TWO-DIMENSIONAL STOCHASTIC AIRFOIL OPTIMIZATION DESIGN METHOD BASED ON NEURAL NETWORKS

Lin Yu, Wang Heping, Peng Runyan

(School of Aeronautics, Northwestern Polytechnical University, Xi'an, 710072, P. R. China)

Abstract: To avoid the aerodynamic performance loss of airfoil at non-design state which often appears in single point design optimization, and to improve the adaptability to the uncertain factors in actual flight environment, a two-dimensional stochastic airfoil optimization design method based on neural networks is presented. To provide highly efficient and credible analysis, four BP neural networks are built as surrogate models to predict the airfoil aerodynamic coefficients and geometry parameter. These networks are combined with the probability density function obeying normal distribution and the genetic algorithm, thus forming an optimization design method. Using the method, for GA(W)-2 airfoil, a stochastic optimization is implemented in a two-dimensional flight area about Mach number and angle of attack. Compared with original airfoil and single point optimization design airfoil, results show that the two-dimensional stochastic method can improve the performance in a specific flight area, and increase the airfoil adaptability to the stochastic changes of multiple flight parameters.

Key words: stochastic airfoil optimization; surrogate model; neural network; uncertain factor; genetic algorithm

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INTRODUCTION

Along with the development of airfoil design technology, many design methods have been investigated, including aerodynamic design by optimization[1-2] and shape parameterization[3-5]. It becomes regular to design new airfoils that match different mission requests. In the process of airfoil numerical design optimization, the optimal solution of airfoil shape is often searched for a certain design point (like mid-cruise). Airfoil shape obtained through this method usually has good aerodynamic performance at the design point, but aircraft actually flights within an operation area. So the optimization running for a single point often brings an evident performance loss when it departures from the design point, which is obviously not expected. Multi-point design provides a tool for this problem. It can mitigate the performance loss at non design state, and has been used in many fields like low speed airfoil design[6], transonic airfoil design[7], reflex airfoil design[8], and so on. The increases of design points are restricted after all, so the multi-point design is hard to completely solve the performance loss at non design state.

Stochastic optimization provides a further way to solve this problem. The optimization design runs for a whole flight area, instead of some design points, and makes the airfoil have better adaptability to actual flight environment, like the design application in drag reduction of transonic airfoil about specific range of Mach number[9-11].

At present, airfoil stochastic optimization design mainly focuses on one stochastic variable[9-11], which can be seen as a one-dimensional stochastic optimization. However, during the actual flight, there is often more than one uncertain factor. One-dimensional stochastic optimization can lack of adaptability to the changes of multi factors, while the design process accounting for multi stochastic variables can provide a better match for airfoil actual application. Also, considering the huge cost of aerodynamic analysis in

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E-mail: adf20001982@126.com

stochastic optimization, it is necessary to find a highly efficient and credible computation method to reduce the analysis cost a lot. In this paper, a multidimensional stochastic optimization method based on neural networks is presented and implemented within a validation process for the redesign of GA(W)-2 airfoil.

1 MULTIDIMENSIONAL STOCHASTIC OPTIMIZATION

As a result of influences caused by uncertain factors, actual flight of aircraft often changes around its design point, for example, the change may appear at Mach number, angle of attack or other parameters. Current stochastic optimization design mainly for Mach number may cause the airfoil have good adaptability to the change of Mach number, but lack of consideration about changes of other flight parameters.

Multidimensional stochastic optimization method can account for this problem. From the point of mathematics, two-dimensional model is a basic model of multidimensional stochastic optimization, so a two-dimensional mathematic model of stochastic optimization is presented and used in the stochastic optimization design of GA (W)-2 airfoil. The higher-dimensional design application of stochastic optimization can be further extended from this model.

One kind of mathematic description of one-dimensional stochastic optimization problem is to minimize the mathematic expectation of object function in the change range of stochastic variable, like the drag minimization about specific range of Mach number, so we have

$$\begin{cases} \min[\int_{Ma} c_d(\boldsymbol{X}, Ma) \times p(Ma)\mathrm{d}Ma] \\ \text{s.t.} \\ \boldsymbol{X} \in \boldsymbol{D} \\ Ma_{\min} \leqslant Ma \leqslant Ma_{\max} \\ g_i(\boldsymbol{X}, Ma) \leqslant 0 \qquad i = 1,2,\cdots,m \\ h_j(\boldsymbol{X}, Ma) = 0 \qquad j = 1,2,\cdots,n \end{cases} \tag{1}$$

where $\boldsymbol{X}$ is the design variable, $\boldsymbol{D}$ the design space, Mach number Ma the stochastic variable, p the probability density distribution about Ma, c_d the object function, and $g_i(\boldsymbol{X}, Ma)$, $h_j(\boldsymbol{X}, Ma)$ are the inequalities and equalities design constraints.

Through the specific range of Ma, the minimal mathematics expectation of c_d is obtained, which makes c_d have good adaptability to the change of Ma. By extending one-dimensional mathematic model, we can obtain a mathematic description of two-dimensional stochastic optimization, that is

$$\begin{cases} \min[\iint_{t_1, t_2} f(\boldsymbol{X}, t_1, t_2) \times p(t_1, t_2)\mathrm{d}t_1\mathrm{d}t_2] \\ \text{s.t.} \\ \boldsymbol{X} \in \boldsymbol{D} \\ t_{1\min} \leqslant t_1 \leqslant t_{1\max} \\ t_{2\min} \leqslant t_2 \leqslant t_{2\max} \\ g_i(\boldsymbol{X}, t_1, t_2) \leqslant 0 \qquad i = 1,2,\cdots,m \\ h_j(\boldsymbol{X}, t_1, t_2) = 0 \qquad j = 1,2,\cdots,n \end{cases} \tag{2}$$

where t_1 and t_2 are the stochastic variables, p is the two-dimensional probability density distribution about t_1 and t_2.

The change from one-dimensional model to multidimensional model greatly improves the adaptability of object function to the environment. In this paper, the two stochastic variables are chosen, i. e., Ma and angle of attack α. The central point of flight area is at $Ma=0.55$, $\alpha=0$. Around this central point, ranges of Ma and α are enlarged, and a two-dimensional flight area of airfoil is obtained, where $Ma \in [0.5, 0.6]$, $\alpha \in [-1.5°, 1.5°]$.

2 GEOMETRY PARAMETERIZATION OF AIRFOIL

There are several methods[3-5] for geometry parameterization of airfoil, including bump function method, PARSEC method and others. The bump function method expresses new foil geometry as the linear combination of a basis airfoil and a set of perturbation functions. The coefficients of perturbation functions are geometry design variables, as shown below

$$y(x) = y_0(x) + \sum_{k=1}^{n} \alpha_k f_k(x) \tag{3}$$

where $y_0(x)$ is the basis airfoil, $f_k(x)$ the bump function and α_k the design variable. Hicks-Henne function is used as bump function, that is

$$f_k(x)=\begin{cases}x^{0.25}(1-x)\mathrm{e}^{-20x} & k=1\\ \sin^3(\pi x^{e(k)}) & k>1\end{cases} \tag{4}$$

where $e(k)=\dfrac{\log 0.5}{\log x_k}$, $0\leqslant x_k\leqslant 1$.

The original foil used in design optimization is GA(W)-2 airfoil, with total 89 points distributed at both surface of airfoil. There are five bump functions distributed on each surface, which makes ten geometry design variables for this problem.

3 CONSTRUCTION OF NEURAL NETWORK

In design practice, huge computation cost of numerical optimization of aerodynamic shape is an important restriction. Under this restriction, the number of design points for multi-point design method is limited. Stochastic optimization needs to compute one or even multiple integral about object function, which makes the computation cost increase at orders of magnitude. This situation makes the highly efficient aerodynamic analysis become very attractive.

Due to the ability of fast computation, surrogate model has already been used in design works, like prediction of aircraft design coefficients[12-13], airfoil design[14-15], aerodynamic optimization[16] and so on. Compared with other response surface methods based on algebraic polynomial, neural network does not depend on accurate mathematic expression, while it can learn from input sample data and have well approximating ability for nonlinear problem. The combination of neural network and stochastic optimization method provides feasibility for multidimensional stochastic airfoil optimization design.

For two-dimensional stochastic optimization in this paper, BP neural network is used to build surrogate model to provide highly efficient analysis. Four independent BP network models are built to simulate lift coefficient C_L, lift-to-drag ratio L/D, pitching moment coefficient C_m, and thickness-to-chord ratio t/c of GA(W)-2 airfoil within its two-dimensional flight area.

3.1 Neural network model

(1) Lift prediction

For lift coefficient C_L, a two-layer network is used, including geometry design variables $\boldsymbol{X}$, Ma, α as inputs, and C_L as output. The two-layer network for C_L is shown in Table 1.

Table 1 Two-layer network for C_L

Layer	Hidden layer-1	Output layer
Neuron	10	1
Transfer function	tansig	Pure linear

(2) Drag prediction

Prediction of lift-to-drag ratio L/D is more difficult than those of other three parameters. First, a multi-layer neural network is established to predict drag coefficient C_D. It has achieved a well prediction precision, with correlation coefficient R of 0.996 9. Most of prediction errors about C_D are within ± 2 drag units. The training result of C_D is shown in Fig. 1.

Unfortunately, in the following optimization, it is found that due to small value of C_D, prediction of L/D through the ratio between lift and drag coefficients, i.e., $L/D=C_L/C_D$, is sensitive to the prediction errors of C_L and C_D, which even considerably affect the optimization result. So, instead of predicting C_D, it prefers to predict L/D directly. A three-layer network is used for this. The inputs are geometry design variables $\boldsymbol{X}$, Ma, α, and the output is L/D. The three-layer network is shown in Table 2.

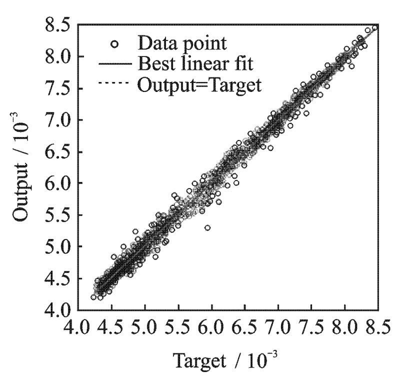

Fig. 1 Training results of C_D

Table 2 Three-layer network for L/D

Layer	Hidden layer-1	Hidden layer-2	Output layer
Neuron	20	10	1
Transfer function	logsig	tansig	Pure linear

(3) Pitching moment prediction

Network structure for pitching moment coefficient C_m is same to that for C_L, and the output is C_m about 1/4 chord. The two-layer network for C_m is same to Table 1.

(4) Airfoil thickness prediction

A two-layer network is used for thickness-to-chord ratio t/c, including geometry design variables $\boldsymbol{X}$ as input, and t/c as output. The two-layer network for t/c is also same to Table 1.

3.2 Sampling and Training

To predict aerodynamic coefficients and thickness accurately, a set of sample data is needed to train the four neural networks established before. This data set is generated according to the design of experiment (DOE) technique named Latin hypercube sampling (LHS). DOE technology improves the quality of data set during sampling, and LHS makes the data set have a reasonable spatial distribution and a sufficient coverage of the sampling space. Under acceptable computation cost, 1 500 samples are generated in the corresponding sampling space, and then analyzed by a subsonic airfoil code XFOIL[17]. As a panel code, XFOIL is widely used in design and analysis of subsonic airfoil, and can provide credible prediction for this kind of problem. Range of each parameter is shown in Table 3.

Table 3 Training set

Parameter	Minimum value	Maximum value
Ten geometry variables	−0.000 3	0.000 3
$\alpha/(°)$	−1.5	1.5
Ma	0.5	0.6

Table 4 shows training results for the four neural networks. It can be seen that for C_L, C_m and t/c, correlation coefficient R is all above 0.99. For L/D, although there are some small departures between the neural network predictions and the samples, the result is still acceptable. And to predict L/D directly is better than the way of $L/D=C_L/C_D$. These four established neural networks provide suitable surrogate models for the following optimization.

Table 4 Training results

Neural network	R
Neural network for C_L	0.999 91
Neural network for L/D	0.983 69
Neural network for C_m	0.998 76
Neural network for t/c	0.999 98

4 TWO-DIMENSIONAL STOCHASTIC OPTIMIZATION

The best way to build probability density distribution is to collect lots of actual flight data of aircraft, but they are often hard to obtain. In natural and social phenomenon, lots of stochastic variables obey or approximately obey normal distribution. So for GA(W)-2 airfoil, we suppose that the probability density p obeys a two-dimensional normal distribution within the two-dimensional flight area about Ma and α, and the peak value of p locates at the central point of flight area, as shown in Fig. 2.

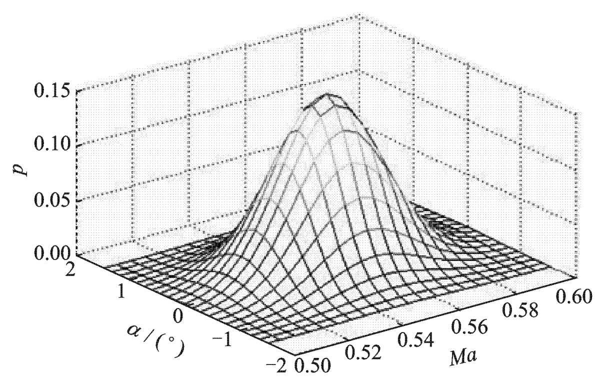

Fig. 2 Probability distribution of Ma and α

$$p(Ma,\alpha) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \times \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(Ma-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(Ma-\mu_1)(\alpha-\mu_2)}{\sigma_1\sigma_2} + \frac{(\alpha-\mu_2)^2}{\sigma_2^2}\right]\right\} \tag{5}$$

where $\mu_1 = 0.55, \mu_2 = 0, \rho = 0, \sigma_1 = \sigma_2 = 1$.

Optimization object is to make the L/D performance of airfoil have the best adaptability to the changes of Ma and α. The four neural networks established before are used to predict aerodynamic coefficients and airfoil thickness. Several design constraints are set at the central point of flight area, about the airfoil point aerodynamic performance T_{pt}. The area aerodynamic performance T_{area} is used to estimate airfoil adaptability to Ma and α, shown as

$$T_{\mathrm{area}} = \iint_{Ma,\alpha} \frac{L}{D}(\boldsymbol{X}_{\mathrm{g}}, Ma, \alpha) \times p(Ma, \alpha)\mathrm{d}Ma\mathrm{d}\alpha \tag{6}$$

where $\boldsymbol{X}_{\mathrm{g}}$ is geometry design variable. So the optimization problem becomes

$$\begin{cases} \max(T_{\mathrm{area}}) \\ \text{s.t.} \\ t/c \geqslant 0.125 \\ C_{\mathrm{L,pt}} \geqslant 0.60 \\ C_{\mathrm{m,pt}} \geqslant -0.13 \end{cases} \tag{7}$$

where subscri "pt" refers to airfoil point aerodynamic performance at the central point.

Genetic algorithm (GA) is used as optimization algorithm. As an artificial intelligence technology, GA simulates the evolution process of biology population under the natural environment, and forms global optimization ability. It dose not depend on the computation of grads information, and has a broad flexibility to design practice. Here the size of population is set to 150, with a crossover fraction of 0.8, and elite count of 3. To speed up computation, three CPUs of four-core processors are used for parallel computing under MATLAB environment. Flowchart of two-dimensional stochastic optimization combined with neural networks and GA is shown in Fig. 3.

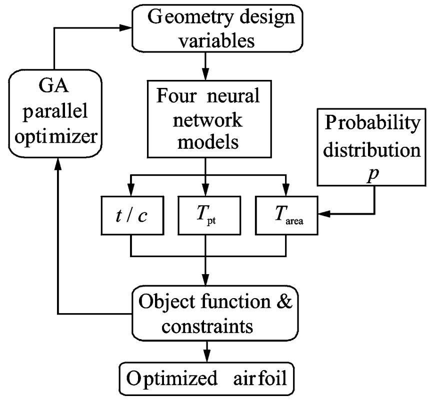

Fig. 3 Flowchart of optimization

5 RESULT AND ANALYSIS

As a comparison, while running the two-dimensional stochastic optimization, another single point design optimization is also running for the central point of flight area, using the same GA settings. At the central point, through XFOIL analysis, Table 5 shows the results of original GA(W)-2 airfoil, single point design optimization airfoil and two-dimensional stochastic optimization airfoil, here $Ma=0.55$ and $\alpha=0$.

Table 5 Performance comparison at flight area central point

Airfoil	$C_{\mathrm{L,pt}}$	L/D_{pt}	$C_{\mathrm{m,pt}}$	$t/c/\%$
GA(W)-2	0.631 9	124.4	−0.137 5	12.9
Single point design	0.629 4	139.2	−0.129 6	13.2
2-D stochastic	0.623 7	136.7	−0.127 3	13.3

At the central point, it can be seen that both optimizations satisfy the design constraints, and improve airfoil performance to some extent. For single point design, the airfoil has the best L/D performance, which increases by 11.9% compared with GA (W)-2. For two-dimensional stochastic optimization, L/D increases by 9.9%, which means a good aerodynamic performance at the max flight probability position. Fig. 4 shows geometries of three airfoils.

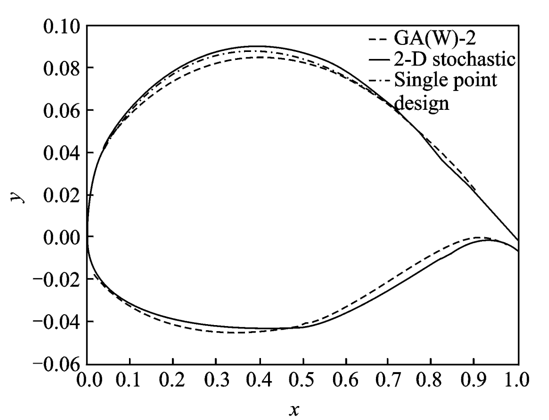

Fig. 4 Geometries of three airfoils

Using XFOIL analysis, Figs. 5, 6 compare area aerodynamic performance among the three airfoils, where $Ma \in [0.5, 0.6]$, $\alpha \in [-1.5°, 1.5°]$.

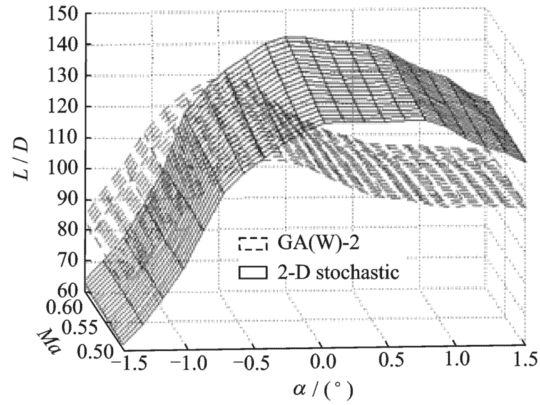

Fig. 5 Comparison of area performance between GA(W)-2 and 2-D stochastic airfoils

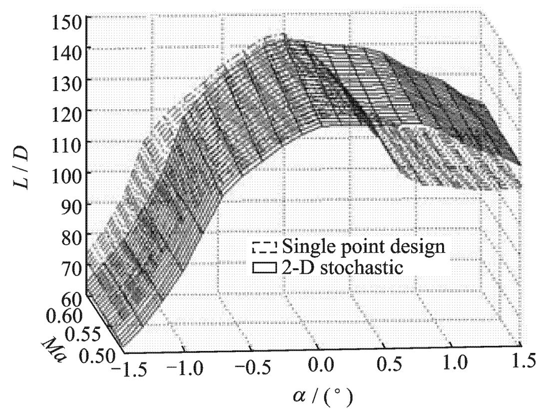

Fig. 6 Comparison of area performance between single point design and 2-D stochastic airfoils

Compared with original GA(W)-2 airfoil, the two-dimensional stochastic optimization efficiently improves the L/D performance in the whole flight area, makes T_{area} increase by 12.5%, and makes the airfoil performance better match high probability flight area. Compared with single point design result, the two-dimensional stochastic optimization method can effectively avoid the performance loss at non design state which exists at single point design airfoil, make a more harmonious performance in the whole flight area, and enhance airfoil adaptability to Ma and α. The optimization results also show that, stochastic optimization based on neural network surrogate model can obtain favorable optimization result under suitable conditions.

6 CONCLUSION

Based on highly efficient analysis of neural networks and global search ability of genetic algorithm, a design method for two-dimensional stochastic airfoil optimization is presented, and validated through the optimization for GA(W)-2 airfoil. The performance comparisons among three airfoils show that the two-dimensional stochastic optimization method can obtain a whole performance improvement in specific flight area, trade off airfoil aerodynamic performance between high and low probability areas in flight, match the airfoil performance to the mission requirement, and enhance airfoil adaptability to stochastic changes of multiple flight parameters.

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基于神经网络的二维随机翼型优化设计方法

林 宇 王和平 彭润艳

（西北工业大学航空学院，西安，710072，中国）

摘要：为避免翼型单点优化设计存在的非设计状态气动性能损失，改进对实际飞行环境中不确定性因素的适应能力，提出了基于神经网络的二维随机翼型优化设计方法。采用4个BP神经网络作为代理模型，用于预测翼型的气动系数与几何参数，以提供高效和可靠的分析。同时联合服从正态分布的概率密度函数和遗传算法构成了优化设计方法。采用该方法，对GA(W)-2翼型，在关于马赫数和迎角的二维飞行区域内进行了随机优化设计。通过与原始翼型和单点优化设计翼型的结果对比，表明该二维随机优化方法能够在指定飞行区域内改进翼型的整体性能，提高了翼型对多个飞行参数随机变化的适应能力。

关键词：随机翼型优化；代理模型；神经网络；不确定因素；遗传算法

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