

# DUAL QUATERNION CURVE INTERPOLATION ALGORITHM FOR FORMATION SATELLITES

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**Abstract:** The traditional algorithms for formation flying satellites treat the satellite position and attitude separately. A novel algorithm combining satellite attitude with position is proposed. The principal satellite trajectory is obtained by dual quaternion interpolation, then the relative position and attitude of the deputy satellite are obtained by dual quaternion modeling on the principal satellite. Through above process, relative position and attitude are unified. Compared with the orbital parameter and the quaternion methods, the simulation result proves that the algorithm can unify position and attitude, and satisfy the precision requirement of formation flying satellites.

**Key words:** formation satellites; dual quaternion; interpolation algorithm; relative position and attitude

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## INTRODUCTION

Compared with traditional satellites, the micro-satellite gives much better performance with lower weight and cost, shorter developing and manufacturing cycles<sup>[1]</sup>. But it also poses many shortcomings such as bad performance in diversity and precision. Thus formation flying is introduced to satellites maneuver for the purpose of overcoming above mentioned shortcomings<sup>[2]</sup>. Such applications can be seen in distributed radar, electronic reconnaissance, 3-D imaging and space interferometer as well. However the bunching problem causes a sharp rise of complexity in flight maneuver. The difficulty expands with the increase of satellite number. Position and attitude information becomes very vital in solving the above mentioned problem.

The most straightforward way to obtain relative position and attitude information of formation

flying satellite is using differential technique of the absolute position and attitude information. The relative position and attitude information gathered by such technique is low precision. The recent researches incline utilizing GPS or GPS-like methods<sup>[3-5]</sup>, or are based on vision meterage technique<sup>[6-7]</sup>. And all separate satellite trajectory and attitude information into translation and rotation. Using vector and direction-cosine matrix or quaternion respectively to solve translation and rotation. This isolated technique increases the possibility of coupling error of translation and rotation, which certainly adds system complexity.

The trajectories of moving objects can be found by interpolation of given positions (points and orientations). Under such assumption, a method utilizing dual quaternion curve is used to interpolate the trajectories of formation flying satellites. At first, the mathematics model of dual quaternion curve is constructed. The trajectory

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of principal satellite can be found by interpolating method such as translation-rotation (TR) or translation-rotation-translation (TRT). Then, the relative position and attitude information of deputy satellite can be found by working out the dual quaternion kinematics equations.

## 1 TRADITIONAL SATELLITE POSITION AND ATTITUDE ALGORITHMS

### 1.1 Orbital parameter method

Orbital parameter method utilizes the number of principal and deputy satellite orbits to get the relative position and velocity information. Suppose that orbital parameters  $a, e, i, \Omega, \omega, f, E$ , and  $M$  denote the long semi-axis, the eccentricity, the orbit inclination angle, the longitude ascending node, the perigee angular, the true anomaly, the eccentric anomaly, and the mean anomaly respectively,  $u = \omega + f$  is the latitude angular. Therefore, under principal satellite frame, the relative position and velocity of deputy satellite are

$$\begin{cases} x = -ae_m \cos(nt - \omega_m) \\ y = 2ae_m \sin(nt - \omega_m) \\ z = a[-\Delta\Omega \sin i_M \cos(nt) + \Delta i \sin(nt)] \end{cases} \quad (1)$$

$$\begin{cases} \dot{x} = -ane_m \sin(nt - \omega_m) \\ \dot{y} = 2ane_m \cos(nt - \omega_m) \\ \dot{z} = an[\Delta\Omega \sin i_M \sin(nt) + \Delta i \cos(nt)] \end{cases} \quad (2)$$

where subscript  $M$  and  $m$  denote the principal and the deputy satellite systems respectively,  $n$  denotes the average orbital angular velocity. Orbital parameter method can describe relative position of formation flying satellites.

### 1.2 Quaternion method

Quaternion method is brought in as a compensation of the orbital method to depict the relative attitude information. A quaternion is made up of one scalar part and three vector parts, shown as

$$\mathbf{q} = q_0 + q_1i + q_2j + q_3k = [q_0 \quad \mathbf{q}^T]^T \quad (3)$$

where  $q_0$  is the scalar part and  $\mathbf{q}$  the vector part. The four parameters satisfy the following obligation equation

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (4)$$

## 2 MATHEMATICAL MODEL OF DUAL QUATERNION CURVE

Dual quaternion curve is based on the quaternion and dual algebra theory. And the curve is powerful tool for rigid body kinematics, since it can easily describe the combination of both translation and rotation<sup>[8]</sup>, shown as

$$\begin{cases} \hat{\mathbf{q}} = \mathbf{q} + \varepsilon \mathbf{q}' \\ \hat{\mathbf{q}} = \left( \cos \frac{\hat{\theta}}{2}, \hat{\mathbf{l}} \sin \frac{\hat{\theta}}{2} \right) \end{cases} \quad (5)$$

where  $\hat{\mathbf{l}}$  is the rotating axis,  $\hat{\theta}$  the dual angle,  $\hat{\theta} = \theta + \varepsilon d$ , and  $\mathbf{q}'$  the dual part. For the dual quaternion curve  $\hat{\mathbf{q}} = \hat{\mathbf{q}}(t)$ , the corresponding translation and rotation parts are  $\text{trans}(\mathbf{q})$  and  $\text{rot}(\mathbf{q})$ .

## 3 DUAL QUATERNION CURVE INTERPOLATION ALGORITHM OF PRINCIPAL SATELLITE TRAJECTORY

Because of the influence caused by disturbing force etc, the satellite trajectory no longer follows the ideal elliptic routine. In order to achieve precise control and meterage, it is crucial to obtain the trajectory information at anytime.

As mentioned before, a movement is made up of several continuous displacements. Take the  $(m+1)$ th displacement for example<sup>[9]</sup>

$$P_i = \underbrace{(2 + \varepsilon \mathbf{s}_i)}_{\text{trans}(P_i)} * \underbrace{(\mathbf{r}_{i,0} + \mathbf{r}_i)}_{\text{rot}(P_i)} \quad \mathbf{s}_i, \mathbf{r}_i \in \mathbf{R}^3, \mathbf{r}_{i,0} \in \mathbf{R} \quad (6)$$

The  $(m+1)$ th interpolated displacement satisfies the following condition

$$\hat{\mathbf{q}}(t_i) = \lambda_i P_i \quad (7)$$

The real factor  $\lambda_i \neq 0$  is arbitrary, the rotation part can be written as follows

$$\hat{\mathbf{q}}_{\text{rot}}(t_i) = \lambda_i \underbrace{(\mathbf{r}_{i,0} + \mathbf{r}_i)}_{\text{rot}(P_i)} \quad i = 0, \dots, m \quad (8)$$

$R_i^{(0)}$  is assumed to be obtained after a normalization on  $\text{rot}(P_i)$ , that is

$$R_i^{(0)} = \pm \frac{1}{\sqrt{\mathbf{r}_{i,0}^2 + \mathbf{r}_i \circ \mathbf{r}_i}} (\mathbf{r}_{i,0} + \mathbf{r}_i) \quad i = 0, \dots, m \quad (9)$$

The signs of  $\lambda_i$  satisfy

$$\begin{aligned} (R_i^{(0)} - \tilde{R}_{i+1}^{(0)}) * (\tilde{R}_i^{(0)} - \tilde{R}_{i+1}^{(0)}) &\leq (R_i^{(0)} + R_{i+1}^{(0)}) * \\ (\tilde{R}_i^{(0)} + \tilde{R}_{i+1}^{(0)}) \quad i = 0, \dots, m-1 \end{aligned} \quad (10)$$

The sign of the first position  $R_i^{(0)}$  is arbitrary, the remaining signs are determined by Eq. (10). The interpolating condition is obtained by solving Eq. (8), that is

$$\sum_{j=0}^k b_j^k(t_i) C_j = R_i^{(0)} \quad i = 0, \dots, m \quad (11)$$

where  $b_j^k$  denotes the  $k$ th Bernstein polynomial, coefficient  $C_j$  is acquired by working out Eq. (11). Then the interpolating function is obtained.

### 3.1 TR method

The interpolating function expression formula is

$$\hat{\mathbf{q}}_{\text{rot}}(t) = \sum_{j=0}^k b_j^k(t) C_j \quad (12)$$

where coefficient  $b_j$  can be obtained by

$$b_j^k(t) b_j^l(t) = \frac{\binom{k}{j} \binom{l}{j}}{\binom{k+l}{j}} b_j^{k+l}(t) \quad (13)$$

Unify the translation and rotation parts into one dual quaternion  $\mathbf{q}(t)$ <sup>[10]</sup>, that is

$$\begin{aligned} \mathbf{q}(t) &= \mathbf{q}_{\text{trans}}(t) * \mathbf{q}_{\text{rot}}(t) = \\ & \left[ 2 + \epsilon \sum_{j=0}^l b_j^l(t) \mathbf{p}_j \right] * \mathbf{q}_{\text{rot}}(t) \end{aligned} \quad (14)$$

where  $\epsilon$  is the real factor, coefficient  $p_j$  unknown. In order to avoid the polar point, the scalar part is normalized to 2. Then the interpolating condition is

$$\sum_{j=0}^l b_j^l(t_i) \mathbf{p}_j = \mathbf{s}_i \quad (15)$$

The algorithm is as follows:

(1) Normalize the rotation part using Eq. (9), then assign the sign using Eq. (10).

(2) Work out Eq. (11) to obtain  $C_j$ .

(3) Work out Eq. (15) to obtain  $p_j$ .

(4) Combine translation and rotation parts using Eq. (14), then work out Eq. (13) to obtain  $b_j$ .

### 3.2 TRT method

The TR method is influenced by the selection of origin. The dual quaternion curve jumps when the interpolating position is close to the origin. A

sharp change of curvature is apparently seen. In order to avert such occasion, a refined TRT method is introduced<sup>[11]</sup>. The rotational motion  $\mathbf{q}_{\text{rot}}(t)$  of degree  $k$  is composed of two translational Q-motions  $\mathbf{q}_{\text{trans}}^{(1)}(t)$  and  $\mathbf{q}_{\text{trans}}^{(2)}(t)$  of degree  $l_1$  and  $l_2$  respectively.

Unify the translation and rotation parts into rotation part, shown as

$$\begin{aligned} \hat{\mathbf{q}}(t) &= \mathbf{q}_{\text{trans}}^{(1)}(t) * \mathbf{q}_{\text{rot}}(t) * \mathbf{q}_{\text{trans}}^{(2)}(t) = \\ & \left[ 2 + \epsilon \sum_{j_1=0}^{l_1} b_{j_1}^{l_1}(t) \mathbf{P}_{j_1}^{(1)} \right] * \mathbf{q}_{\text{rot}}(t) * \\ & \left[ 2 + \epsilon \sum_{j_2=0}^{l_2} b_{j_2}^{l_2}(t) \mathbf{p}_{j_2}^{(2)} \right] \end{aligned} \quad (16)$$

The interpolating condition is

$$\begin{aligned} (\mathbf{q}_{\text{rot}}(t_i) * \tilde{\mathbf{q}}_{\text{rot}}(t_i)) \sum_{j_1=0}^{l_1} b_{j_1}^{l_1}(t) \mathbf{P}_{j_1}^{(1)} + \epsilon \sum_{j_2=0}^{l_2} b_{j_2}^{l_2}(t) \mathbf{p}_{j_2}^{(2)} = \\ (\mathbf{q}_{\text{rot}}(t_i) * \tilde{\mathbf{q}}_{\text{rot}}(t_i)) \mathbf{s}_i \end{aligned} \quad (17)$$

The refined TRT method can effectively avoid the strong vibration around the origin. However, for object like satellites whose size is far smaller than its trajectories, the error is quite big. So the TRT method is used only when the deputy satellite is close to the principal satellite trajectory. Fig. 1 shows the trajectories obtained by interpolating with both TR and TRT methods. The curvature is smooth and has no jump.

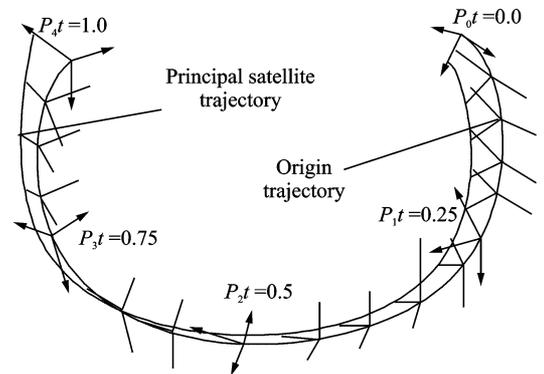


Fig. 1 Principal satellite trajectories obtained by interpolating

## 4 DUAL QUATERNION CURVE MODELING OF DEPUTY SATELLITE POSITION AND ATTITUDE

### 4.1 Transform of deputy to principal satellites

Dual quaternion is a powerful tool in rigid

body kinematics and coordinate transform. As can be seen in Fig. 2, coordinate system  $O$  revolves an angle of  $\theta$  around special vector  $\hat{l}$ , and makes a translation of  $d$  along  $\hat{l}$  at the same time, then becomes a new coordinate system  $N$ .

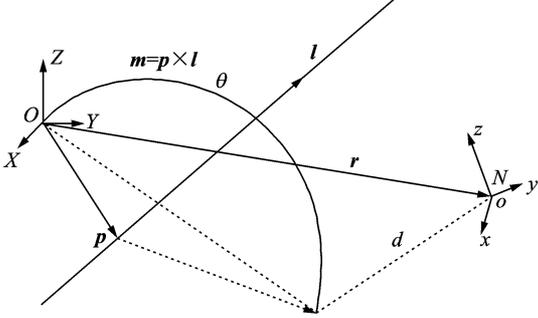


Fig. 2 Geometrical display of screw

The above mentioned rigid body movement can easily be expressed with dual quaternion curve, shown as

$$\hat{q} = \left( \cos \frac{\hat{\theta}}{2}, \hat{l} \sin \frac{\hat{\theta}}{2} \right) \quad (18)$$

where  $\hat{l} = l + \varepsilon(p \times l)$ ,  $\hat{\theta} = \theta + \varepsilon d$ ,  $p$  is the vector pointing from coordinate system  $O$  to one point on the rotation axis. Judging from Fig. 2, if dual vector  $\hat{l}$  and dual angle  $\hat{\theta}$  are given, the traditional complex coordinate system transform becomes quite simple using dual quaternion curve. Define the following equation<sup>[12]</sup>

$$\begin{aligned} \hat{q} &= q + \varepsilon q' = q + \frac{\varepsilon}{2} r^O \circ q = \\ & q + \frac{\varepsilon}{2} q \circ r^N \end{aligned} \quad (19)$$

where  $r$  denotes the position vector between two origins.

Therefore, the kinematics equation can be expressed as

$$\dot{\hat{q}} = \frac{1}{2} \hat{\omega}_{ON}^O \circ \hat{q} = \frac{1}{2} \hat{q} \circ \hat{\omega}_{ON}^N \quad (20)$$

where dual vector  $\hat{\omega}_{ON}^O$  and  $\hat{\omega}_{ON}^N$  are called spinor. The spinor is expressed as<sup>[13]</sup>

$$\hat{\omega}_{ON}^N = \omega_{ON}^N + \varepsilon(\dot{r}^N + \omega_{ON}^N \times r^N) \quad (21)$$

The relative movement between deputy and principal satellites is depicted by the dual quaternion  $\hat{q}_{Mm} = q_{Mm} + \varepsilon q'_{Mm}$ , and each centroid is the origin. They can be rewritten as

$$\begin{aligned} \hat{q}_{Mm} &= q_{Mm} + \varepsilon q'_{Mm} = \\ & q_{Mm} + \frac{\varepsilon}{2} r_{Mm}^M \circ q_{Mm} \end{aligned} \quad (22)$$

where  $\hat{q}_{Mm}$  denotes the relative attitude quaternion curve,  $r_{Mm} = r_M - r_m$  the relative centroid position information between deputy and principal satellites, and  $r_{Mm}^M$  the projection of position vector on principal satellite frame.

## 4.2 Dual quaternion curve updating

This paper chooses spiral vector method to calculate the renovated dual quaternion curve between deputy and principal satellites<sup>[14]</sup>, that is

$$\Delta \hat{q}_{Mmt_k} = \left[ \cos \frac{\hat{\sigma}(t_k)}{2}, \frac{\hat{\sigma}(t_k)}{\sigma(t_k)} \sin \frac{\hat{\sigma}(t_k)}{2} \right] \quad (23)$$

where  $\hat{\sigma}$  denotes the spiral vector along spiral axis, that is

$$\dot{\hat{\sigma}} = \hat{\omega}_{Mm}^M + \frac{1}{2} \hat{\sigma} \times \hat{\omega}_{Mm}^M \quad (24)$$

The fourth order trigonometric series approximation is used

$$\begin{cases} \cos \frac{\hat{\sigma}(t_k)}{2} = 1 - \frac{\hat{\sigma}(t_k)^2}{8} + \frac{\hat{\sigma}(t_k)^4}{384} \\ \frac{1}{\sigma(t_k)} \sin \frac{\hat{\sigma}(t_k)}{2} = \frac{1}{2} - \frac{\hat{\sigma}(t_k)^2}{48} \end{cases} \quad (25)$$

And the kinematics equation for  $\hat{q}_{Mm}$  is

$$\dot{\hat{q}}_{Mm} = \frac{1}{2} \hat{\omega}_{Mm}^M \circ \hat{q} = \frac{1}{2} \hat{q} \circ \hat{\omega}_{Mm}^m \quad (26)$$

From above equations, we can obtain the updating formula

$$\hat{q}_{Mmt_{k+1}} = \hat{q}_{Mmt_k} \circ \Delta \hat{q}_{Mmt_k} \quad (27)$$

where  $\hat{q}_{Mmt_k}$  and  $\hat{q}_{Mmt_{k+1}}$  are the former and the current dual quaternion curves respectively,  $\Delta \hat{q}_{Mmt_k}$  is the updating dual quaternion.

After updating, position and attitude dual quaternion of deputy to principal satellites is

$$\begin{aligned} \hat{q}_{Mmt_{k+1}} &= q_{Mmt_{k+1}}^* \circ \hat{q}_{Mmt_{k+1}} = \\ & q_{Mmt_{k+1}} + \frac{\varepsilon}{2} r_{Mmt_{k+1}}^M \circ q_{Mmt_{k+1}} = \\ & q_{Mmt_{k+1}} + \varepsilon q'_{Mmt_{k+1}} \end{aligned} \quad (28)$$

where  $q_{Mmt_{k+1}}$  is the attitude quaternion, and relative position vector of deputy to principal satellites is

$$r_{Mmt_{k+1}} = 2q'_{Mmt_{k+1}} \circ q_{Mmt_{k+1}}^* \quad (29)$$

Judging from Eq. (29), if the initial position and attitude information of two satellites is given, and the relative velocity and angular rate are measured, it is realistic to obtain the real-time relative position and attitude information of deputy to principal satellites.

## 5 DATA SIMULATION

Orbital parameters of two satellites are as follows.

Principal satellite:

$$a_M = 6\,800 \text{ km}, e_M = 0, i_M = 30^\circ,$$

$$\Omega_M = 0, \omega_M = 60^\circ, f_M = 0^\circ.$$

Deputy satellite:

$$a_m = 6\,800 \text{ km}, e_m = 0.000\,000\,01,$$

$$i_m = 30.005^\circ, \Omega_m = 0, \omega_m = 60^\circ, f_m = 300.025^\circ$$

The relative attitude angular velocities to the inertial system are  $\omega_M = [0\ 0\ 1]$ ,  $\omega_m = [0\ 0\ 1]$ , respectively. The initial attitude angles are  $\theta_{M0} = [0.05^\circ\ 0.05^\circ\ 0.05^\circ]$ ,  $\theta_{m0} = [0.05^\circ\ 0\ 0]$ .

The simulating interval for position and attitude error is  $\Delta t = 0.1 \text{ s}$ , and simulation lasts for  $t = 6\,000 \text{ s}$ .

The real value of relative position and attitude is denoted by  $\bar{\mathbf{r}}$ , and the calculated value is  $\hat{\mathbf{r}}$ , the simulation error is  $\delta\mathbf{r} = \bar{\mathbf{r}} - \hat{\mathbf{r}}$ . Comparing the ideal position and attitude data calculated by orbital parameter and quaternion methods with the data obtained by the algorithm in this paper, the position and attitude error is obtained. Figs. 3-6 illustrate the position and the attitude angle errors obtained using orbital parameter and dual quaternion methods respectively. The attitude angle includes rolling, pitch and yaw angles.

In Fig. 3, the errors along axes  $x$ ,  $y$  and  $z$  are  $0-3.9$ ,  $0-2.1$ ,  $0-3.7 \text{ cm}$  respectively. In Fig. 4, the corresponding errors are  $0-1.1$ ,  $0-0.9$ ,  $0-1.8 \text{ cm}$  respectively. Comparison of Figs. 3, 4 shows an obvious improvement in error using the dual quaternion method. The position error curve shows a periodical trend, which can satisfy long time functioning formation flying requirement.

Comparing Figs. 5, 6, the estimation precision of the roll and pitch angle is basically equivalent. However in the estimation of yaw angle, the precision using dual quaternion method is much better, which confines the attitude angle error to within  $0.03^\circ$ . Results show that using dual quaternion method, the convergence rate is much faster and stability of the estimation error is much better.

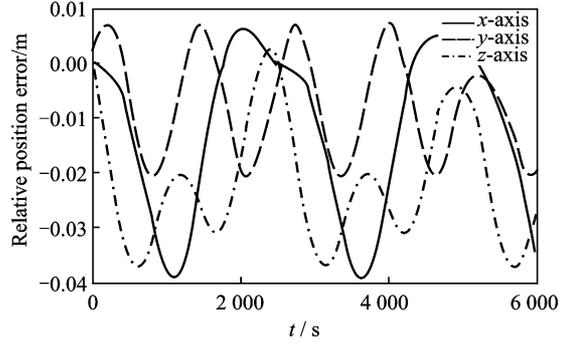


Fig. 3 Position errors using orbital parameters method

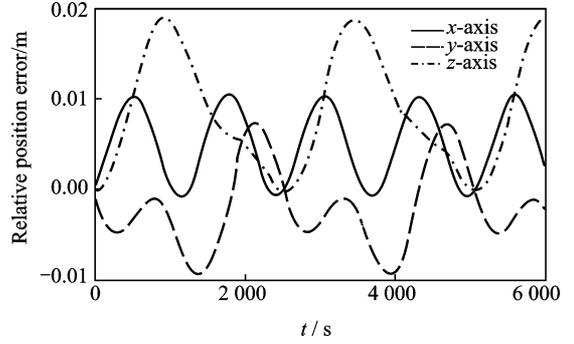


Fig. 4 Position errors using dual quaternion method

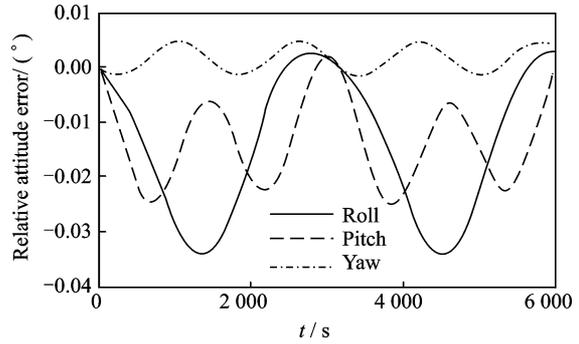


Fig. 5 Attitude angle errors using quaternion method

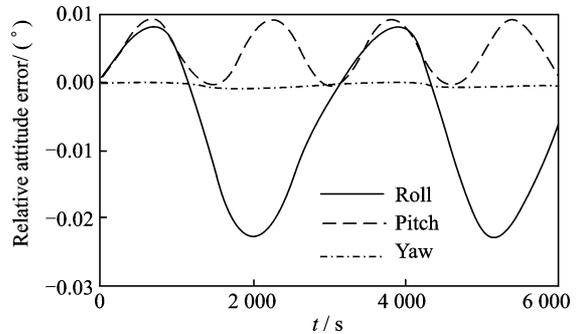


Fig. 6 Attitude angle errors using dual quaternion method

## 6 CONCLUSION

This paper constructs a position and attitude

unified model for satellite formation flying based on the dual quaternion method. First TR and TRT interpolation methods are used to calculate the principal satellite trajectory during formation flying. Then using dual quaternion modeling, the position and attitude information integration of deputy satellite is accomplished. The simulation results show that the proposed algorithm can obviously improve both position and attitude precision.

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## 对偶四元数曲线插值算法在编队卫星中的应用

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**摘要:**传统的编队卫星位置和姿态算法是分开处理的。提出一种新的对偶四元数曲线插值算法,将卫星的姿态结合到轨道中去。首先采用曲线插值算法计算主星的运动轨迹,然后建立从星相对于主星的对偶四元数模型来确定从星的相对位置和姿态,将卫星的位置和姿态进行统一处理。与传统的轨道参数法及四元数法进行误差比较,仿真结果

表明该方法不但可以实现编队卫星位置和姿态的统一确定,而且可以满足卫星编队的精度要求。

**关键词:**编队卫星; 对偶四元数; 插值算法; 相对位姿  
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