ADRC FRACTIONAL ORDER PID CONTROLLER DESIGN OF HYPERSONIC FLIGHT VEHICLE

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Abstract: Active disturbance rejection controller (ADRC) uses tracking-differentiator (TD) to solve the contradiction between the overshoot and the rapid nature. Fractional order proportion integral derivative (PID) controller improves the control quality and expands the stable region of the system parameters. ADRC fractional order (ADRFO) PID controller is designed by combining ADRC with the fractional order PID and applied to reentry attitude control of hypersonic vehicle. Simulation results show that ADRFO PID controller has better control effect and greater stable region for the strong nonlinear model of hypersonic flight vehicle under the influence of external disturbance, and has stronger robustness against the perturbation in system parameters.

Key words: hypersonic flight vehicle; active disturbance rejection controller (ADRC); fractional order PID; D-decomposition method

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INTRODUCTION

Han Jingqing has proposed active disturbance rejection controller (ADRC)[1] and developed ADRC proportion integral derivative (PID) using tracking-differentiator. ADRC PID has the good control effect by organizing the transition process and solves the conflict between the overshoot and the rapid nature of traditional PID controller[2-3]. In the case of input signal with a strong disturbance, the ADRC controller has good robustness and filtering property. The stability domain of parameters of the controlled system represents the robustness of the controller. At present, the study of the stability domain[4-5] is only focused on the stability of the controller and rather than the stablility of the system parameters. The stability domain is consistent with the traditional PID as the core of ADRC is an improved PID controller.

Fractional order PID[6] extends the integer order of the traditional PID to fractional order and obtains a stronger robustness and better control effect than the traditional PID without losing the simple structure characteristics of the traditional PID. Fractional order PID has been applied to the aerospace field and others[7-9]. Fractional order PID has rapid response, but the overshoot is large. However ADRC has a good solution to solve the conflict between the rapid nature and overshoot. ADRC is formed by the extended state observer (ESO) and the nonlinear controller. ESO only obtains feedback from output signal in real-time, so the nonlinear controller output is the actual controller output. When using the traditional PID controller, ADRC inherits the stability region of the traditional PID. Fractional order PID extends the stability domain of the controlled system parameters and is more robust to the time-varying parameters of the controlled system. ADRC and fractional order PID are combined and compensated each other in this paper.

In this paper, the pitch channel model of the hypersonic vehicle is taken as the research object, and D-decomposition is used to analyze the effect on Mach number and angle of attack stability region with fractional order PID. The fractional or-

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der PID has a broader stability domain, namely stronger robustness is also demonstrated. Then ADRC fractional PID is designed by combining ADRC with the fractional order PID and uses the tracking-differentiator to solve the conflict between the overshoot and the rapid nature of the traditional PID controller through organizing the transition process. Finally the superiorities of ADRC fractional PID in controlling quality and stability domain are demonstrated by simulation.

1 STABILITY DOMAIN ANALYSIS USING D-DECOMPOSITION METHOD

Take pitch channel model of Hypersonic flight vehicle as example[10-11]

$$\begin{aligned} \dot{\theta} &= \frac{L}{mV} - \frac{(\mu - V^2 r)}{V r^2}\cos\theta \\ \dot{\alpha} &= \omega_z - \dot{\theta} \\ \dot{\omega}_z &= \frac{M_z}{I_z} \end{aligned} \tag{1}$$

where

$$L = \frac{1}{2}\rho V^2 S C_{\mathrm{L}}$$

$$M_z = \frac{1}{2}\rho V^2 S\bar{c}[C_M(\alpha) + C_M(\delta_e) + C_M(\omega_z)]$$

$$r = h + R_E$$

$$C_{\mathrm{L}} = 0.620\,3\alpha$$

$$C_{\mathrm{M}}(\alpha) = -0.035\alpha^2 + 0.036\,617\alpha + 5.326\,1 \times 10^{-6}$$

$$C_{\mathrm{M}}(\delta_e) = c_e(\delta_e - \alpha)$$

$$C_{\mathrm{M}}(\omega_z) = \left(\frac{\bar{c}}{2V}\right)\omega_z(-6.796\alpha^2 + 0.301\,5\alpha - 0.228\,9)$$

where V, θ, h, α, and ω_z represent the velocity, the flight-path angle, the altitude, the angle of attack, and the velocity of path angle; ρ represents the atmospheric density; L and M_z represent the lift and the pitching moment, m, I_z, μ, S, and R_{E} the mass, the moment of inertia, the gravitational constant, the reference area, and the radius of the Earth, C_{L}, $C_{\mathrm{M}}(\alpha)$, $C_{\mathrm{M}}(\delta_e)$, and $C_{\mathrm{M}}(\omega_z)$ the lift coefficient, the moment coefficient about angle of attack, the moment coefficient about elevator deflection, and the moment coefficient about pitch rate, $\bar{c}=80$ is the mean aerodynamic chord and $c_e \approx 0.03$ the mean constant coefficient.

The pitch channel mode of the hypersonic flight vehicle is expressed by a nonlinear transfer function of Mach number and angle of attack.

$$G = \frac{\alpha}{\delta_e} = f(Ma, \alpha) = \frac{D\alpha}{A\alpha + AC - B} \tag{2}$$

where

$$A = s^2 + 0.040\,78 Ma\alpha^2 s - 0.001\,809 Ma\alpha s + 0.001\,373\,4 Mas$$

$$B = -0.001\,578\,5 Ma^2\alpha^2 + 0.000\,334\,4 Ma^2\alpha + 2.402 \times 10^{-7} Ma^2$$

$$C = \frac{0.000\,87 Ma\alpha}{s} - \frac{0.032\,36}{Mas} + \frac{0.000\,046\,82 Ma}{s}$$

$$D = 0.001\,317 Ma^2$$

and s represents the differential operator.

The fractional PID controller model is chosen as follows

$$C = 100(1 + s^{-\lambda} + s^{\mu})$$

Therefore, the system closed-loop transfer function is

$$\frac{GC}{1+GC} = \frac{D\alpha \cdot 100(s^{\lambda+\mu} + s^{\lambda} + 1)}{s^{\lambda}(A\alpha + AC - B) + D\alpha \cdot 100(s^{\lambda+\mu} + s^{\lambda} + 1)} \tag{3}$$

The characteristic polynomial is

$$P(s; Ma, \alpha) = s^{\lambda}(A\alpha + AC - B) + D\alpha \cdot 100(s^{\lambda+\mu} + s^{\lambda} + 1) \tag{4}$$

Definition 1 Set the stability domain of the Mach number and the angle of attack to be S_{d}, when $K = (Ma, \alpha) \in S_{\mathrm{d}}$, the stability condition of the control system is that characteristic roots are in the left-hand of the s plane. The boundaries of the stability domain S_{d} are described by the real root boundary (RRB), the infinite root boundary (IRB) and the complex root boundary (CRB), and can be determined by the D-decomposition method. The three root boundaries are shown as follows

$$\text{RRB: } P(0; K) = 0$$

$$\text{IRB: } P(\infty; K) = 0$$

$$\text{CRB: } P(\pm \mathrm{j}\omega; K) = 0$$

The RRB border line equation can be obtained by putting $s=0$ into the characteristic polynomial $Ma=0$.

As numerator and denominator of transfer function of the most high-end times of hypersonic flight vehicle pitch channel are not equal, there is no IRB boundary line.

The CRB border line equation can be obtained by putting $s = j\omega$ into the characteristic polynomial.

The formula of fractional power for complex number is

$$(\sigma + j\omega)^{\gamma} = (\sqrt{\sigma^2 + \omega^2})^{\gamma}[\cos(\gamma \arctan\frac{\omega}{\sigma}) + j\sin(\gamma \arctan\frac{\omega}{\sigma})] \quad (5)$$

where σ, ω and γ are the real part, the imaginary part and the order, respectively.

The following function can be obtained

$$j^{\gamma} = \cos\left(\gamma\frac{\pi}{2}\right) + j\sin\left(\gamma\frac{\pi}{2}\right)] \quad (6)$$

With substituting Eq. (6) into the characteristic polynomial and setting the real and the imaginary parts to be 0, the stability domain of Ma and α can be calculated. During calculating $\omega \in (0,\infty)$, it takes the results of $Ma>0$. Then the stability region boundaries for RRB connect with CRB when $\alpha=\infty$. The stable region is above the boundary curve for CRB.

1.1 Effect of λ on stability region

Taking $\mu=1$ and $\lambda=0$, 0.2, 0.4, 0.6, 0.8, 1.0, the stability domain of the hypersonic flight vehicle is shown in Fig. 1.

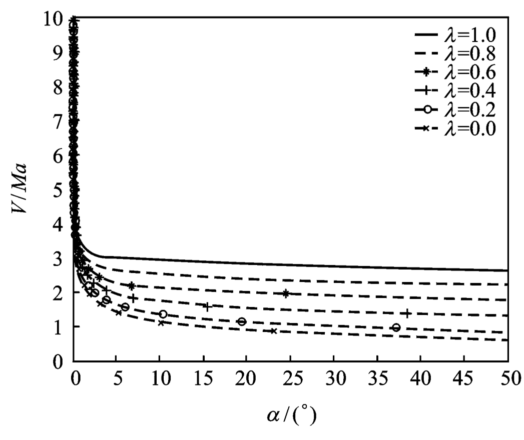

Fig. 1 Effect on stability region when λ changes

In Fig. 1, the stability domain reduces as λ increases. Changes are apparent when $\alpha \in [5, 30]$. The stability domain of traditional PID is smallest and the fractional order PID expands the stability domain. Results show that the fractional order PID is less sensitive to change system parameters than that of the traditional PID. And the application is broader. Therefore, λ should be taken the fractional order and a smaller value. Here select $\lambda=0.2$.

1.2 Effect of μ on stability region

Taking $\lambda=1$ and $\mu=0.2$, 0.4, 0.6, 0.8, 1.0, the stability domain of the hypersonic flight vehicle is shown in Fig. 2.

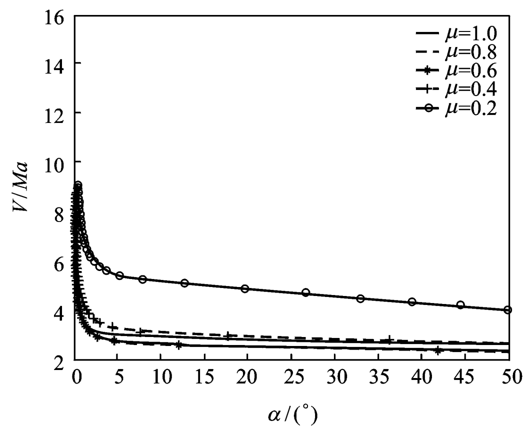

Fig. 2 Effect on stability region when μ changes

In Fig. 2, the stability region first decreases and then increases as μ increases. The stable region is smaller than that of the traditional PID when μ is smaller. The stable region will remain when μ is 0.6—0.8 and the stable region is bigger than that of the traditional PID. Therefore, μ should be taken a larger value. Finally the value of μ is determined by the combination of integrated time absolute error (ITAE) index or taken $\mu=1$ to simple the fractional PID design and combine with the ADRC technology when the system parameters are not charged with the curves of critical circumstances.

2 ADRFO CONTROLLER

Tracking differentiator (TD) is an important component of ADRC control technology. In the practical problems of engineering, TD mainly takes the tracking signal and the differential signal of the discontinuous or noisy signal effectively. Namely TD gives out the tracking signal and the differential signal of the input by differentiating the tracking signal. For the discrete system, the specific algorithm is given as follows

$$\begin{cases} x_1(k+1) = x_1(k) + hx_2(k) \\ x_2(k+1) = x_2(k) + hu \quad |u| \leqslant r \end{cases} \tag{7}$$

An integrated function of optimal control $fhan\ (x_1, x_2, r, h)$ algorithm is constructed as follows

$$\begin{cases} d = rh \\ d_0 = hd \\ y = x_1 + hx_2 \\ a_0 = \sqrt{d^2 + 8r|y|} \\ a = \begin{cases} x_2 + \dfrac{a_0 - d}{2}\text{sign}(y) & |y| > d_0 \\ x_2 + \dfrac{y}{h} & |y| \leqslant d_0 \end{cases} \\ fhan = -\begin{cases} r\text{sign}(a) & |a| > d \\ r\dfrac{a}{d} & |a| \leqslant d \end{cases} \end{cases} \tag{8}$$

The tracking differentiator is discretized by replacing $x_1(k)$ with $x_1(k) - v(k)$, then we have

$$\begin{cases} fh = fhan(x_1(k) - v(k), x_2(k), r, h) \\ x_1(k+1) = x_1(k) + hx_2(k) \\ x_2(k+1) = x_2(k) + hfh \end{cases} \tag{9}$$

where $v(k)$ is the input signal, $x_1(k)$ the tracking signal of the input signal, $x_2(k)$ the differential signal of $x_1(k)$, by which the differential signal of $v(k)$ can be approximated, r the fast factor, and h the filter factor. And $x_1(k)$ tracks $v(k)$ faster as r is greater.

The ADRFO PID controller model of pitch channel of the hypersonic vehicle is shown in Fig. 3.

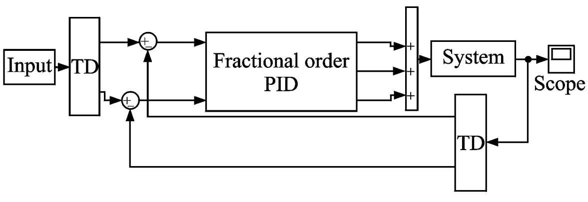

Fig. 3 Pitch channel control block diagram of hypersonic vehicle

3 SIMULATION

The simulation parameters are selected as follows: $r_0 = r_1 = 300$, $h_0 = h_1 = 0.1$, $t = 0.001$, where r_0, h_0 are respectively the fast factor and the filter factor of forward path, r_1, h_1 respectively the fast factor and the filter factor of feedback path, t is the sampling time. The simulation results are shown in Fig. 4 under the conditions: (1) $Ma = 15$; (2) Expected $\alpha = 10$ with Gaussian white noise; (3) Pulse amplitude of 10° in 5 s in the input signal.

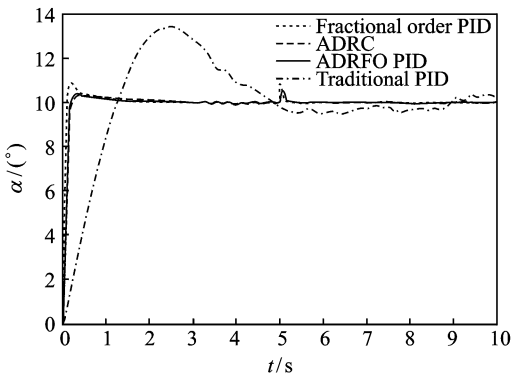

Fig. 4 Simulation results of controller

The control effects and quality of fractional order PID, ADRC and ADRFO PID are better than those of the traditional PID. The control time and the overshoot are superior to the traditional PID. Compared with the fractional order PID and ADRC, ADRFO PID reduces the control time, but the overshoot is slightly larger; The overshoot of ADRC is smallest, but the control time is slightly larger; And ADRFO PID is between them.

Fig. 5 shows that when $\alpha = 10$, $Ma = 2.7$, the system is unstable for the traditional PID and ADRC, while stable for the fractional order PID.

ADRC simulation results are divergent and unstable, while the ADRFO PID simulation results are convergent and stable. Since the ADRC sets the time of the transition process, and overshoot and control time have been greatly improved. But the stability domain inherits the characteristics of the traditional PID. The frac-

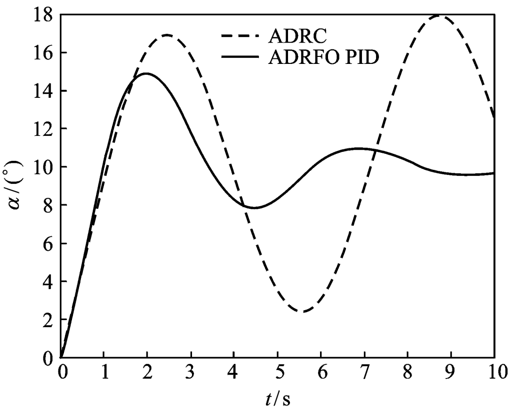

Fig. 5 Control system stability analysis

tional order PID expands the stable domain of the traditional PID and improves the robustness of the controller. Therefore, combining the advantages of ADRC controller with fractional order PID controller can improve the control quality and the stability domain of the controlled system, and also improve the robustness of the controller.

4 CONCLUSION

Since the ADRC and fractional PID controllers are extension of the traditional PID, they inherit the advantages of the traditional PID and obtain the better control quality and the greater robustness. ADRFO PID combines the advantages of ADRC and the fractional order PID controller. In the case of noise and large perturbation, the controller still has a better control quality. And the controller resolves the contradiction between the rapidity and the overshoot and generates larger stability region at the same time. The controller has stronger robustness against the perturbation of system parameters and it is convenient to be used in engineering application.

References:

[1] Han Jingqing. From the PID technology to "Auto-disturbance rejection control" technology[J]. Control Engineering, 2002,9(3):13-18. (in Chinese)

[2] Zhao Chunzhe, Huang Yi. ADRC based integrated guidance and control scheme[J]. Systems Science and Mathematics, 2010, 30 (6): 742-751. (in Chinese)

[3] Lian Ming, Han Zhenyu, Fu Hongya. Application of active disturbances rejection control technique to satellite attitude simulation system[J]. Optics and Precision Engineering, 2010, 18(3): 616-622. (in Chinese)

[4] Hamamci S E. An algorithm for stabilization of fractional-order time delay systems using fractional-order PID controllers[J]. IEEE Transactions on Automatic Control, 2007,52(10):1964-1969.

[5] Hamamci S E, Kanthabhabha P, Vaithiyanathan K. Computation of all stabilizing first order controllers for fractional-order systems[C]//Proceedings of the 27th Chinese Control Conference. Kunming, Yunnan, China: [s. n.], 2008:123-128.

[6] Xue Dingyu, Zhao Chunna. Fractional order PID controller design of fractional-order system[J]. Control Theory and Applications, 2007,24(5):771-776. (in Chinese)

[7] Zhang Bangchu, Zhu Jihong, Pan Shushan, et al. Using fractional-order PIλDμ controller for control of aerodynamic missile[J]. Journal of China ordnance, 2006,2(2):127-131.

[8] Masoud K G, Majid Z, Nasser S, et al, An optimal fractional order controller for an AVR system using particle swarm optimization algorithm[C]//Power Engineering, 2007 Large Engineering Systems Conf. Montreal, Que: IEEE Press, 2007:244-249.

[9] Tar J K, Bito J F, Bencsik A L, et al. Preliminary design of a fractional order controller for an active car body suspension system[C]//The 6th International Symposium on Applied Machine Intelligence and Informatics. Herlany: IEEE Press, 2008: 297-302.

[10] Xu H J, Ioannou P A, Mirmirani M. Adaptive sliding mode control design for a hypersonic flight vehicle[J]. Journal of Guidance, Control and Dynamics, 2004,25(5):829-838.

[11] Gao Daoxiang, Sun Zengqi, Luo Xiong, et al. Adaptive fuzzy control of hypersonic vehicle based on Backstepping[J]. Control Theory and Applications, 2008,25(5):805-810. (in Chinese)

高超声速飞行器自抗扰分数阶 PID 控制器设计

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摘要：自抗扰控制器利用跟踪微分器可解决超调量及快速性之间的矛盾，分数阶 PID 控制器在提高控制品质的同时扩大了被控系统参数的稳定域。结合自抗扰技术及分数阶 PID 控制器设计了自抗扰分数阶 PID 控制器，并应用于高超声速飞行器再入姿态控制。仿真结果表明，自抗扰分数阶 PID 控制器对于高超声速飞行器非线性模型及强外干扰的影响下具有很好的控制效果，并且有较大的稳定域，即针对被控系统参数变化具有更强的鲁棒性。

关键词：高超声速飞行器；自抗扰控制器；分数阶 PID；D-分解法
中图分类号：V448.2

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