AUCTIONING METHOD FOR AIRSPACE CONGESTING RESOURCE ALLOCATION AND GAME EQUILIBRIUM ANALYSIS

Liu Fangqin, Hu Minghua

(College of Civil Aviation, NUAA, 29 Yudao Street, Nanjing, 210016, P.R. China)

Abstract: The airspace congestion is becoming more and more severe. Although there are traffic flow management (TFM) initiatives based on CDM widely applied, how to reschedule these disrupted flights of different airlines integrating TFM initiatives and allocate the limited airspace resources to these airlines equitably and efficiently is still a problem. The air traffic management (ATM) authority aims to minimizing the systemic costs of congested airspaces. And the airlines are self-interested and profit-oriented. Being incorporated into the collaborative decision making (CDM) process, the airlines can influence the rescheduling decisions to profit themselves. The airlines maybe hide the flight information that is disadvantageous to them, but is necessary to the optimal system decision. To realize the coincidence goal between the ATM authority and airlines for the efficient, and equitable allocation of airspace resources, this paper provides an auction-based market method to solve the congestion airspace problem under the pre-tactic and tactic stage of air traffic flow management. Through a simulation experiment, the rationing results show that the auction method can decrease the total delay costs of flights in the congested airspace compared with both the first schedule first service (FSFS) tactic and the ration by schedule (RBS) tactic. Finally, the analysis results indicate that if reallocate the charges from the auction to the airlines according to the proportion of their disrupted flights, the auction mechanism can allocate the airspace resource in economy equitably and decrease the delay losses of the airlines compared with the results of the FSFS tactic.

Key words: air traffic control; resource allocation; sealed-price auction; airspace; flow constrained area (FCA); game equilibrium analysis

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INTRODUCTION

Due to the limited civil airspace resources and the uneven flow distributions in China, the flight delays caused by the airspace congestion are becoming more and more severe. At present, traffic flow management (TFM) with the collaborative decision making (CDM) aid is applied widely to help to resolve the traffic congestion and balance demand and capacity when the airspace system is disrupted. According to the schedule time, different traffic management initiatives such as rerouting, ground delay program (GDP) and airspace flow program (AFM) can be used to revise the disrupted flight schedules and make the schedule demand adapt to the decreased capacities of the airspace system.

However, how to reschedule these disrupted flights of different airlines and allocate the limited airspace resources equitably and efficiently integrating these traffic management initiatives is a hard problem for the ATM authority. ATM authority aims to minimizing system delay time or cost under some assured fairness rules when it reschedules the disrupted flights of different users. Minimizing system delay time does not reflect the lowest total delay cost. Being incorporat-

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E-mail: liuliuqing2003@yahoo.com.cn

ed into the collaborative decision making (CDM) process, the airlines could influence the rescheduling decisions to profit themselves[1-2]. Because the total delay cost does not include the delay costs of airlines and the delay costs of travelers, the goals of different decision makers including airlines and ATM authority may conflict and the available information for good decision makings varies among these decision makers. The airlines maybe hide the flight information that is disadvantageous to them, but is necessary to the optimal system decision. It is hard for ATM authority to get the decision aim that reschedules these disrupted flights and allocates the limited airspace resources to airlines equitably and efficiently.

The auction is a resource rationing method of the market mechanism. The bid price is the reflection of the value of a scarce resource for the bidder. The successful use of auctions for telecommunication spectrum, energy and other commodities provide valuable insight into how to design auctions for the airspace resources[3-5]. Due to the fast progress of network and web technologies, traditional trading systems can be operated well on the internet. It unchained the technical barrier for the auction applied to the air traffic management[4, 6]. There are some references about market-based approaches using auctions to ration the congesting airport slots[7]. Grether et al used the competitive sealed-bid auctions for primary market, complemented by the oral double auction for the secondary market[8]. Rassenti presented the combinatorial auction model to allocate the arrival slots and take-off slots of airport together[9]. Wang Fei presented the artificial fish school algorithm to compute the combinatorial auction model[10]. Ball preliminarily investigated the auction mechanism for allocating slot resources[3]. He thought that allocating airspace capacity resources to flights in a finite specified period of time is more likely to provide an efficient ATM system, under some concrete slot allocation mechanisms of assigning them to the flights that can generate the greatest benefits from the use of slots. Ball suggested applying the hybrid auction mechanism to the slot rationing about the airport congestion problem.

And, the airspace congestion problem is different from the airport congestion problem. The airspace operational capacity is more uninsurably predicted than the airport. The airspace operation situation is often changes very quickly. This condition requires that air traffic flow management (ATFM) authority and the airlines must make flexible traffic management decisions and resource ration mechanisms adapted to the constantly shifting airspace states. The sealed-price auction without iterative interactive process is adapted to the pre-tactic and tactic air traffic flow management due to the airspace congestion.

In this paper, the auction method is applied to solve the airspace congestion problem in the pre-tactic and tactic stage of air traffic management. A first sealed-price auction method is presented based on dynamic Stackelberg equilibrium to realize the coincidence goal between the ATM authority and airlines. The market-based and user self-decision ATM mechanism is set up. For improving the system performance, ATM authority announces the specific congestion toll schedules that internalize the congesting external cost into the flight operational cost of airlines. ATM authority considers the global impact of dynamic congestion tolls that encourages the profit-oriented airlines to shift their low marginal profit flights to the non-peak traffic period or other legs which may be not charged by congesting fees or charged a little. Each airline is assumed to reschedule its disrupted flights by themselves according to the maximizing self-interested rule, considering the pre-announced toll schedules and allocated capacity which is preferentially sold to the airline. Those elastic flights may be delayed or rerouted from the congesting airspace to other airspace.

1 AUCTION PROCESS DESIGN

To achieve capacity, efficiency, and flexibility gains, the proposed auction mechanism solutions focus on improving planning at the level between ATM/ATFM authority and airline operational center (AOC) rather than at the level between air traffic controllers and pilots. The mechanism scheme involves AOCs as bidders and the ATM authority as the auctioneer. Implementation scheme consists of four steps. The completing deadline of every step except Step 1 is within 5 min. The main framework is shown in Fig. 1.

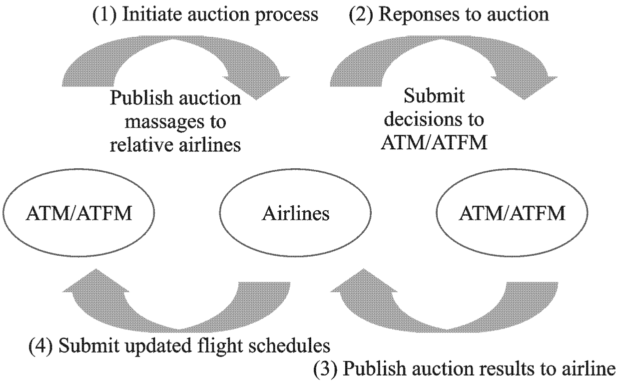

Fig. 1 Auction process based on CDM infrastructure

The auction process is as follows:

Step 1 Initiate the auction process and publish the auction massages to the relative airlines.

The ATM/ATFM authorities (the auctioneer) initiate the airspace flow program (AFP) and change the airspace capacities into the competitive airspace slots. And publish these messages to the related airliners that have the disrupted flights.

Step 2 Reponses to the auction and submit the decisions to ATM/ATFM.

Airlines assess the reschedule losses and risks for choosing the different slots related to the different delay costs of flights, and make the optimal biding prices for their flights under the auction game. And then submit the biding information to the auctioneer. The airline must finish the submitting in the given time, or else it is forced to give up this time of the bid.

Step 3 Publish the auction results.

ATM/ATFM authorities assure and publish the biding results and the competitive resource allocation in the given time.

Step 4 Submit updated flight schedules.

Airlines reschedule the disrupted flights based on the slots bided, and submit the reschedule information. The airline must finish the submitting in the given time.

2 GAME EQUILIBRIUM ANALYSIS

2.1 Auction equilibrium model for assured capacity

The decision behaviors of users under competition are the key factors affecting the auction success. Bayesian Nash equilibrium theory is applied to analyze the game between the airlines existing in the first-price sealed bid auction.

The usage cost of airspace r, the expectation delay cost of flight f_i and the opportunity cost of flight f_i are defined.

Let the marginal usage cost of the assured capacity of airspace r as

$$\mathrm{MC}_{a_i}^1(f_i,t) = p_{a_i}^1(t) \tag{1}$$

where $p_{a_i}^1(t)$ denotes the biding price of the assured capacity that airline a_i submit for flight f_i.

Let the lower one of the expectation delay cost and the rerouting cost of flight f_i as the opportunity cost of f_i using the assured capacity

$$\mathrm{OP}_{f_i}(t) = \begin{cases} \mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) + \\ (1-\mathrm{prob}_{\mathrm{gdp}}(t)) * p_r(t+\Delta t_{f_i}) \\ \qquad \mathrm{Edelay}_{f_i}(t) \leqslant \mathrm{reroute}_{f_i}(t) \\ \mathrm{reroute}_{f_i}(t) \quad \mathrm{reroute}_{f_i}(t) < \mathrm{Edelay}_{f_i}(t) \end{cases} \tag{2}$$

where $\mathrm{Edelay}_{f_i}(t)$ denotes the expectation delay cost of flight f_i, that is

$$\begin{aligned} \mathrm{Edelay}_{f_i}(t) &= \mathrm{prob}_{\mathrm{gdp}}(t) * [\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \\ &\mathrm{delay}_{f_i}(t)] + \overline{\mathrm{prob}_{\mathrm{gdp}}(t)} * [p_r(t+\Delta t_{f_i}) + \\ &\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)] = \\ &\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) + \\ &(1-\mathrm{prob}_{\mathrm{gdp}}(t)) * p_r(t+\Delta t_{f_i}) \approx \\ &\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) + \\ &(1-\mathrm{prob}_{\mathrm{gdp}}(t)) * p_r(t) \end{aligned} \tag{3}$$

where the expectation delay cost of flight f_i at the future departing interval $(t+\Delta t_{f_i})$ is the operational cost of flight f_i after ATM/ATFM authorities implementing the AFP program, if the flight do not depart at the current interval. If the congestion situation will disappear at the future departing interval $(t+\Delta t_{f_i})$, that predicted by the operator of the flight, the AFP program will be canceled and the delay time will be the ground delay time from the current interval to the future departing interval $(t+\Delta t_{f_i})$. But if the congestion situation has not disappeared, the cost should include the ground delay cost and the usage cost of airspace r at the future departing interval $(t+\Delta t_{f_i})$. $\text{prob}_{\text{gdp}}(t)$ is the probability that the AFP initiative will be canceled during the $(t+\Delta t_{f_i})$th period. So, the delay cost of flight f_i is the expectation value including the ground delay cost with probability $\text{prob}_{\text{gdp}}(t)$ and the usage cost at the future departing interval $(t+\Delta t_{f_i})$ with probability1-$\text{prob}_{\text{gdp}}(t)$.

If the airliner wins the assured capacity biding game, the payoff utility of flight f_i is

$$\text{OP}_{f_i}(t)-\text{MC}^1_{a_i}(f_i,t)=$$
$$\begin{cases}\text{Edelay}_{f_i}(t)-p^1_{a_i}(t) \\ \qquad \text{Edelay}_{f_i}(t)\leqslant \text{reroute}_{f_i}(t) \\ \text{reroute}_{f_i}(t)-p^1_{a_i}(t) \\ \qquad \text{reroute}_{f_i}(t)<\text{Edelay}_{f_i}(t)\end{cases} \approx$$
$$\begin{cases}\text{delay}_{f_i}(t+\Delta t_{f_i})-\text{delay}_{f_i}(t)-\text{prob}_{\text{gdp}}(t)*p^1_{a_i}(t) \\ \qquad \text{Edelay}_{f_i}(t)\leqslant \text{reroute}_{f_i}(t) \\ \text{reroute}_{f_i}(t)-p^1_{a_i}(t) \\ \qquad \text{reroute}_{f_i}(t)<\text{Edelay}_{f_i}(t)\end{cases} \tag{4}$$

Because the unknown expectation delay cost is the empirical data, for simplifying the problem we take the delay cost of flight instead of the expectation delay cost.

Only if is the marginal usage cost of the assured capacity lower than its opportunity operational cost, the airline will attend the auction for flight f_i. So, the payment utility value is always a positive number. Simply, assume that $p_{a_i}(t+\Delta t_{f_i})$ is approximate to $p^1_{a_i}(t)$ if there will be still the congestion during the future departing interval $(t+\Delta t_{f_i})$. Both opportunity operational cost $\text{OP}_{f_i}(t)$ of flight f_i. and marginal cost of the uninsured capacity are the delay cost of the flight. Assume that the delay cost, caused by congestion-related events, of each flight that takes part in the auction is independent and uniform random variable on the same interval$(0, T)$. Because all of users in set A are profit-oriented, assume that in civil aviation industry there is a common maximum $\text{delay}_A(T)$of the delay cost of flight. Each of the bidders who auction the same resource submits a nonnegative biding price. The bidder submitting the highest bid price will win and pay his bid. Other bidders pay and receive nothing. Bidders are risk-neutral and all of the information is common knowledge. If bidder a_i wins and pays the biding price, bidder $a_i$payoff is

$$\text{bid}_{a_i,f_i}(a_i,a_j,v_{a_i,f_i}(t))=$$
$$\begin{cases}OP_{f_i}(t)-p^1_{a_i}(t) \\ \qquad p^1_{a_i}(t)>p^1_{a_j}(t) \qquad \underset{i\neq j}{\forall} a_j\in A \\ 0 \qquad \text{else}\end{cases} \tag{5}$$

Because the bid game is peer to peer, we just need to analyzing the equilibrium strategy of a_i: $p^1_{a_i}(t)=p^{1*}_{a_i}(\text{OP}_{f_i}(t))$. Given the equilibrium solution $\text{OP}_{f_i}(t)-p^{1*}_{a_i}(t)$, the expected payoff function is

$$\text{Ebid}_{a_i,f_i}(p^1_{a_i}(t))=(\text{OP}_{f_i}(t)-p^1_{a_i}(t))*$$
$$\left[\prod_{j\neq i}\text{probability }(p^1_{a_j}(t)<p^{1*}_{a_i}(t))\right] \tag{6}$$

where the first part before the multiplicative sign is the payoff of a_i, and the second part the probability that a_i wins all of the others.

The probability that bidder a_i wins bidder a_j is

$$\text{probability}(p^1_{a_j}(t)<p^{1*}_{a_i}(t))=$$
$$\text{probability}(p^{1*}_{a_j}(\text{OP}_{f_i}(t))<p^{1*}_{a_i}(t))=$$
$$\text{probability}(\text{OP}_{f_i}(t))_{a_j}<\Phi(p^{1*}_{a_i}(t))=$$
$$\Phi(p^{1*}_{a_i}(t))/\text{delay}_A(T) \tag{7}$$

where $\Phi(p^{1*}_{a_i}(t))$ is the inverse function of $p^{1*}_{a_i}(t)$, which denotes that the delay cost saving is $\Phi(p^{1*}_{a_i}(t))$ if airline $a_i$submits bid price $p^{1*}_{a_i}(t)$. So we have

$$\mathrm{Ebid}_{a_i,f_i}(p_{a_i}^{1*}(t)) = (\mathrm{OP}_{f_i}(t) - p_{a_i}^{1}(t)) * [\Phi(p_{a_i}^{1*}(t))/\mathrm{delay}_A(T)]^{n-1} \tag{8}$$

For maximizing the expected payoff, we get

$$\frac{\partial \mathrm{Ebid}_{a_i,f_i}(p_{a_i}^{1*}(t))}{\partial p_{a_i}^{1*}(t)} = 0$$

If $\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) < \mathrm{reroute}_{f_i}(t)$, we get

$$\frac{\partial \mathrm{Ebid}_{a_i,f_i}(p_{a_i}^{1*}(t))}{\partial p_{a_i}^{1*}(t)} = - \mathrm{prob}_{\mathrm{gdp}}(t) * [\Phi(p_{a_i}^{1*}(t))]^{n-1} + [\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) - \mathrm{prob}_{\mathrm{gdp}}(t) * p_{a_i}^{1}(t)] * (n-1)\Phi^{n-2}\Phi'(p_{a_i}^{1*}(t)) = 0$$

If $\mathrm{reroute}_{f_i}(t) < \mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)$, we get

$$\frac{\partial \mathrm{Ebid}_{a_i,f_i}(p_{a_i}^{1*}(t))}{\partial p_{a_i}^{1*}(t)} = - [\Phi(p_{a_i}^{1*}(t))]^{n-1} + [\mathrm{reroute}_{f_i}^{*}(t) - p_{a_i}^{1*}(t)] * (n-1)\Phi^{n-2}\Phi'(p_{a_i}^{1*}(t)) = 0$$

Due to $\Phi(p_{a_i}^{1*}(t)) = \min(\mathrm{OP}_{f_i}(t), \mathrm{reroute}_{f_i}(t))$, we get the equilibrium bid price of a_i

$$p_{a_i}^{1*}(t) = \begin{cases} \frac{n-1}{n * \mathrm{probgdp}(t)}[\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)] - \frac{1}{n}[\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)]^{1-n} & \mathrm{Edelay}_{f_i}(t) \leqslant \mathrm{reroute}_{f_i}(t) \\ \frac{n-1}{n}\mathrm{reroute}_{f_i}(t) & \mathrm{reroute}_{f_i}(t) < \mathrm{Edelay}_{f_i}(t) \end{cases} \tag{9}$$

where under the equilibrium condition, the airline who gets the highest value from the resource will give the highest price. According to the first price sealed bidding principle, the player who gives the highest price will get the resource. The equilibrium price of this bid game relies on the number of bidders, the value $v_{a_i,f_i}^{*}(t)$ and their own estimations about the airspace congestion situation. Each airline bidding price is determined by the value from the bidding resource.

Here the equilibrium value is just theory results. In practice, the behaviors of airlines in the bid games are hard to be assumed. However, the equilibrium bid price is direct correlative to the delay cost. The equilibrium bid price of flight could reflect the true delay cost of flight in the assumption that every airline is rational and profit-oriented . We get the derivative of $p_{a_i}^{1*}(t)$ on $v_{a_i,f_i}(t)$ as

$$\frac{\partial p_{a_i}^{1*}(t)}{\partial v_{a_i,f_i}(t)} = \frac{n-1}{n * \mathrm{prob}_{\mathrm{gdp}}(t)} - \frac{1-n}{n}[v_{a_i,f_i}(t)]^{-n} \geqslant 0 \tag{10}$$

where $v_{a_i,f_i}(t) = \mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)$.

Obviously, $p_{a_i}^{1*}(t)$ is the increasing function about the variable $v_{a_i,f_i}(t)$. So the airlines whose flights suffer the more delay or rerouting cost will give the higher price about the auctioned resources, and will get more chance of wining.

When the biding resources are more than one, if the biding flights has the consistent utility for each resource unit in the same decision period, based on Eqs. (2-7), we get

$$\mathrm{Ebid}_{a_i,f_i}(p_{a_i}^{1*}(t)) = (\mathrm{GDP}_{f_i}(t) - p_{a_i}^{1}(t)) * [\Phi(p_{a_i}^{1*}(t))/\mathrm{delay}_A(T)]^{n-m} \tag{11}$$

Because there are m available resources, if only the biding price of the flight is above to any of $(n-m)$ other bidders, the flight could get one capacity slot.

Likewise in Eq. (9), we get the equilibrium biding price under the condition that the biding resources are more than one

$$p_{f_i,a_i}^{1*}(t) = \begin{cases} \frac{n-(m-1)-1}{n-(m-1) * \mathrm{prob}_{\mathrm{gdp}}(t)}[\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)] - \frac{1}{n-(m-1)}[\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t)]^{1-n+(m-1)} & \mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) \leqslant \mathrm{reroute}_{f_i}(t) \\ \frac{n-(m-1)-1}{n-(m-1)}\mathrm{reroute}_{f_i}(t) & \mathrm{reroute}_{f_i}(t) < \mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) \end{cases} \tag{12}$$

where m denotes the number of the assured capacities auctioned.

2.2 Auction equilibrium model for uninsured capacity

Because this paper tries to solve the airspace resource allocation problem, the uninsured factors of airspace capacity assessment is considered. Assume that the capacity prediction including the assured capacity and the uninsured capacity. The uninsured capacity is fluctuated with the time close to the operational points of the flights which plan to use the uninsured one. So, the flights have to add the unexpected operational cost because the unpredicted decrease of the capacity will lead to the ground stop or air holding initiatives implemented on them. Let the marginal cost of the uninsured capacity of airspace r_j as

$$\mathrm{MC}^2_{a_i}(f_i,t) = p^2_{a_i}(t) + \mathrm{GPdelay}_{f_i}(t) \quad (13)$$

where $p^2_{a_i}(t)$ denotes the biding price of the uninsured capacity that airliner a_i submits for flight f_i, $\mathrm{GPdelay}_{f_i}(t)$ the expectation ground stop cost or air holding cost of flight f_i (Eq, (14)). The maximum delay time is less than 1 h.

$$\begin{aligned}&\mathrm{GPdelay}_{f_i}(t) = \\ &\overline{\mathrm{prob}_{\Delta C_r(t)}} * \mathrm{prob}_{\Delta C_r(t+1)} * \mathrm{GSdelay}_{f_i}(\tau) + \\ &\overline{\mathrm{prob}_{\Delta C_r(t)}} * \overline{\mathrm{prob}_{\Delta C_r(t+1)}} * \mathrm{prob}_{\Delta C_r(t+2)} * \\ &\mathrm{GSdelay}_{f_i}(2\tau) + \overline{\mathrm{prob}_{\Delta C_r(t)}} * \overline{\mathrm{prob}_{\Delta C_r(t+1)}} * \\ &\overline{\mathrm{prob}_{\Delta C_r(t+2)}} * \mathrm{prob}_{\Delta C_r(t+3)} * \mathrm{GSdelay}_{f_i}(3\tau) \approx \\ &\overline{\mathrm{prob}_{\Delta C_r(t)}} * \mathrm{prob}_{\Delta C_r(t)} * \mathrm{GSdelay}_{f_i}(\tau) + \\ &\overline{\mathrm{prob}_{\Delta C_r(t)}} * \overline{\mathrm{prob}_{\Delta C_r(t+1)}} * \mathrm{prob}_{\Delta C_r(t+2)} * \mathrm{GSdelay}_{f_i}(2\tau)\end{aligned} \quad (14)$$

In the first part of Eq. (14), $\overline{\mathrm{prob}_{\Delta C_r(t)}} * \mathrm{prob}_{\Delta C_r(t+1)}$ is the predicting probability that the uninsured capacity will not exist during the tth period, but exist during $(t+1)$th period. If the uninsured capacity would not exist in the tth period, but exist during the $(t+1)$th period, the flight allocated to the capacity in the tth period would be executed the ground stop initiative, and be delayed to the next period. It will use the uninsured capacity of the next period without another charge. The other two parts have the analogical meaning. Here, τ is assumed to be less than 15 min. The maximum time that the flight executes the ground stop initiative is less than 1 h.

If the airliner wins the uninsured capacity biding game, the payoff utility of flight f_i is

$$\mathrm{OP}_{f_i}(t) - \mathrm{MC}^2_{a_i}(f_i,t) = \begin{cases}\mathrm{Edelay}_{f_i}(t) - p^2_{a_i}(t) - \mathrm{GPdelay}_{f_i}(t) & \mathrm{Edelay}_{f_i}(t) \leqslant \mathrm{reroute}_{f_i}(t) \\ \mathrm{reroute}_{f_i}(t) - p^2_{a_i}(t) - \mathrm{GPdelay}_{f_i}(t) & \mathrm{reroute}_{f_i}(t) < \mathrm{Edelay}_{f_i}(t)\end{cases} \approx \begin{cases}\mathrm{delay}_{f_i}(t+\Delta t_{f_i}) - \mathrm{delay}_{f_i}(t) - \mathrm{GPdelay}_{f_i}(t) - \mathrm{prob}_{\mathrm{gdp}}(t) * p^2_{a_i}(t) & \mathrm{Edelay}_{f_i}(t) \leqslant \mathrm{reroute}_{f_i}(t) \\ \mathrm{reroute}_{f_i}(t) - \mathrm{GPdelay}_{f_i}(t) - p^2_{a_i}(t) & \mathrm{reroute}_{f_i}(t) < \mathrm{Edelay}_{f_i}(t)\end{cases} \quad (15)$$

where the payoff utility is positive, or the airline will not attend the auction for flight f_i. For simply, assume that $\mathrm{MC}^2_{a_i}(f_i, t+\Delta t_{f_i})$ is approximate to $\mathrm{MC}^2_{a_i}(f_i,t)$ if there is a congestion during the future departing interval $(t+\Delta t_{f_i})$.

Both opportunity operational cost $\mathrm{OP}_{f_i}(t)$ and marginal usage cost of flight are the delay cost of flight. Assume that the delay cost, caused by congestion-related events, of each flight that takes part in the auction, is independent and uniform random variable on the same interval $(0, T)$. Because all of users in set A are profit-oriented, assume that in civil aviation industry there is the common maximum $\mathrm{delay}_A(T)$ of the delay cost of flight. Each of the bidders who competes the same airspace resource submits a nonnegative biding price. The bidder submitting the highest bid price will win and pay his bid. Other bidders pay and receive nothing. Bidders are risk-neutral and all of the information is common knowledge. Bidder a_i payoff, if wining and paying the biding price, is

$$\mathrm{bid}^2_{a_i,f_i}(a_i, a_j, v_{a_i,f_i}(t)) = \begin{cases}\mathrm{OP}_{f_i}(t) - \mathrm{GPdelay}_{f_i}(t) - p^2_{a_i}(t) & p^1_{a_i}(t) > p^1_{a_j}(t) \quad \forall_{i\neq j} a_j \in A \\ 0 & \text{else}\end{cases} \quad (16)$$

Like the analysis in Section 2.1, we get the bid price of the uninsured capacity

$$p_{a_i}^{2*}(t)=\begin{cases}\frac{n-1}{n*\text{prob}_{\text{gdp}}(t)}[\text{delay}_{f_i}(t+\Delta t_{f_i})-\text{delay}_{f_i}(t)-\\ \text{GPdelay}_{f_i}(t)]-\frac{1}{n}[\text{delay}_{f_i}(t+\Delta t_{f_i})-\\ \text{delay}_{f_i}(t)-\text{GPdelay}_{f_i}(t)]^{1-n}\\ \qquad \text{Edelay}_{f_i}(t)\leqslant \text{reroute}_{f_i}(t)\\ \frac{n-1}{n}[\text{reroute}_{f_i}(t)-\text{GPdelay}_{f_i}(t)]\\ \qquad \text{reroute}_{f_i}(t)<\text{Edelay}_{f_i}(t)\end{cases} \tag{17}$$

The reason is that using the uninsured resource will risk ground stop events. It makes the expected operational costs of the flights using the uninsured resources higher than the ones of the flights using the assured capacity . So , price $p_{a_i}^{2*}(t)$ of the uninsured capacity should be lower than price $p_{a_i}^{1*}(t)$ of the assured capacity (Eq. (9)).

3 SIMULATION EXPERIMENT

The experiment is applied to illustrate the concepts and the models presented in the paper.

3.1 Assumption and simulation testing data presentation

Because collect the data about the real operational and delay costs of flights is hard, some assumptions about these costs are given. First, the delay cost functions and the rerouting cost function for the disrupted flights of different types simply are given as

If the flight is the airliner, the delay cost function is

$$\text{delay}_{f_i}(\tau)=\begin{cases}0 & \tau<15\text{ min}\\ \alpha_1*\tau & 15\text{ min}\leqslant\tau<\tau_{\max}\\ \alpha_2 & \tau_{\max}\leqslant\tau\end{cases} \tag{18}$$

where assume that if the flight delay time is less than 15 mins, the flight will not suffer the delay cost. Let $\tau_{\max}$ denotes the limit of tolerable delay time of the flight. If the delay time is between 15min and $\tau_{\max}$, the delay cost is the linear function about delay time τ. Linear cost coefficient α_1 depends on the flight type. If the delay time of the flight is no less than $\tau_{\max}$, the flight may miss its connection flights at the arrival airport or even the flight will be cancelled. In this condition, the delay cost will reach the maximum value α_2. Of course, $\tau_{\max}$ and α_2 depend on the flight type.

The ground stop or air holding cost function is Eq. (19). It includes two parts: The first part denotes the operational cost per minute due to the ground stop initiative; The second part the air holding cost per minute.

$$\text{GSdelay}_{f_i}(\tau_{\text{GS}})=0.5*\alpha_3*\tau_{GS}+0.5*\alpha_4*\tau_{\text{GS}} \tag{19}$$

where α_3 is the delay cost per minute caused by the ground stop event, α_4 the delay cost per minute caused by the air holding event, τ_{GS} the delay time due to the ground stop or the air holding.

The additional rerouting operational cost function is given as

$$\text{reroute}_{f_i}(\tau_{\text{re}})=\beta*\tau_{\text{re}}+\text{delay}_{f_i}(\tau_{\text{re}}) \tag{20}$$

where β is the cost per additional flying minute caused by the rerouting event, τ_{re} the possible additional flying time than the plan time due to rerouting. The rerouting cost of the flight includes two parts. The first part is the additional rerouting operational cost in air and the second part the delay cost due to the rerouting.

An example of FCA composed of two en route sectors caused by the en route convective weather in South China area is presented. The capacities of these sectors in the effected airspace would have been decreased. The metering point on the border of FCA began to be as the mile-in-trail decision point restricting the flight flow into the FCA. Lots of flights that planed to pass through the airspace were disrupted and must be rescheduled. The airlines and the ATM authority decision-making interaction process in schedule disruptions initiated according to Section 1.

The experimental data about the flight schedule and FCA is from the historical data from 9:00 to 10:00 on May 23, 2009 at the South China airspace by South China Air Traffic Control Service Center, as shown in Tables 1,2. The experiment involves 23 flights of 5 airlines during 1 h excluding the flights of the foreign airlines supposed to have the higher priority. Some assumptions are made to the predicted airspace capacity to adapt it to the framework of our models.

Table 1 Prediction information of airspace operational state

Congestion time period	Assured capacity	Uninsured-capacity and prediction probability	Operational capacity	Predicted delay time Δt_{f_i}/min and prediction probability/%
9:00—9:15	2	1,80%	3	60,80
9:15—9:30	3	1,80%	4	45,80
9:30—9:45	3	1,80%	4	30,80
9:45—10:00	4	1,80%	5	15,80

Table 2 Data about linear coefficients and time constants of flights

Airlines	*A*	*B*			*C*							
Flight No.	*A*1	*B*1	*B*2	*B*3	*C*1	*C*2	*C*3	*C*4	*C*5	*C*6	*C*7	*C*8
Aircraft type	B767	738	738	767	ATR42	767	319	320	320	738	757	737-3
Scheduled time	9:33	9: 36	9:46	9:48	9:03	9:18	9:22	9:25	9:29	9:35	9:43	9:50
α_1^* /	142.2	99.4	99.4	142.2	31.3	0	75.2	90.1	90.1	99.4	122.5	74.4
α_2^+ /	30 000	20 000	25 000	30 000	5 000	60 000	25 000	35 000	20 000	20 000	20 000	20 000
α_3/	162.6	110.7	110.7	162.6	33.3	162.6	84.9	100.9	100.9	110.7	137.1	88.6
α_4^* /	148.3	103.1	103.1	148.3	31.7	148.3	78.4	93.3	93.3	103.1	126.9	78.2
β^* /	167	112.7	112.7	167	33.6	167	87.3	102.4	102.4	112.7	139.9	87.2
τ_{max}/min	120	120	120	120	40	300	120	120	120	120	120	120
τ_{GS}/min	10	10	10	10	15	15	15	15	15	10	10	10
τ_{re}/min	15	15	15	15	15	15	15	15	15	15	15	15

Airlines	*D*						*E*				
Flight No.	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*E*1	*E*2	*E*3	*E*4	*E*5
Aircraft type	747	320	737-5	757	767	320	747	737-3	737-5	737-3	ATR72
Scheduled time	9:12	9:16	9:20	9:40	9:55	9:59	9:08	9:14	9:27	9:31	9:38
α_1^* /	238.8	90.1	62.7	122.5	142.2	90.1	238.8	74.4	62.7	74.4	40.8
α_2^+ /	30 000	30 000	20 000	25 000	30 000	20 000	30 000	30 000	28 000	20 000	20 000
α_3/	276.6	100.9	75.8	137.1	162.6	100.9	276.6	88.6	75.8	88.6	43.4
α_4^* /	252.9	93.3	66.4	126.9	148.3	93.3	252.9	78.2	66.4	78.2	41.9
β^* /	289.1	102.4	74.3	139.9	167	102.4	289.1	87.2	74.3	87.2	44.1
τ_{max}/min	120	120	120	120	120	120	120	120	120	110	120
τ_{GS}/min	15	15	15	10	10	10	15	15	15	10	10
τ_{re}/min	15	15	15	15	15	15	15	15	15	15	15

Note: ″∗″ denotes that the costs with flight network effect are from Ref. [11]; ″+″ denotes that the costs are the stimulation data.

In Table 1, the predictions about the capacity of FCA and the congestion duration time in each period are assumed according to the real post-event operational data. The real operational capacity is presented in fourth column of Table 1. These predictions in different periods are not made at the same time point, but the predicted information of every period is updated as close to the decision deadline of each period as possible. Every airline has the different cognitive knowledge about the congestion situation. For simplicity, different airlines have the consistent prediction situation about their disrupted flights, whose average ground delay time in each period is predicted with the same probability of 80%. The linear coefficients and time constants and parameters of the flights in the cost functions (Eqs. (18-20)) are given in Table 2.

3.2 Game equilibrium results

Here, we give the analysis of the decision behaviors of airlines about the reschedule of the disrupted flights in the interacting decision process between the ATM authority and them.

According to Eq. (9,12,17) in Section 2, the equilibrium biding prices of airlines for the capacity slots in each period are obtained. According to the auction rule, the reschedule results are presented in Table3.

Table 3 Rescheduled results based on auction method

Airlines	A	B			C							
Flight No.	*A*1	*B*1	*B*2	*B*3	*C*1	*C*2	*C*3	*C*4	*C*5	*C*6	*C*7	*C*8
Scheduled time	9:33	9: 36	9:46	9:48	9:03	9:18	9:22	9:25	9:29	9:35	9:43	9:50
Rescheduled time	9:50	9:56	9:27	9:31	Reroute	10:02	9:29	9:21	9:25	9:46	9:40	10:12
Delay costs including rerouting costs /	0	0	0	0	1 512	0	0	0	0			148.8
Payments /	4 266	2 635.3	1 237.5	2 133	Reroute	0	2 809	3 801.1	3 801.1	0	3 675	0

Airlines	D						E				
Flight No.	*D*1	*D*2	*D*3	*D*4	*D*5	*D*6	*E*1	*E*2	*E*3	*E*4	*E*5
Scheduled time	9:12	9:16	9:20	9:40	9:55	9:59	9:08	9:14	9:27	9:31	9:38
Rescheduled time	9:12	9:16	10:05	9:36	9:53	9:56	9:08	9:14	10:06	10:07	10:10
Delay costs including rerouting costs /	0	0	2 696	0	0	0	0	0	2 508	2 604	1 224
Payments /	8 130.9	3 801.1	0	3 675	2 133	1 351.5	5 420.6	2 369.1	0	0	0

Fig. 2 illustrates the graphical comparison of delay costs of airlines and the system under four different tactics including first schedule first service (FSFS), ration by schedule (RBS), the auction, and the optimal ration. The variation tendency and regularity in the delay costs and social benefits under different prices are indicated in Fig. 2. The four bars on the right of Fig. 2 show the decreasing tendency of the total delay costs under different tactics: FSFS has the highest delay cost, RBS has a little fewer delay cost, both of the optimal ration and the auction have the same lowest delay cost. This accords with the expected result.

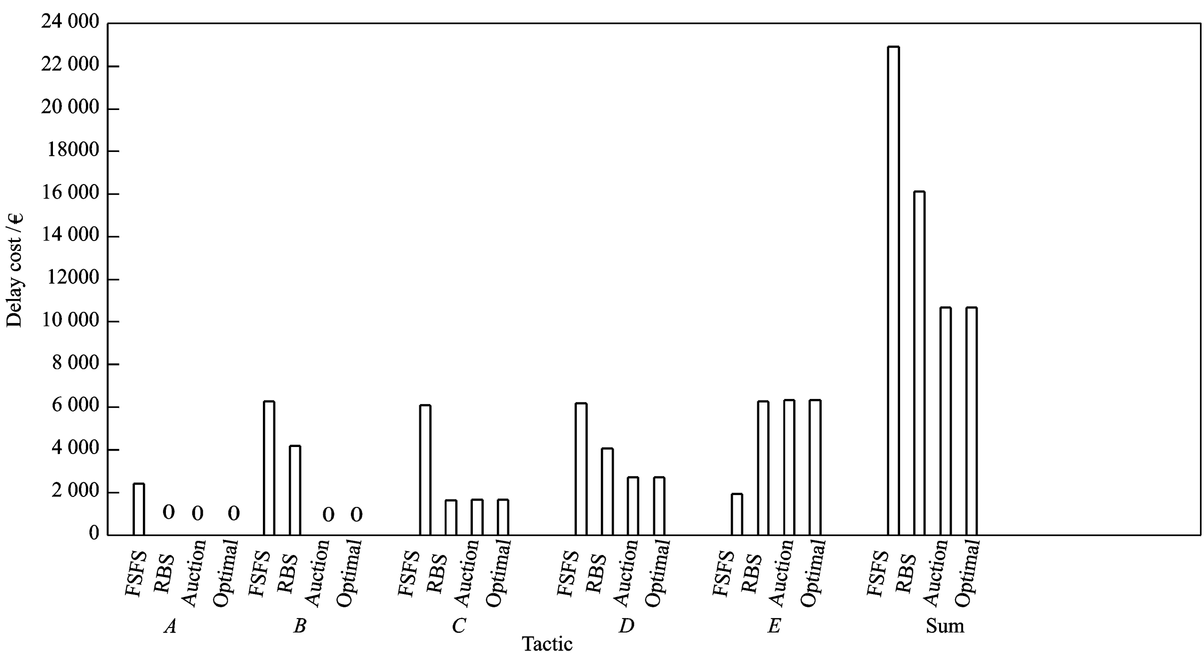

Fig. 2 Comparison of delay costs of each airline and system under different ration tactics

Comparing the results of the different tactics in Figs. 2-3, the delay times and costs of Airline *A* and Airline *B* are decreased very much, even to zero under the auction tactic. The delay time and cost of Airline *D* are decreased too, but not so much. The delay time and cost of Airline *E* are increased largely. The reason is that it has more flights with low delay costs than other airlines. Airline *C* is special that its delay time under congestion charges is increased than the ones under no charge, but its delay costs is decreased. The reason is that the flights of Airline *C* with the low delay cost under FSFS takes up the allocated resources, and under congestion charges, these flights are allocated to be delayed and the flights with higher delay costs can use the allocated resources, as shown in Tables 2,3.

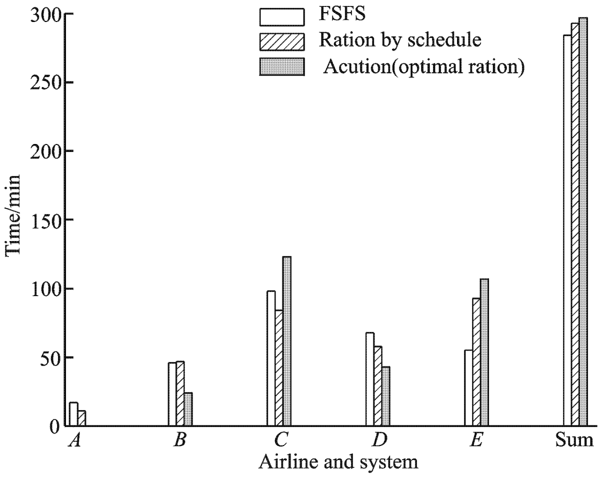

Fig. 3 Delay times of each airline and system under different tactics

Wojcik (2004) presented that "assessment of the benefits and risks of introducing new concepts will require an understanding of how multiple airspace users will interact and how their performance will be affected by implementation of the concepts, because performance and fairness of the resource rationing among users are important to understanding system impact of future concepts". We introduce the proportion of scheduled profit (POSP) value as the performance and equity metric (Wojcik, 2004). POSP is according to the internalized delay costs realized as a fraction of the delay costs that would obtain if the disrupted flights of all airlines ran their intended schedule of flights through airspace. Likewise, we define a congestion pricing performance metric (PM) that is the ratios of internalized delay costs to the delay costs according to the FSFS tactic, because the first-schedule-first-service (FSFS) tactic is the current main rationing rule widely accepted by airlines in China. PM value is used to assess the performance and fairness of the market mechanism. The lower PM value of the airline is, the higher the performance of the airline is, and the more the airline likes accepting the allocation mechanism. PM values are defined as

$$\mathrm{PM}_a = \frac{\mathrm{delay}_{a,1} + \mathrm{charge}_a}{\mathrm{delay}_{a,2}} \tag{21}$$

where $\mathrm{delay}_{a,1}$ is the total delay costs of airline a under the auction, $\mathrm{delay}_{a,2}$ the total delay costs of airline a under FSFS tactic, and charge_a the payments of airline a for the congested slots under the auction.

Fig. 4 gives the PM metric results of each airline. Black bars under different tactics in Fig. 4 show that the metric values of the airlines except Airline B are all more than 1. It means that the internalized delay costs of most airlines are more than the non-internalized under FSFS rule. From the view of the airlines, the pricing mechanism should not be popular by the airlines although the delay costs could be decreased.

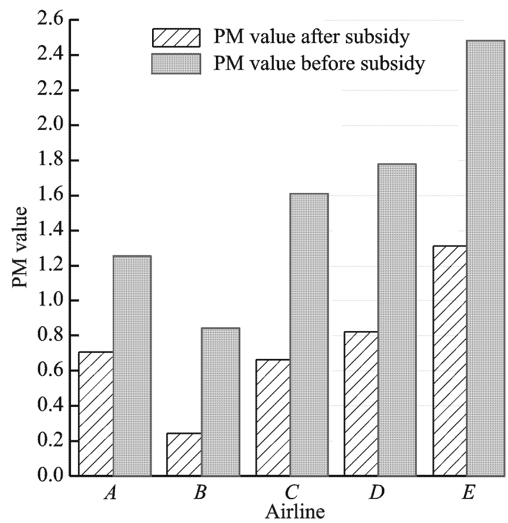

Fig. 4 Comparison of PM values of each airline between unsubsidized and subsidized

And, there is another problem relating to how to utilize the congestion fees. According to the current air traffic control service charge, we propose the suggestion that the moneys can averagely subsidize the ATC service charge of each involved flight scheduled in the charged airspace and time period. If so, the real internalized delay cost of the disrupted flight should be that the internalized delay cost subtracts the subsidy from the congestion charges. The real payment of airline a for the congesting event after subsidy is

$$\text{payment}_a = \text{delay}_{a,1} + \text{charge}_a - n_a * \left(\frac{\text{charge}_{\text{total}}}{n_{\text{total}}}\right) \tag{22}$$

where $\text{charge}_{\text{total}}$ are the total charges from the five airlines. n_a is the number of the disrupted flights of airline a and n_{total} the number of the disrupted flights of airline a.

Sidetrack bars in Fig. 4 shows the PM results of each airline after the airlines got the subsidies. The results of the airlines except Airline E are all lower than 1. This means that the internalized delay costs of most airlines are lower than the one under FSFS rule. Under this situation, most airlines would be prone to accept the mechanism. And Table 4 shows that the variance values are all less than the values without subsidies. We take the variance of PM values of airlines as the equity metric across airlines. More closer to 0 the variance is, the higher the equity across airlines is.

Table 4 Variance values of PM of all airlines

Auction	No subsidy	Subsidy
Variance of PM	0.347 49	0.167 53

4 CONCLUSION

Given that the delay costs of flights are important components of the airline decision-making process, how the economic costs of flights under the different air traffic management tactics influence the airline decision behaviors have not analyzed in precious research. Our models allow for a test of the market mechanism effects on the airline decision behaviors in the context of ATM that carefully optimizes the airspace system costly. The main contribution of this paper is to develop the auction method of the congested airspace resource allocation. The method makes an attempt to solve the capacity allocation of FCA in the pre-tactic or tactic stage of the air traffic flow management, ensuring the airspace systemic benefit and equity and efficiency.

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空域拥挤资源分配的拍卖方法研究和博弈均衡分析

刘方勤　胡明华

（南京航空航天大学民航学院，南京，210016，中国）

摘要：基于协调决策的各种流量管理策略已被广泛应用于空域拥挤问题，但是如何整合这些流量管理策略重排拥挤的航班，实现空域资源公平有效地分配，一直是一个难题。流量管理部门对拥塞空域的主要管理目标是系统总延误成本最小，而航空公司则是以自身获利最大化为目标。航空公司可在协调决策过程中，隐藏必要的信息，以实现自身利益的最大化，但是这样会增加系统的总延误成本。为实现流量管理部门与航空公司的目标一致性，提高空域资源的分配效率，文中提出了基于拍卖的方法解决预战术和战术流量管理阶段的空域拥挤问题，并利用博弈理论对航空公司的决策进行了分析建模。通过一个仿真实验，验证了拍卖的分配结果能够降低拥挤空域的总延误成本。最后，根据实验数据分析了通过按拥挤航班数量比例重分配拍卖收入给航空公司方案，该方案能有效地降低航空公司的航班延误损失，提高空域资源分配的公平性和有效性。

关键词：空中交通管制；资源分配；密封价格拍卖；空域；流量限制空域；博弈分析

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