of homogenous linear equations. Kulkarni and Pande<sup>[9]</sup> used a feature-based model and generated initial setups by performing systematic reasoning of part model, inter feature relations, tolerance relations and expert rules. Michael<sup>[10]</sup> introduced a method to compute the optimal set of locators, among an initial finite discrete set of candidates, which minimized the position inaccuracy of the work part. This inaccuracy was shown to depend on the so-called information matrix of the locator scheme. Ong and Nee<sup>[11]</sup> used fuzzy set theory together with production rules in automatic setup planning for machining of prismatic parts on vertical machining centre. Raghu and Melkote<sup>[12]</sup> presented a model to analyze the position and orientation of the workpiece after loading based on fixture geometric error, then to calculate the final position of component from the deformation of the fixture-workpiece system.

Most of the above studies use a mathematical model to analyze some locating planning, then optimize these locating planning with linear or non-linear programming methods. All of the fixture layout optimization procedures start with an initial feasible layout. Solutions from these methods are dependent on the initial fixture layout. The literature review reveals few researches on the automated initial locating layout planning.

The paper presents a fuzzy-evaluation-based algorithm for initial planning of locating points, then for optimum locating planning with GA. Minimizing locating errors as the goal, a set of locating positions are decided for a 3-D object with arbitrary shape.

## 1 AUTOMATION OF LOCATOR LAYOUT SCHEME

The fixture design is a complex process including a few uncertain factors. It is difficult to obtain the best fixture design results. Therefore a fuzzy judgment matrix is first constructed to analyze every candidate surface. When degree of freedom(DOF) of the workpiece can correctly be constrained, the initial locator number is determined. Otherwise, the candidate surface is continued to be analyzed until the constrained freedom meets the machining requirements. To some special workpieces, for example sheet metal, the above steps is able to constrain all the undesired DOFs of the workpiece. But it is not well satisfied with the machining requirement. The design should add the auxiliary supporter. Finally, the locator position is optimized by GA. The whole design process is shown in Figs. 1-2.

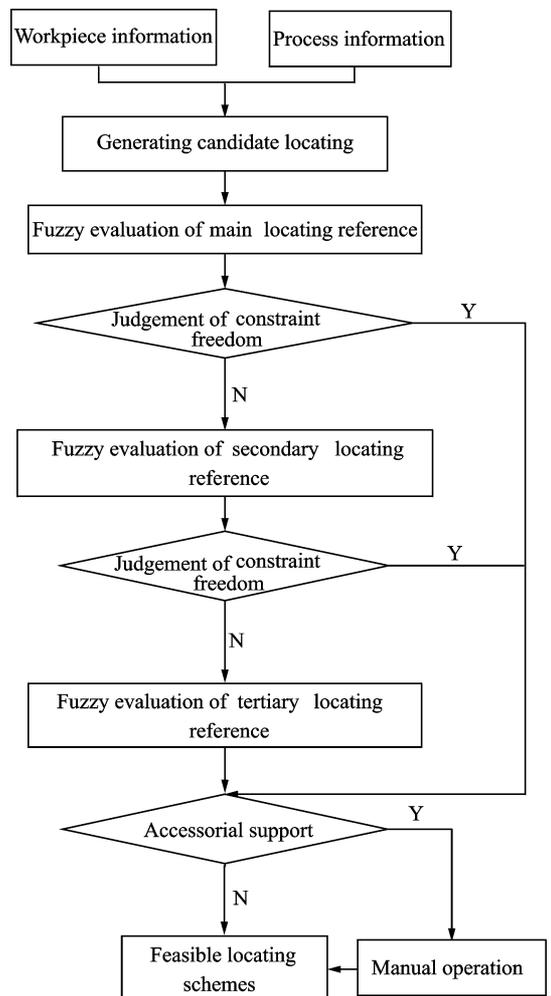


Fig. 1 Flowchart of selection of locating reference

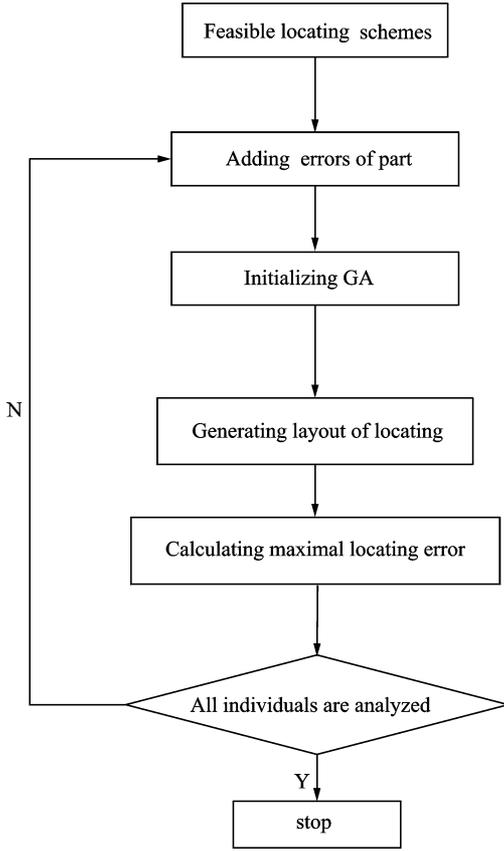


Fig. 2 Flowchart of GA

## 2 DOF CONSTRAINT MODEL OF WORKPIECE

A workpiece has six DOFs in orthogonal coordinate system. In order to guarantee the design specification of the machining feature of the workpiece, some DOFs must be constrained to obtain the reasonable location of the workpiece with respect to the cutting tool. The essential constrained DOFs are named as the theoretical constrained DOFs. The relationship between the theoretical constrained DOFs and the design specification of the machining feature is represented as

$$\delta \mathbf{r}_p = \mathbf{E}_p \mathbf{f}_{r_1} \quad (1)$$

where  $\delta \mathbf{r}_p$  is the machining error measuring the design specification,  $\mathbf{f}_{r_1}$  the theoretical constrained DOFs, and  $\mathbf{E}_p$  the configuration matrix at the process point  $\mathbf{r}_p = [x_p, y_p, z_p]^T$  whose expression is

$$\mathbf{E}_p = \begin{bmatrix} 1 & 0 & 0 & 0 & z_p & -y_p \\ 0 & 1 & 0 & -z_p & 0 & x_p \\ 0 & 0 & 1 & y_p & -x_p & 0 \end{bmatrix} \quad (2)$$

It is well known that the theoretical constrained DOFs are eliminated by a feasible locating scheme. Here, an arbitrary locating scheme is assumed to consist of  $k$  ( $i=1, 2, \dots, k$ ) locators, as shown in Fig. 3<sup>[13]</sup>. In Fig. 3, the workpiece coordinate system (WCS) and the global coordinate system (GCS) are given.  $\mathbf{n}_i = [n_{ix}, n_{iy}, n_{iz}]^T$  is the unit normal vector of the workpiece surface at the  $i$ th contact point  $\mathbf{r}_i = [x_i, y_i, z_i]^T$ . Thus, its practical constrained DOFs of the workpiece whose formulation can be referred to Ref. [14] in detail is rewritten as

$$\mathbf{J} \mathbf{f}_{r_2} = 0 \quad (3)$$

where  $\mathbf{f}_{r_2}$  is the practical constrained DOFs and  $\mathbf{J}$  the locating Jacobian matrix, and its expression can be concluded as

$$\mathbf{J} = \begin{bmatrix} -n_{1x} & -n_{1y} & -n_{1z} & n_{1z}y_1 - n_{1y}z_1 & n_{1x}z_1 - n_{1z}x_1 & n_{1y}x_1 - n_{1x}y_1 \\ -n_{2x} & -n_{2y} & -n_{2z} & n_{2z}y_2 - n_{2y}z_2 & n_{2x}z_2 - n_{2z}x_2 & n_{2y}x_2 - n_{2x}y_2 \\ \vdots & \dots & \dots & \dots & \dots & \vdots \\ -n_{kx} & -n_{ky} & -n_{kz} & n_{kz}y_k - n_{ky}z_k & n_{kx}z_k - n_{kz}x_k & n_{ky}x_k - n_{kx}y_k \end{bmatrix} \quad (4)$$

The correctness of the designed locator layout depends on the logic relationship between  $\mathbf{f}_{r_1}$  and  $\mathbf{f}_{r_2}$ . If  $\mathbf{f}_{r_1} \cap \mathbf{f}_{r_2} = \mathbf{f}_{r_1}$ ,  $f_i = 0.9$ . If  $\mathbf{f}_{r_1} \cap \mathbf{f}_{r_2} = \mathbf{f}_{r_2}$ ,  $f_i = 0.8$ . If  $\mathbf{f}_{r_1} \cap \mathbf{f}_{r_2} \neq \emptyset$ ,  $f_i = 0.4$ . Otherwise,  $f_i = 0$ .

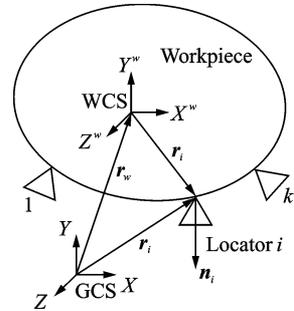


Fig. 3 Fixture locating scheme

## 3 DETERMINATION OF OTHER EFFECT FACTORS

In addition to the logic relationship between the theoretical constrained DOFs and the practical constrained DOFs, generally speaking, the effect factors of locator layout design also include following five aspects.

### 3.1 Distance

If the mass center of a workpiece is projected

onto the  $i$ th locating reference, the obtained point is defined as the projected mass center. The ratio of the distance from the projected mass center to the geometric center of the locating reference is written as

$$d_i = 1 - \frac{l_i}{l_{\max}} \quad (5)$$

where  $l_i$  is the distance from the projected mass center to the geometric center of the  $i$ th locating reference, and  $l_{\max}$  the maximum distance.

### 3.2 Surface roughness

The smaller the roughness value is, the larger the performance norm is. Compute the degree of membership with ascending exponent fuzzy distributing function as

$$r_i = \begin{cases} 0 & R_i > 100 \\ 1 - \frac{\log R_i}{\log 100} & 1 < R_i \leq 100 \\ 1 & R_i < 1 \end{cases} \quad (6)$$

where  $R_i$  is the roughness value of the  $i$ th locating reference.

### 3.3 Position tolerance

The position tolerance is usually ignored when the designer presents the locator layout. In fact, the smaller the position tolerance is, the higher the locating accuracy is. The value can be determined as

$$p_i = \begin{cases} 0.9 & 0 < \sigma_i < 0.05 \\ 0.8 & 0.05 < \sigma_i < 0.10 \\ 0.7 & 0.10 < \sigma_i < 0.15 \\ 0.6 & 0.15 < \sigma_i < 0.20 \\ 0.4 & \text{other} \end{cases} \quad (7)$$

where  $\sigma_i$  is the position tolerance of the  $i$ th locating reference.

### 3.4 Surface type

A workpiece consists of a variety of surfaces. Here, these surfaces are categorized into plane surface, cylindrical surface, conical surface, curved surface and other surfaces. Thus, the ratio of surface type denoted by  $t_i$  is shown in Table 1.

**Table 1 Matrix element with surface type**

Type	Matrix element
Plane surface	0.8
Cylindrical surface	0.8
Conical surface	0.7
Curved surface	0.3
Other surfaces	0.2

### 3.5 Surface area

Supposing that  $s_i$  is the surface area of the  $i$ th locating surface, the area ratio of locating reference can be defined as

$$q_i = \frac{s_i}{s_{\max}} \quad (8)$$

## 4 DETERMINATION OF WEIGHT SET

Letting  $x_i$  be an effect factor on locator layout determination (i. e.,  $x_i = d_i, p_i, t_i, q_i$  and  $f_i$ ),  $x_{ij}$  is defined as the important factor of  $x_i$  with respect to  $x_j$ . Their values can be selected from 1 to 9 according to the comparison of their importance with each other, as listed in Table 2. Thus, the ration matrix can be constructed as

$$\mathbf{c} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mm} \end{bmatrix} \quad (9)$$

with  $x_{ii} = 1, x_{ij} = 1/x_{ji}$ .

**Table 2 Scaling value**

$x_{ij}$	Condition
1	$x_i$ is as important as $x_j$
3	$x_i$ is less important than $x_j$
5	$x_i$ is more important than $x_j$
7	$x_i$ is more important than $x_j$
9	$x_i$ is the most important than $x_j$
2, 4, 6, 8	The middle value of 1-3, 3-5, 5-7, 7-9

It is known that the eigenvalues of the ratio matrix can be used to identify the consistency of  $x_{ij}$  in the practical problem. If the ratio matrix has good consistency, the weight coefficient of all effect factors can be obtained. Otherwise,  $x_{ij}$  should be adjusted until its value makes the ratio matrix to be good consistency.

Let  $\mathbf{y} = [y_1, y_2, \dots, y_m]^T$  an eigenvector of the ratio matrix  $\mathbf{c}$ , a new vector can be constructed as

$$\mathbf{d} = \mathbf{c}\mathbf{y} \quad (10)$$

If the non-negative components number in  $\mathbf{d}$  equal to the column number of  $\mathbf{c}$ , the consistency of the ratio matrix with the practical problem is

good. Otherwise, the component in the matrix  $c$  must be adjusted until the matrix meet the requirement of consistency.

In the traditional analytic hierarchy process (AHP), the weight set of the matrix with consistency is solved by the root method or the least square method. The algorithm is shown by Eqs. (11-12).

$$g_i = \left( \prod_{j=1}^m x_{ij} \right)^{1/m} \quad (11)$$

$$\omega_i = \frac{g_i}{\sum_{i=1}^m g_i} \quad (12)$$

## 5 LOCATING LAYOUT OPTIMIZATION

Consider the locating problem of the 3-D workpiece. As shown in Fig. 4, a certain surface is discretized to a series of points. The locator position is the design variable.

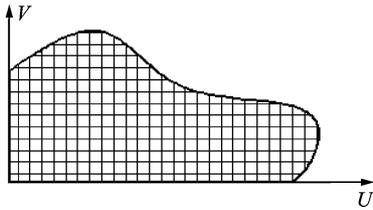


Fig. 4 Generation of candidate locating points

The position can be optimally selected with GA as follows:

**Step 1** Generation of initial locator positions.

By random selecting initial points from the discretized points, the initial locator positions are generated. Of course, three points laid out on the uniform surface cannot be collinear. In addition, the mass center must be projected within the triangle configured by the three points.

**Step 2** Improvement of initial locator positions.

Because the initial locator positions can cause the poor locating accuracy, GA is used to regenerate the initial locator positions so that the locating error can be improved to be minimal.

**Step 3** Selection of optimal solution.

Applying GA for each feasible set of locators we will end up with several distinct solutions in the layout scheme of locating positions. Thus, the final solution may not be unique, due to the fact that the optimization problem may have many local optima. The procedure operates on step-by-step basis to seek for an optimal solution. Depending on the objective function pursued the best solution can be evident or might need the designer's final decision.

## 6 CASE STUDY

In this section, a typical example is illustrated to validate the proposed approach. The workpiece is made up of some complex surfaces which are accessible to the locators, as shown in Fig. 5. The features to be produced are a step and a hole with the diameter of 12 mm. The candidate locators are restricted to a set of discrete positions, which are generated as uniformly as possible on the part surfaces. The determination procedure of locator layout is as follows:

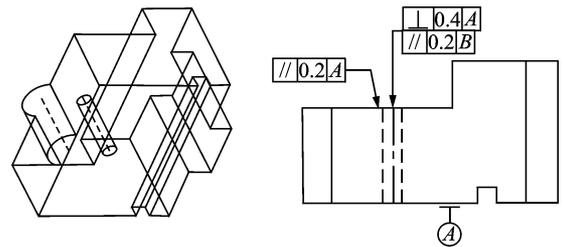


Fig. 5 Test workpiece

**Step 1** Determination of effect factors.

According to the effect factors  $d_i$ ,  $r_i$ ,  $p_i$ ,  $t_i$ ,  $a_i$  and  $f_i$ , a effect factor matrix  $V$  can be constructed. Here, surfaces  $A$  and  $B$  are initially selected as the first locating reference and the second locating reference, respectively. Thus, their corresponding effect factor matrixes  $V_1$  and  $V_2$  are obtained as

$$V_1 = \begin{bmatrix} 0.923\ 2 & 0.897\ 9 & 0.8 & 0.8 & 0.765\ 3 & 0.5 \\ 0 & 0.747\ 4 & 0.4 & 0.8 & 0.8 & 0.5 \\ 0 & 0.747\ 4 & 0.4 & 0.8 & 0.8 & 0.5 \\ 0.978\ 7 & 0.897\ 9 & 0.6 & 0.8 & 0.510\ 2 & 0.6 \\ 0.961\ 5 & 0.747\ 4 & 0.4 & 0.8 & 1 & 0.6 \\ 0.961\ 5 & 0.747\ 4 & 0.4 & 0.8 & 1 & 0.6 \\ 0.882\ 1 & 0.747\ 4 & 0.4 & 0.8 & 0.801\ 4 & 0.7 \end{bmatrix}$$

(13)

$$V_2 = \begin{bmatrix} 0.8979 & 0.5102 & 0.8 & 0.6 & 0.7 \\ 0.7474 & 1 & 0.8 & 0.4 & 0.7 \\ 0.7474 & 1 & 0.8 & 0.4 & 0.7 \\ 0.7474 & 0.8014 & 0.8 & 0.4 & 0.7 \end{bmatrix} \quad (14)$$

### Step 2 Determination of weight set.

According to Eq. (9), the ratio matrices can be written as

$$c_1 = \begin{bmatrix} 1 & 2 & 2 & 3 & 5 & 8 \\ 1/2 & 1 & 1 & 2 & 3 & 5 \\ 1/2 & 1 & 1 & 2 & 3 & 5 \\ 1/3 & 1/2 & 1/2 & 1 & 2 & 3 \\ 1/5 & 1/3 & 1/3 & 1/2 & 1 & 2 \\ 1/8 & 1/5 & 1/5 & 1/3 & 1/2 & 1 \end{bmatrix} \quad (15)$$

$$c_2 = \begin{bmatrix} 1 & 1 & 2 & 3 & 7 \\ 1 & 1 & 2 & 3 & 7 \\ 1/2 & 1/2 & 1 & 2 & 4 \\ 1/3 & 1/3 & 1/2 & 1 & 2 \\ 1/7 & 1/7 & 1/4 & 1/2 & 1 \end{bmatrix} \quad (16)$$

Consequently, the corresponding eigenvectors of  $c_1$  and  $c_2$  can be calculated as

$$y_1 = [6.0225, 0.0047, 0.0047, -0.0160, -0.0160, 0]^T \quad (17)$$

$$y_2 = [5.0100, -0.0050, -0.0050, 0, 0, 0]^T \quad (18)$$

The maximal eigenvectors of the two matrices are both close to the exponent number of matrix and other eigenvectors are all close to zero. So without adjustment of the element of the matrix, the weight set can be directly computed as

$$w_1 = [0.3650, 0.2048, 0.2048, 0.1162, 0.0692, 0.0400]^T \quad (19)$$

$$w_2 = [0.3342, 0.3342, 0.1820, 0.1020, 0.0480]^T \quad (20)$$

### Step 3 Selection of locating reference.

Here, a new concept is introduced to select the locating reference. If the effect factor matrix  $V$  and the weight set  $w$  are known, the selection set can be defined as

$$Z = Vw \quad (21)$$

Therefore, we can obtain the following selection sets

$$Z_1 = [0.8528, 0.3538, 0.3595, 0.8163, 0.7721, 0.7721, 0.7334]^T \quad (22)$$

$$Z_2 = [0.7317, 0.6645, 0.6645, 0.6490]^T \quad (23)$$

### Step 4 Optimization of locator layout.

The entire flowchart of GA is shown in Fig. 2. GA input parameters used in this study is given in Table 3. The value of objective function is shown in Fig. 6. Results are given in Table 4.

**Table 3 GA input parameters**

Parameter	Value
Number of iteration	150
Population size	20
Crossover probability	0.85
Mutation probability	0.2
Number of variable	12

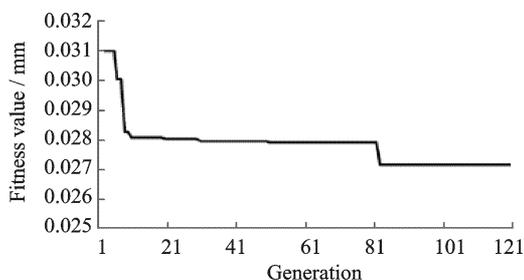


Fig. 6 Convergence of GA for locator layout optimization procedure

**Table 4 Optimum layout obtained by GA**

Locator	Layout
$L_1$	(20, 63, 0)
$L_2$	(75, 75, 0)
$L_3$	(20, -63, 0)
$L_4$	(-5.67, -5.67, 30)
$L_5$	(-5.67, 5.67, 30)
$L_6$	(140, 0, 45)

## 7 CONCLUSION

The accurate position of the workpiece with respect to the cutting tool is crucial to guarantee the design specification. However, this precise position depends on acceptable locator layout. A GA-based approach to locator layout optimization is concerned. An initial locator layout is first obtained based on fuzzy mathematics. And then the optimal locator layout is fast achieved with GA. The approach can be used to determination of locator layout of a workpiece with arbitrary surfaces.

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## 基于遗传算法的定位布局优化新方法

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**摘要:**在夹具设计过程中,建立了一种定位元件布局确定的新方法。该方法基于模糊评判,考虑候选表面特征类型、表面面积、位置公差等影响因素,确定定位参考面和定位点的数量。以定位误差最小为优化目标通过遗传算法确定定位点的具体位置。最后,通过实例验证该方法优于检验设计,

并且能有效提高定性精度。

**关键词:**定位布局; 定位误差; 模糊评判; 遗传算法  
**中图分类号:**TG702

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