

# OPTIMIZATION DESIGN METHOD FOR INPUT IMPEDANCE MATCHING NETWORK OF LOW NOISE AMPLIFIER

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**Abstract:** According to the theories of optimal noise match and optimal power match, a method for calculating the optimal source impedance of low noise amplifier(LNA) is proposed based on the input reflection coefficient  $S_{11}$ . Moreover, with the help of Smith chart, the calculation process is detailed, and the trade-off between the lowest noise figure and the maximum power gain is obtained during the design of LNA input impedance matching network. Based on the Chart 0.35- $\mu\text{m}$  CMOS process, a traditional cascode LNA circuit is designed and manufactured. Simulation and experimental results have a good agreement with the theoretical analysis, thus proving the correctness of theoretical analysis and the feasibility of the method.

**Key words:** low noise amplifier; power match; noise match; Smith chart

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## INTRODUCTION

As illustrated in the "analog design octagon", the design of amplifiers is full of compromises<sup>[1]</sup>, especially for a low noise amplifier (LNA), which is the first active stage in a receiver. LNA must operate at the same high frequency as carriers and must achieve impedance matching, low noise figure (NF), high power gain, good linearity and so on. Therefore, it is necessary to trade these parameters each other during the LNA design, which also becomes its design difficulties.

According to the reported papers, many LNA circuits have been designed for different systems<sup>[2-5]</sup>, and some optimization techniques have been considered. In Ref. [6], for example, a T-type input matching network was used to achieve a wideband input impedance matching. A novel noise optimization technique for inductively de-generated CMOS LNA was presented in Ref. [7]. And in Ref. [8], a design method us-

ing mathematics optimization technology was proposed to obtain optimal transistor sizes and component values in input matching network and load network. For a certain LNA, a power-constrained optimization of simultaneous noise match and input match was given in Ref. [9]. Different from those previously reported works, a convenient method based on Smith chart is advanced in this paper to obtain the optimal source impedance for lower NF and the higher gain.

## 1 OPTIMAL NOISE MATCH AND POWER MATCH

As we all know, in the design of LNA, there exist an optimal source impedance to provide the lowest noise figure and an optimal source impedance to provide a maximum power gain. However, these two optimal source impedances are not equal, in other words, the optimal noise match and the power match cannot be achieved simultaneously. Therefore, a trade-off should be

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made between them. After a brief review of optimal noise match and optimal power match, a method for calculating the optimal source impedance is proposed to achieve the lowest noise figure and the maximum power gain under the constraint of input reflection coefficient  $S_{11}$ .

### 1.1 Optimal noise match

A typical equivalent representation of a noisy two-port is shown in Fig. 1, where  $Z_s$  is the impedance of input source,  $Z_L$  the load impedance,  $Z_{in}$  the input impedance of the two-port network,  $v_n^2$  and  $i_n^2$  are the equivalent input noise voltage and current respectively,  $v_s$  is the voltage source and  $\overline{v_{ns}^2}$  the noise voltage source. The noise factor ( $F$ ) in terms of  $F_{min}$  and the source admittance is given by<sup>[10]</sup>

$$F = F_{min} + \frac{R_n}{G_s} [(G_s - G_{opt})^2 + (B_s - B_{opt})^2] \quad (1)$$

where  $F_{min}$  is the minimum noise factor,  $R_n$  the equivalent resistance,  $G_s$  and  $B_s$  are the conductance and the susceptance of the input source, while  $G_{opt}$  and  $B_{opt}$  the conductance and the susceptance of the input source at the minimum noise factor, respectively. The noise figure is the simple noise factor expressed in dB ( $NF = 10 \log F$ ).

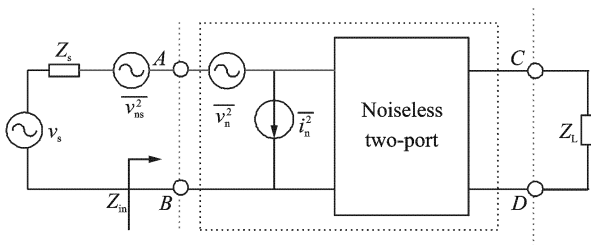


Fig. 1 Equivalent representation of noisy two-port

The expression in Eq. (1) means that the contours of a constant noise figure are circles centered about  $(G_{opt}, B_{opt})$  on a Smith chart. Fig. 2 shows an example of constant noise figure circles plotted on a Smith chart. Some important conclusions can be drawn as follows: The centers of all constant noise circles locate along a line drawn from the origin of the Smith chart to the point  $Y_{opt}$  ( $Y_{opt} = G_{opt} + jB_{opt}$ ). For the smaller noise figure, the center of noise figure circle is farther to the o-

rigin and the radius of the circle is smaller. When  $F = F_{min}$ , the constant noise figure circle becomes a point and  $Y_s = G_{opt} + jB_{opt}$ . It means that if a matching network transfers the source impedance  $Y_s$  to  $Y_{opt}$ , the circuit is in the state of best noise match. It is called the optimal noise match, so  $Y_{opt}$  is the optimal noise source admittance and  $Z_{opt}$  the optimal noise source impedance.

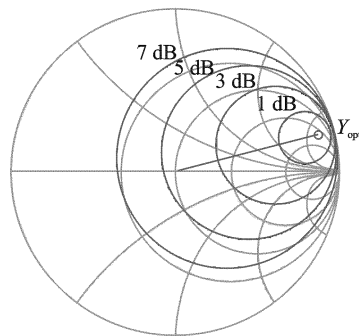


Fig. 2 Bunch of constant noise figure circles

### 1.2 Optimal power match

The optimal power match for LNA means that the input impedance and the source impedance should be conjugate matched. In other words, the input matching network of LNA circuit should transform the input impedance to  $50 \Omega$  or transform  $50 \Omega$  to the conjugate impedance of input. For there always exists a certain mismatch in real systems, the return loss, or  $S_{11}$ , is used to specify this deviation. As shown in Fig. 2,  $S_{11}$  can be expressed in terms of impedance<sup>[11]</sup> as

$$|S_{11}| = \left| \frac{Z_{in} - Z_s^*}{Z_{in} + Z_s} \right| \quad (2)$$

When  $Z_s = Z_{in}^*$ ,  $S_{11}$  has the minimum value and  $Z_{in}^*$  is called the optimal power source impedance. Substituting  $Z_{in} = R_{in} + jX_{in}$  and  $Z_s = R_s + jX_s$  into Eq. (2), we have

$$|S_{11}|^2 = \frac{(R_{in} - R_s)^2 + (X_{in} + X_s)^2}{(R_{in} + R_s)^2 + (X_{in} + X_s)^2} \quad (3)$$

so that

$$R_{in}^2 - 2R_{in}R_s \left( \frac{1 + |S_{11}|^2}{1 - |S_{11}|^2} \right) + R_s^2 + (X_{in} + X_s)^2 = 0 \quad (4)$$

The expression in Eq. (4) denotes a set of constant  $S_{11}$  circles on the Smith chart, as shown in Fig. 3, where  $Z_s = R_s + jX_s$  is set as the variable. Similar to a constant noise figure circle and

its conclusions, the centers of all constant  $S_{11}$  circles locate along a line drawn from the origin of the Smith chart to the point  $Z_{in}^*$  ( $Z_{in}^* = R_{in} - jX_{in}$ ). For the smaller  $S_{11}$ , the center of the circle is farther to the origin and the radius of the circle is smaller. In other words, when the source impedance is closer to the optimal power source impedance  $Z_{in}^*$ , the  $S_{11}$  is smaller.

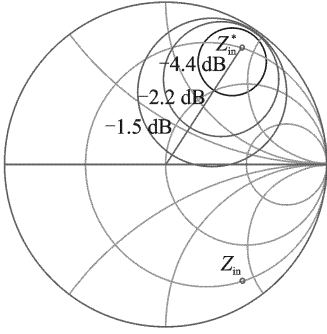


Fig. 3 Bunch of constant  $S_{11}$  circles

### 1.3 Trade-off between optimal noise match and power match

As mentioned in previous section, the noise match or the power match can be achieved by moving source impedance to the optimal noise source impedance or the optimal power source impedance. Then, there must be a suitable source impedance to trade power match with noise match<sup>[12]</sup>. At specified  $S_{11}$ , a method for calculating this source impedance is introduced as follows.

According to the location of optimal noise source impedance on the Smith chart, there exist two cases.

(1) Optimal noise source impedance is in the constant  $S_{11}$  circle.

As shown in Fig. 4, when the optimal noise source impedance  $Z_{opt}$  is in the constant  $S_{11}$  circle, which is the ideal condition, the LNA circuit can meet the  $S_{11}$  specification while achieving minimum noise figure. In this case, there is no need for compromise, just matching the source impedance to the optimal noise source impedance directly.

(2) Optimal noise source impedance is out of constant  $S_{11}$  circle.

As an example, assume that the required  $S_{11}$

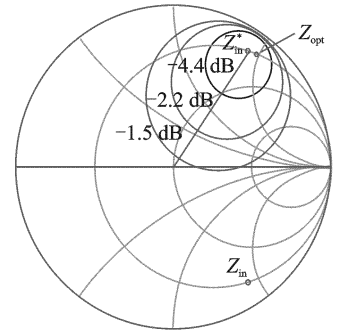


Fig. 4  $Z_{opt}$  in constant  $S_{11}$  circle

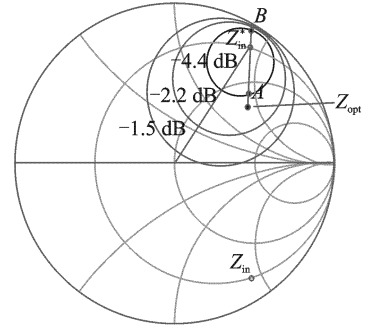


Fig. 5  $Z_{opt}$  out of constant  $S_{11}$  circle

is  $-4.4$  dB and the optimal power source impedance is  $Z_{in}^*$ , while the optimal noise source impedance  $Z_{opt}$  is out of  $-4.4$  dB circle, as shown in Fig. 5. By drawing a straight line between  $Z_{in}^*$  and  $Z_{opt}$ , there exist two points, denoted by  $A$  and  $B$ , where the line intersects the contour of  $S_{11}$  circle with the constant value  $-4.4$  dB. As we can see, point  $A$  is closer to  $Z_{opt}$ , so it is the point of lowest noise figure at this  $S_{11}$  circle. By moving source impedance to point  $A$ , the optimal noise match is achieved at this specified  $S_{11}$ . A detailed calculation process is shown as follows.

The input impedance

$$Z_{in} = R_{in} + jX_{in} \quad (5)$$

The optimal power source impedance

$$Z_{in}^* = R_{in} - jX_{in} \quad (6)$$

The optimal noise source impedance

$$Z_{opt} = R_{opt} + jX_{opt} \quad (7)$$

The source impedance

$$Z_s = R_s + jX_s \quad (8)$$

The linear equation connected  $Z_{in}^*$  and  $Z_{opt}$

$$X_s = \frac{-X_{in} - X_{opt}}{R_{in} - R_{opt}} \times (R_s - R_{opt}) + X_{opt} \quad (9)$$

The circle equation of constant  $S_{11}$

$$|S_{11}|^2 = \frac{(R_{in} - R_s)^2 + (X_{in} + X_s)^2}{(R_{in} + R_s)^2 + (X_{in} + X_s)^2} \quad (10)$$

The distance from the cross point to the optimal noise source impedance is

$$d^2 = (R_s - R_{opt})^2 + (X_s - X_{opt})^2 \quad (11)$$

From Eqs. (9,10), the impedances at point A ( $Z_A = R_A + jX_A$ ) and point B ( $Z_B = R_B + jX_B$ ) can be solved respectively. Then, using Eq. (11), the distance can be calculated. The point which has the closer distance is the desired solution. With the help of mathematical tools, the calculate process is easier and faster. Mathcad, one of engineering calculation software, is used in the design.

## 2 DESIGN EXAMPLE

Based on the Chart 0.35- $\mu\text{m}$  CMOS process, a cascode LNA circuit without input/output matching network is designed to verify the method. The total die area is  $368 \mu\text{m} \times 554 \mu\text{m}$ . Fig. 6 shows its schematic and micrograph with test probes. And its on-chip measured S-parameters are shown in Fig. 7.

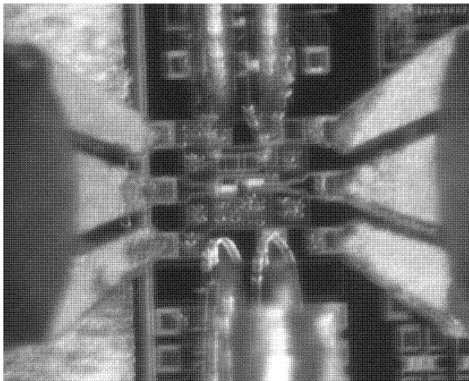
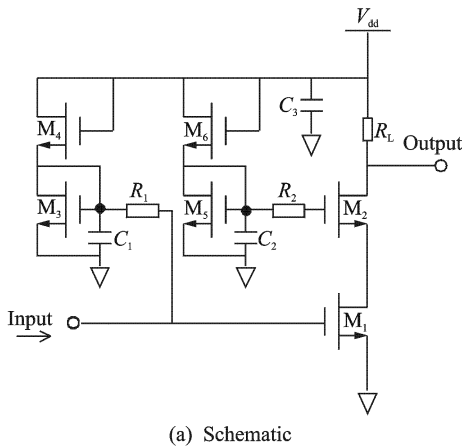


Fig. 6 Cascode LNA without input/output matching network

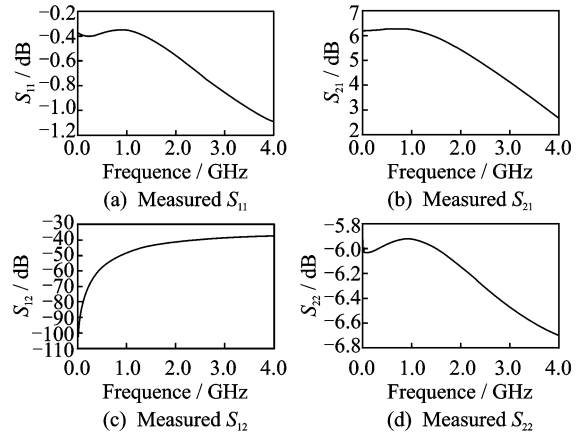


Fig. 7 Measured S-parameters of LNA without matching network

### 2.1 Calculation of required optimal source impedance

In this work, the designed LNA circuit is aimed at UHF RFID systems, and the centre operation frequency is 922.5 MHz. After an SP simulation in Cadence Spectre at 922.5 MHz, we have:

The input impedance

$$Z_{in} = 74.533 - j253.182 \Omega$$

The equivalent resistance

$$R_n = 34.281 \Omega$$

The optimal conductance of input source

$$G_{opt} = 0.664 \text{ ms}$$

The optimal susceptance of input source

$$B_{opt} = -1.514 \text{ ms}$$

The minimum noise figure

$$NF = 0.345 \text{ dB}$$

The noise factor

$$F = 2.374$$

Therefore, the optimal power and noise source impedances of this designed LNA at 922.5 MHz are

$$Z_{in}^* = R_{in} - jX_{in} = 74.533 + j253.182 \quad (12)$$

$$Z_{opt} = 1/Y_{opt} = 1/(G_{opt} + B_{opt}) = 242.777 + j553.754 \quad (13)$$

Assuming that the specified  $S_{11} < -10 \text{ dB}$ , from Eqs. (9, 10), we obtain  $Z_A = 53.975 + j216.232$  and  $Z_B = 102.921 + j304.206$ . Further, from Eq. (11), we have

$$d_A^2 = (R_A - R_{opt})^2 + (X_A - X_{opt})^2 = 1.49 \times 10^5 \quad (14)$$

$$d_B^2 = (R_B - R_{opt})^2 + (X_B - X_{opt})^2 = 8.18 \times 10^4 \quad (15)$$

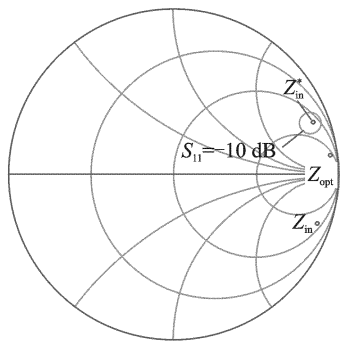
As we can see,  $d_B^2 < d_A^2$ , the cross point  $Z_B$  is the required optimal source impedance. The noise figures and the corresponding  $S_{11}$  at different source impedances are simulated in Cadence Spectre and the results are listed in Table 1.

**Table 1 NF and  $S_{11}$  at different source impedances**

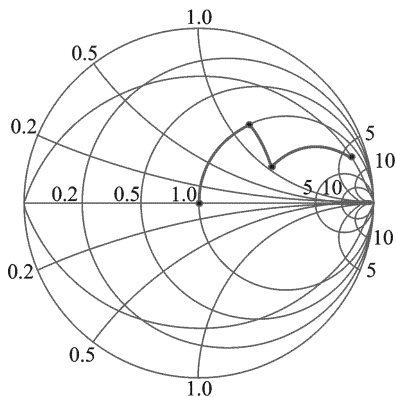
Source impedance	Simulated NF/dB	Simulated $S_{11}$ /dB
$Z_{in}^* = 74.533 + j253.182$ (optimal power match)	0.915	-40.00
$Z_{opt} = 242.777 + j553.754$ (optimal noise match)	0.345	-2.10
$Z_B = 102.921 + j304.206$ (trade-off match)	0.646	-10.01

## 2.2 Design of input impedance matching network

According to the calculated results, the input impedance matching process is to pull  $Z_B$  to  $50 \Omega$  or to pull  $50 \Omega$  to  $Z_B$ . There are many different topologies of matching network, the T-type is chosen in this design and a matching sketch is shown in Fig. 8.



(a) Parameters located on Smith chart



(b) Locus of matching process

Fig. 8 Sketch of input impedance matching network

Based on the on-chip measured S-parameters of LNA without matching network (Fig. 7) and the model parameters of passive components (inductors and capacitors) from Murata Manufacturing Co. Ltd, the input and output impedance matching networks are accomplished in ADS and the experimental results are shown in Figs. 9, 10. We can see that at 922.5 MHz, NF is 0.656 dB (point  $m_2$ ) and  $S_{11}$  is -13.094 dB (point  $m_1$ ). Here, the value of NF is 0.01 dB higher than the one listed in Table 1 while  $S_{11}$  is 3 dB lower.

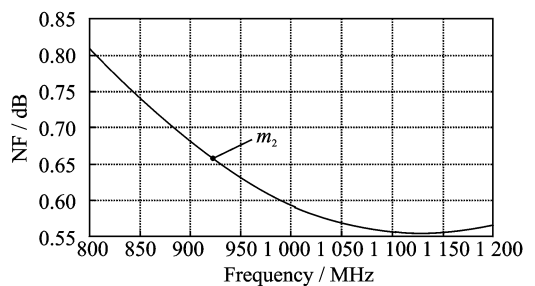


Fig. 9 Curve of noise figure

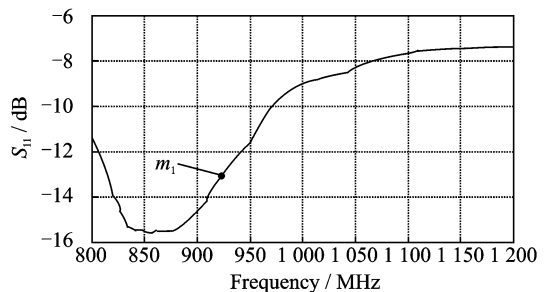


Fig. 10 Curve of  $S_{11}$

## 3 CONCLUSION

A clearer and more straightforward understanding of optimal noise match and optimal power match is presented. With the help of Smith chart, the detailed calculation of desired source impedance is advanced for trade-off match at specified  $S_{11}$ . Based on Chart 0.35- $\mu\text{m}$  CMOS process, LNA for UHF RFID systems is designed and its simulation and experimental results are analyzed. Finally the design example proves the feasibility of the calculation method.

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## 低噪声放大器输入阻抗匹配网络设计优化方法

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**摘要:**通过分析电路的最佳噪声匹配和最佳功率匹配,研究了根据给定的输入反射系数( $S_{11}$ )计算低噪声放大器(LNA)最佳源阻抗的方法。借助史密斯圆图给出了详细的计算过程,实现了LNA输入匹配网络设计时的最佳噪声和最佳功率之间的折衷匹配。基于Chart 0.35- $\mu\text{m}$  CMOS工艺完成了一种cascode结构LNA的核心电路设计,仿真和

实验结果与理论分析相吻,表明了理论分析的正确性和方法的可行性。

**关键词:**低噪声放大器;功率匹配;噪声匹配;史密斯圆图  
**中图分类号:**TN722.3

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