CALCULATION FOR KERNEL OF INTERVAL GREY NUMBER BASED ON BARYCENTER APPROACH

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Abstract: The kernel of interval grey number is most likely the real number, which can be used to represent whitenization value of interval grey number. A novel method for calculating kernel of interval grey number is constructed based on the geometric barycenter of whitenization weight function in the two-dimensional coordinate plane, and the calculation of kernel is converted to the calculation of barycenter in geometric figures. The method fully considers the effect of all information contained in whitenization weight function on the calculation result of kernel, and is the extension and perfection of the existing methods in the scope of application.

Key words: grey theory; interval grey number; kernel; whitenization weight function; barycenter approach

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INTRODUCTION

Grey numbers are the elementary "atoms" or "cells", and they are the basic representation style of behavior characteristics of grey system[1-2]. There are all kinds of grey number, including interval grey number, discrete grey number, continuous grey number, and so on. Thereinto, the interval grey number is one of the most common and the most broadly applied grey numbers [3-5]. A whitenization weight function is used to describe the degree of preference of an interval grey number to take values in its range[2]. Some frequently-used whitenization weight functions include trapezoid whitenization weight function, and triangular whitenization weight function [6-7]. In 1987, in order to research grey matrix, Professor Liu Sifeng proposed a concept of mean value whitenization number, and realized that it would play an important role in the operations of grey number[5]. Based on this, Professor Liu referred to a whitenization number as the kernel of interval grey number in Ref. [8]. In other words, a kernel is the most possible real number which can be used to represent the whitenization number of interval grey number on the basis of full consideration of known information. At the moment, kernels of interval grey numbers have become an important concept in grey system theory. It is the foundation of building operation algorithms, rela-

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tional models and prediction models of interval grey number[9-13].

Ref. [8] discussed calculation methods for kernel only when whitenization weight function was rectangle. If whitenization weight function is non-trapezoid or non-triangle, the current methods cannot solve it. This paper proposes a novel method through the barycenter of graph that is formed by interval grey number and its whitenization weight function in two-dimensional rectangular planar system, and it effectively solves calculation problem of kernel when whitenization weight functions are non-trapezoid or non-triangle. Therefore, results are important reference for other whitenization weight functions.

When a grey number equally takes values in its range, its whitenization weight function is rectangular actually. In order to uniformly define the calculation method for kernel, the unknown whitenization weight function is regarded as a rectangular one in this paper.

1 PRIMARY CONCEPTS

Definition 1 A grey number with both a lower limit a_k and an upper limit b_k is called an interval grey number, denoted as $\otimes(t_k)$, $\otimes(t_k)\in[a_k, b_k]$ and $a_k \leqslant b_k$.

Definition 2 A function that is used to depict the degree of preference of $\otimes(t_k)$ to take values in its range $[a_k, b_k]$ is named the whitenization weight function of $\otimes(t_k)$, denoted as $f^{(k)}(x)$. A continuous function with rising on the left, declining on the right, and certain start and end points is called a typical whitenization weight function (namely trapezoid whitenization weight function)[2], in which, a_k, b_k are called the start and end points of $\otimes(t_k)$, and a'_k, b'_k the second start and end points of $\otimes(t_k)$, shown as Fig. 1.

Definition 3 In Fig. 1, when a'_k is coincident with b'_k, the whitenization weight function changes from trapezoid into triangle (Fig. 2). When a_k, b_k are coincident with a'_k, b'_k respectively, the whitenization weight function changes from trapezoid into rectangle (Fig. 3), and it means that $\otimes(t_k)$ can equally take value in its range.

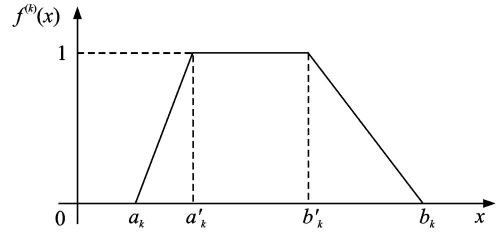

Fig. 1 Typical whitenization weight function of $\otimes(t_k)\in[a_k, b_k]$

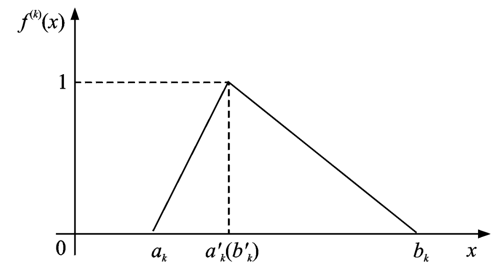

Fig. 2 Triangular whitenization weight function

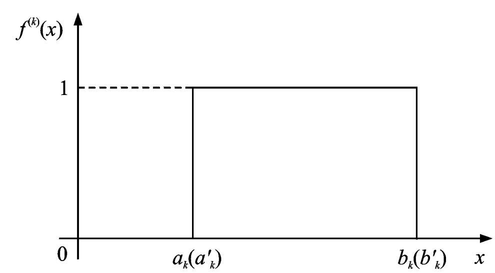

Fig. 3 Rectangular whitenization weight function

Definition 4 The whitenization result based on $f^{(k)}(x)$ of $\otimes(t_k)$ is called the kernel of $\otimes(t_k)$, denoted as $\tilde{\otimes}(t_k)$.

2 CALCULATION METHOD FOR KERNELS

Kernels are the most possible real numbers which can be used to represent interval grey numbers. Whitenization weight functions are used to define the preference of an interval grey number to take values in its range. If the value of $f^{(k)}(a_t)$ is bigger, the effect of a_t on kernel is greater, then the point a_t is more close to the kernel. The range of interval grey number is continuous, so a geometric graph is formed by interval grey number and its whitenization weight function in two-dimensional coordinate plane, and the barycenter of this graph is undoubtedly the kernel of the interval grey number. According to the above analysis, the barycenters method is used to research the kernels of interval grey numbers.

For whitenization weight functions with the trait of symmetry characteristics $f^{(k)}(x)$, such as rectangle, isosceles trapezoid or isosceles triangle whitenization weight functions, their kernels $\tilde{\otimes}(t_k)$ can be easily computed, and the symmetric points are the kernels of interval grey numbers, that is

$$\tilde{\otimes}(t_k)=\frac{a_k+b_k}{2}=\frac{a'_k+b'_k}{2} \qquad (1)$$

Positions of kernels are shown in Fig. 4.

However, for non-symmetry whitenization weight functions, the calculation process of kernels is very complicated, because midpoints of interval grey numbers are no longer the kernels. In this section, kernels are computed by the barycenter method, and the main process is: Geometric graph→barycenter→x-coordinate→kernel.

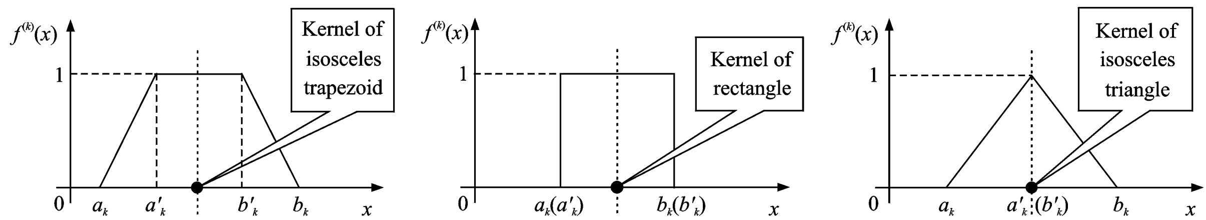

Fig. 4 Kernels of symmetry whitenization weight functions of $\otimes(t_k)\in[a_k,b_k]$

2.1 Kernel of non-symmetry triangle

According to the barycenter theorem of triangle, the coordinates of a triangle barycenter are the arithmetic mean value of three vertex coordinates of triangle. In Fig. 5, coordinates of A, B, C are $A(a_k,0)$, $B(b_k,0)$, $C(c_k,1)$, and G is the barycenter of ΔABC, then x-coordinate of point G is just the kernel $\tilde{\otimes}(t_k)$ of $\otimes(t_k)$, that is

$$\tilde{\otimes}(t_k)=X_G=\frac{a_k+b_k+c_k}{3} \qquad (2)$$

It can be seen from Fig. 5, when whitenization weight function is non-symmetry triangle, the kernel does not equal the mean value of upper limit and lower limit of interval grey number, but slopes toward the side with greater value of whitenization weight function.

2.2 Kernel of non-symmetry trapezoid

Kernel and barycenter of non-symmetry trapezoid are shown in Fig. 6, where G_A, G_B are the barycenters of ΔACD and ΔABC, O_1, O_2 the midpoints of upper and base of trapezoid, and G is the barycenter of trapezoid.

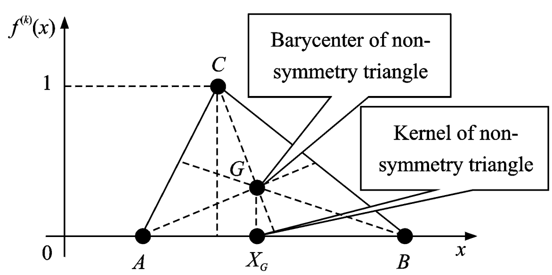

Fig. 5 Kernel and barycenter of non-symmetry triangle

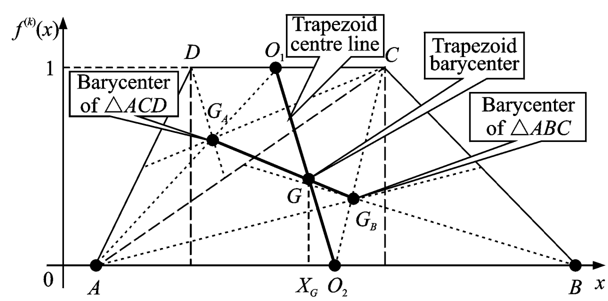

Fig. 6 Kernel and barycenter of non-symmetry trapezoid

According to Fig. 6, coordinates of A, B, C, D are as follows

$$A(a_k,0),B(b_k,0),C(b'_k,1),D(a'_k,1)$$

Then the horizontal coordinate X_{GA} and vertical coordinate Y_{GA} of the point G_A are as follows

$$X_{GA}=(a_k+b'_k+a'_k)/3$$
$$Y_{GA}=(0+1+1)/3=2/3$$

Similarly, the horizontal coordinate X_{GB} and the vertical coordinate Y_{GB} of the point G_B are

$$X_{GB}=(a_k+b_k+b'_k)/3$$
$$Y_{GB}=(0+0+1)/3=1/3$$

According to the principle of "two points determine one line", the formulas of line G_AG_B are as follows

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}\Rightarrow$$

$$\frac{x-\frac{a_k+b'_k+a'_k}{3}}{\frac{a_k+b_k+b'_k}{3}-\frac{a_k+b'_k+a'_k}{3}}=$$

$$\frac{y-\frac{2}{3}}{\frac{1}{3}-\frac{2}{3}}\Rightarrow\frac{3x-(a_k+b'_k+a'_k)}{a'_k-b_k}=$$

$$3y-2\Rightarrow\frac{3x-(a_k+b'_k+a'_k)}{a'_k-b_k}=$$

$$3y-2\Rightarrow 3y=\frac{3x-(a_k+b'_k+a'_k)}{a'_k-b_k}+2$$

Then

$$y=\frac{x}{a'_k-b_k}-\frac{a_k-a'_k+b'_k+2b_k}{3(a'_k-b_k)} \tag{3}$$

In which: $G_A\left(\frac{a_k+b'_k+a'_k}{3},\frac{2}{3}\right)$, $G_B\left(\frac{a_k+b_k+b'_k}{3},\frac{1}{3}\right)$.

Similarly, the formulas of line O_1O_2 are as follows

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}\Rightarrow\frac{x-\frac{a'_k+b'_k}{2}}{\frac{a_k+b_k}{2}-\frac{a'_k+b'_k}{2}}=$$

$$\frac{y-1}{0-1}\Rightarrow\frac{2x-(a'_k+b'_k)}{(a'_k+b'_k)-(a_k+b_k)}=y-1$$

Then

$$y=\frac{2x}{(a'_k+b'_k)-(a_k+b_k)}-\frac{a_k+b_k}{(a'_k+b'_k)-(a_k+b_k)} \tag{4}$$

In which: $O_1\left(\frac{a'_k+b'_k}{2},1\right)$, $O_2\left(\frac{a_k+b_k}{2},0\right)$.

Combining Eqs. (3, 4), we can calculate x-coordinate of point G, which is the intersection of the lines G_AG_B and O_1O_2.

$$\frac{x}{a'_k-b_k}-\frac{a_k-a'_k+b'_k+2b_k}{3(a'_k-b_k)}=\frac{2x}{(a'_k+b'_k)-(a_k+b_k)}-\frac{a_k+b_k}{(a'_k+b'_k)-(a_k+b_k)}\Rightarrow$$

$$\left[\frac{1}{a'_k-b_k}-\frac{2}{(a'_k+b'_k)-(a_k+b_k)}\right]\cdot x=\frac{a_k-a'_k+b'_k+2b_k}{3(a'_k-b_k)}-$$

$$\frac{a_k+b_k}{(a'_k+b'_k)-(a_k+b_k)}\Rightarrow\left[\frac{(a'_k+b'_k)-(a_k+b_k)-2(a'_k-b_k)}{(a'_k-b_k)(a'_k+b'_k-a_k-b_k)}\right]\cdot x=$$

$$\frac{(a_k-a'_k+b'_k+2b_k)(a'_k+b'_k-a_k-b_k)-3(a'_k-b_k)(a_k+b_k)}{3(a'_k-b_k)(a'_k+b'_k-a_k-b_k)}$$

Then

$$\tilde{\otimes}(t_k)=X_G=x=\frac{(a_k-a'_k+b'_k+2b_k)(a'_k+b'_k-a_k-b_k)-3(a'_k-b_k)(a_k+b_k)}{3(a'_k+b'_k)-3(a_k+b_k)-6(a'_k-b_k)} \tag{5}$$

It can be seen from Fig. 6 that, when whitenization weight function is non-symmetry trapezoid, the kernel does not equal the mean value of upper limit and lower limit of interval grey number, but slopes toward the side with greater value of whitenization weight function.

3 EXAMPLES

Assume that an interval grey number $\otimes(t_1)\in[104,130]$, and its whitenization weight function is shown in Fig. 7. The kernels of $\tilde{\otimes}(t_1)$ with different whitenization weight functions are computed.

(1) When whitenization weight function of $\otimes(t_1)$ is rectangle, according to Eq. (1), we have

$$\tilde{\otimes}(t_1)=\frac{a_1+b_1}{2}=\frac{104+130}{2}=117.000\ 0$$

(2) When whitenization weight function of $\otimes(t_1)$ is non-symmetry trapezoid, according to Eq. (5), we have

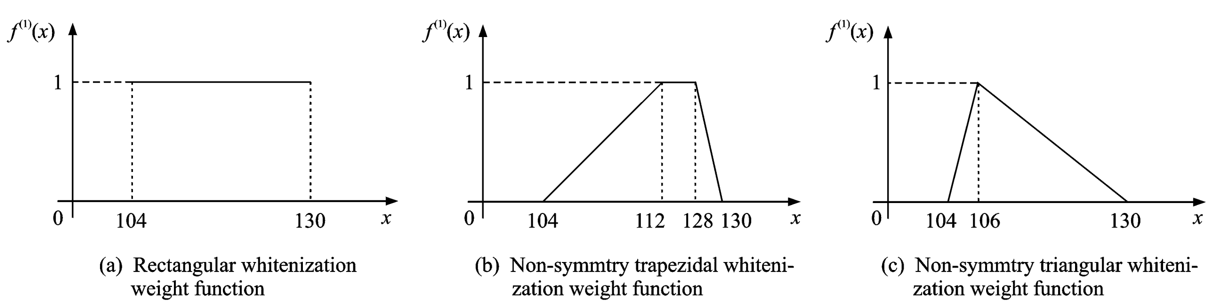

Fig. 7 Whitenization weight functions of $\otimes(t_1)$

$$\tilde{\otimes}(t_k) = X_G = x = \frac{(a_k - a_k' + b_k' + 2b_k)(a_k' + b_k' - a_k - b_k) - 3(a_k' - b_k)(a_k + b_k)}{3(a_k' + b_k') - 3(a_k + b_k) - 6(a_k' - b_k)}$$

$$\tilde{\otimes}(t_1) = \frac{(104 - 112 + 128 + 2 \times 130)(112 + 128 - 104 - 130) - 3 \times (112 - 130)(104 + 130)}{3 \times (112 + 128) - 3 \times (104 + 130) - 6 \times (112 - 130)} = 118.381\,0$$

(3) When whitenization weight function of $\otimes(t_1)$ is non-symmetry triangle, according to Eq.(2), we have

$$\tilde{\otimes}(t_1) = \frac{a_1 + b_1 + c_1}{3} = \frac{104 + 106 + 130}{3} = 113.333\,3$$

It is easy to see from the calculation results that, for the same interval grey number with different whitenization weight functions, the kernel is not the same, but traditional methods can only aim at one special situation that whitenization weight function is a symmetry graph.

4 CONCLUSION

Kernel of interval grey number is an important concept in grey system theory, however the existing researches in this field only discussed the calculation method for kernel when the whitenization weight function of interval grey number is rectangle, not unsymmetrical triangle or trapezoidal. This paper proposes a novel calculation method for kernel by geometric barycenter. The method fully considers the effect of information contained in whitenization weight function on the kernel, and effectively solves the calculation problems of kernel when whitenization weight function is asymmetric graph. It is the extension and perfection of the existing methods in the scope of application.

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