HYBRID CARTESIAN GRID/GRIDLESS METHOD FOR CALCULATING VISCOUS FLOWS OVER MULTI-ELEMENT AIRFOILS

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Abstract: A hybrid Cartesian grid/gridless method is developed for calculating viscous flows over multi-element airfoils. The method adopts an unstructured Cartesian grid to cover most areas of the computational domain and leaves only small region adjacent to the aerodynamic bodies to be filled with the cloud of points used in the gridless methods, which results in a better combination of the computational efficiency of the Cartesian grid and the flexibility of the gridless method in handling complex geometries. The clouds of points in the local gridless region are implemented in an anisotropic way according to the features of the thin boundary layer of the viscous flows over the airfoils, and the clouds of points at the vicinity of the interface between the grid and the gridless regions are also controlled by using an adaptive refinement technique during the generation of the unstructured Cartesian grid. An implementation of the resulting hybrid method is presented for solving two-dimensional compressible Navier-Stokes (NS) equations. The simulations of the viscous flows over a RAE2822 airfoil or a two-element airfoil are successfully carried out, and the obtained results agree well with the available experimental data.

Key words: multi-element airfoil; gridless method; Cartesian grid; viscous flow; NS equations

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INTRODUCTION

During the early study of the numerical solution methods for partial differential equations (PDE), Cartesian grid is utilized as a common tool to discretize the physical domain. This kind of grid is regular and it has no issues associated with grid skewness and distortion, therefore, it is very popular with many researchers[1]. However, when dealing with complex configurations, Cartesian grid lines usually cross the physical boundary while do not fit the body, so it is difficult to implement the boundary condition in this kind of gird method directly. To simplify the treatment of the boundary condition, structured grid is then proposed. This kind of method transforms the physical body-fitted grid to regular grid in the computational domain and then solves the governing equations in the computational domain. However, the transformed equations in the computational domain are usually much more complicated than the original ones in the physical domain.

Recently, a new type of numerical method called gridless method has been proposed[2]. This method is capable of directly estimating the derivatives without transforming the problem from physical domain to computational domain. It is simple and flexible, because only a set of points are required to be distributed in the physical domain without considering the connectives among these points. However, when this method is applied to deal with different configurations, even if there is only a little change between the configu-

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rations, all of the points have to be redistributed once again, and this redundant work probably slows down the computational speed and results in low efficiency.

To overcome the difficulty of Cartesian grid implementing the physical boundary condition and to avoid the redundant work of point redistribution for gridless method, we proposed a hybrid Cartesian grid/gridless method in our own previous work[3]. The whole physical domain is covered by a base Cartesian grid and only a few of clouds are introduced into the adjacent region of the physical boundary. Because of Cartesian grid's regularity and simplicity, the computing time for generating grid is almost negligible and numerical schemes are easy to be implemented on the grid, and the local gridless treatment technique only requires a small amount of clouds to be introduced to represent the configuration. Therefore, the hybrid method not only reduces the redundant work, but also provides the flexibility to handle arbitrary configurations. The proposed hybrid method has been used to solve Euler equations, and the inviscid flows over the bump, cylinder and airfoils are successfully simulated respectively.

In this paper, the hybrid Cartesian grid/gridless method is further developed to simulate viscous flows over multi-element airfoils. In order to simulate the thin boundary layer of the viscous flows, the cloud of anisotropic points is implemented in the local gridless region. A cloud overlap-free procedure is proposed, which enables the hybrid method to handle close-coupled bodies including multi-element airfoils conciderded in this paper. The unstructured Cartesian grid requiring the hybrid method is generated based on an adaptive refinement technique, which can help to control the quality of the clouds at the vicinity of the interface between the grid and the gridless region. The resulting hybrid method is applied to solve two-dimensional compressible Navier-Stokes (NS) equations. The viscous flow over a RAE2822 airfoil is first simulated, and the numerical result obtained is compared with the available experimental data, then the viscous flow over a two-element airfoil is simulated, which demonstrates the ability of the present method for treating more complicated flows over multi-bodies.

1 GOVERNING EQUATIONS

The governing equations of this study are the compressible NS equations in Cartesian coordinates, which can be written as

$$\frac{\partial \boldsymbol{W}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}+\frac{\partial \boldsymbol{F}}{\partial y}-\left(\frac{\partial \boldsymbol{E}_{\mathrm{V}}}{\partial x}+\frac{\partial \boldsymbol{F}_{\mathrm{V}}}{\partial y}\right)=0 \tag{1}$$

where $\boldsymbol{W}$ is the vector of conservative variables, $\boldsymbol{E}$ and $\boldsymbol{F}$ are the convective flux terms, $\boldsymbol{E}_{\mathrm{V}}$ and $\boldsymbol{F}_{\mathrm{V}}$ the viscous flux terms

$$\begin{aligned}
\boldsymbol{W}&=[\rho,\ \rho u,\ \rho v,\ \rho E]^{\mathrm{T}}\\
\boldsymbol{E}&=[\rho u,\ \rho u^2+p,\ \rho uv,\ \rho uH]^{\mathrm{T}}\\
\boldsymbol{F}&=[\rho v,\ \rho uv,\ \rho v^2+p,\ \rho vH]^{\mathrm{T}}\\
\boldsymbol{E}_{\mathrm{V}}&=[0,\ \tau_{xx},\ \tau_{yx},\ \Theta_x]^{\mathrm{T}}\\
\boldsymbol{F}_{\mathrm{V}}&=[0,\ \tau_{xy},\ \tau_{yy},\ \Theta_y]^{\mathrm{T}}
\end{aligned} \tag{2}$$

where ρ, p, E, H are the density, the pressure, the total energy per unit mass, the total enthalpy per unit mass, respectively, u and v the cartesian components of the velocity vector, τ is the viscous stress and Θ the term describing the work of viscous stresses and the heat conduction in the fluid. The laminar viscosity coefficient μ_{L} requiring the calculation of $\boldsymbol{E}_{\mathrm{V}}$ and $\boldsymbol{F}_{\mathrm{V}}$ is computed with the Sutherland formula[4] and the turbulence viscosity coefficient μ_{T} is obtained from the Spalart-Allmaras turbulence model[5]. The NS equations are non-dimensionalized by free stream density ρ_∞, free stream pressure p_∞, reference length L, and viscosity $\mu_\infty\left(\frac{Re_\infty}{(\sqrt{\gamma}\cdot Ma_\infty)}\right)$, where Re_∞ and Ma_∞ represent the Reynolds number and Mach number of the free stream.

2 DECOMPOSITION OF COMPUTATIONAL DOMAIN

The entire flow domain is decomposed into two types of sub-domains, one is discretized by Cartesian grid and the other is filled with clouds of points, as shown in Fig. 1. First, the surfaces of the aerodynamic bodies are broken into edges for a full description. Then, the unit normal vector of all the points on the surfaces are calculated, and new points are produced along the normal vectors layer to layer until a user specifies the number of layers. The neighbors of any point in the gridless zone can be easily defined since these points are generated in the above regular manner. When two or more bodies are closely coupled in the domain, for example, the main element and the flap of GA(W)-1 two-element airfoil in Fig. 1, their gridless zones overlap. The overlap region can be deleted by determing the distances between the point and each body. It can be easily noted that the point spacing normal to the wall can be controlled during the generation of the gridless zones by the user simulating the boundary layer. The remaining part of the flow domain is discretized by an unstructured Cartesian grid using the adaptive refinement technique[1].

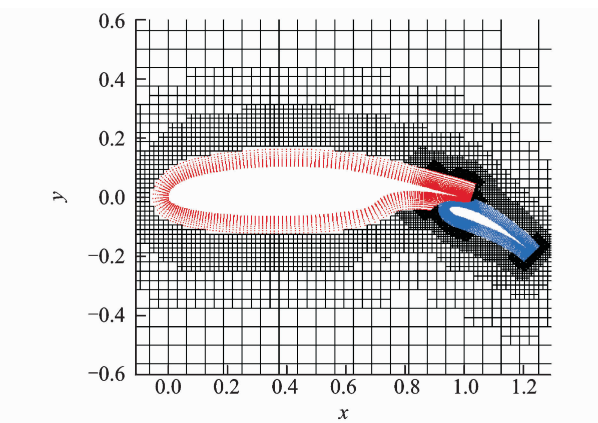

Fig. 1 Points and Cartesian grid around GA(W)-1 airfoil

As both Cartesian grid and gridless clouds are used to discretize the flow field, these two different regions need to exchange the flow information in order to obtain the physical solution. If the interpolation technique is adopted to exchange the flow information, truncation error may be caused especially in the vicinity of critical flow features such as the shock waves. In this paper, we follow our previous work[3] to use dual points method for exchanging flow information, which means the first two layers of the Cartesian points near the gridless region(the square points in Fig. 2 are selected and considered for gridless zone computations, and the satellite points of these dual points can be found using point-selection strategies[6], as shown in Fig. 2.

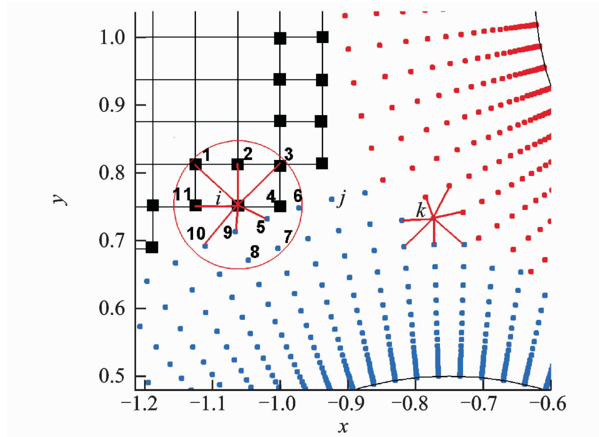

Fig. 2 Cloud of points at interface

3 NUMERICAL DISCRETIZATION OF GOVERNING EQUATIONS

For gridless method, the spatial derivatives of any quantities are evaluated with linear combinations of certain coefficients and the quantities in the cloud of points. For example, in the cloud of points $C(i)$ shown in Fig. 2, the first spatial derivatives of function f at point i are evaluated with the following linear combination forms[7]

$$\left.\frac{\partial f}{\partial x}\right|_i = \sum_{k=1}^{m} \alpha_{ik} f_{ik}, \quad \left.\frac{\partial f}{\partial y}\right|_i = \sum_{k=1}^{m} \beta_{ik} f_{ik} \tag{3}$$

where m is the number of satellite points in the cloud of $C(i)$, and f_{ik} the value at the midpoint between points i and k. The coefficients α_{ik} and β_{ik} can be obtained with a weighted least-squares curve fit to the following linear equation

$$f = a + bx + cy \tag{4}$$

On the Cartesian grid, suppose h_i is the space step along x and y axes at point i, then the first spatial derivatives of function f at point i can be computed using the central difference scheme

$$\left.\frac{\partial f}{\partial x}\right|_i = \frac{f_{iE} - f_{iW}}{h_i}, \quad \left.\frac{\partial f}{\partial y}\right|_i = \frac{f_{iN} - f_{iS}}{h_i} \tag{5}$$

where f_{iE}, f_{iW}, f_{iS}, f_{iN} are the values at the midpoints between points i and its neighboring points in the east, the west, the south and the north directions.

If Eq. (3) is applied to the convective flux of the NS equations, the following expression can be obtained

$$\frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{F}}{\partial y} = \sum_{k=1}^{m} (\alpha_{ik} \boldsymbol{E}_{ik} + \beta_{ik} \boldsymbol{F}_{ik}) = \sum_{k=1}^{m} \boldsymbol{G}_{ik} \tag{6}$$

The numerical flux $\boldsymbol{G}_{ik}$ at the midpoint between points i and k can be obtained by using Roe′s approximate Riemann solver

$$G_{ik} = \frac{1}{2}(G(W_R) + G(W_L) - |\bar{A}_{\mathrm{Roe}}|_{ik}(W_R - W_L)) \quad (7)$$

where A are the flux Jacobian matrices of G. The conservative variables at the midpoint are reconstructed with

$$\begin{cases} W_L = W_i + \frac{1}{2}\varphi_i(\nabla W_i \cdot r_{ik}) \\ W_R = W_k - \frac{1}{2}\varphi_k(\nabla W_k \cdot r_{ik}) \end{cases} \quad (8)$$

where ∇W is the gradient of the conservative variables, and φ the Venkatakrishnan′s flux limiter employed to prevent nonlinear instability[4].

The viscous terms of the NS equations are evaluated using Eq. (3) at each point

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)\bigg|_i = \sum_{k=1}^{m}\alpha_{ik}\left(\mu\frac{\partial u}{\partial x}\right)_{ik} \quad (9)$$

The first derivative at the midpoint between points i and k is obtained with[7]

$$\left(\frac{\partial u}{\partial x}\right)_{ik} = \frac{\Delta x}{\Delta s^2}(u_k - u_i) + \frac{1}{2}\frac{\Delta y}{\Delta s^2}\left(\Delta y\left(\frac{\partial u}{\partial x}\bigg|_i + \frac{\partial u}{\partial x}\bigg|_k\right) - \Delta x\left(\frac{\partial u}{\partial y}\bigg|_i + \frac{\partial u}{\partial y}\bigg|_k\right)\right) \quad (10)$$

where Δx, Δy, and Δs^2 are given as

$$\Delta x = x_k - x_i,\ \Delta y = y_k - y_i,\ \Delta s^2 = \Delta x^2 + \Delta y^2 \quad (11)$$

After the spatial discretization, the semi-discretization form of the NS equations at point i can be expressed as

$$\frac{\partial W}{\partial t}\bigg|_i + R_i = 0 \quad (12)$$

where R_i represents the residual error at point i. In order to obtain the steady solution, an explicit five-stage Runge-Kutta time integration schems is used

$$\begin{aligned} W^{(0)} &= W^{(n)} \\ W^{(m)} &= W^{(0)} - \alpha_m \Delta t R^{(m-1)} \\ m &= 1, \cdots, 5 \\ W^{(n+1)} &= W^{(5)} \end{aligned} \quad (13)$$

where the superscript n denotes the current time level, m the internal step and $n+1$ the next new time level. The factor α_m can be found in Ref. [4].

4 NUMERICAL RESULTS

To evaluate the accuracy of the presented hybrid method, the transonic viscous flow over a RAE2822 airfoil is first considered. The points and the used Cartesian grid are shown in Fig. 3. The total number of points is 23 866 with 310 points on the airfoil. The point spacing normal to the airfoil surface is 1.0×10^{-5}. The density contours of the flow field obtained with $Ma=0.73$, $\alpha=2.79°$, $Re=6.5\times10^6$ are shown in Fig. 4. We can find that the contours change smoothly at the interface. The pressure coefficient c_{p} and friction coefficient c_{f} distributions on the airfoil surface are compared with the experimental data in Ref. [8] in Fig. 5, which indicates good agreement between the numerical results and the experimental data. From the residual history presented in Fig. 6, the hybrid method has a reasonable convergence character. Then the viscous flow over a GA(W)-1 airfoil is simulated. The points and the used Cartesian grid has been shown in Fig. 1. The total number of points is 29 768 with 250 points on the main element surface and 190 points on the flap surface. The density contours

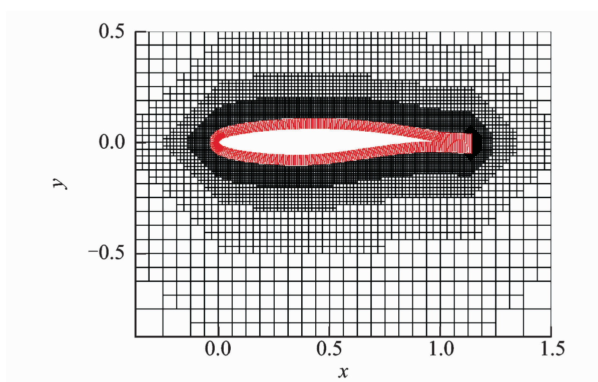

Fig. 3 Points and Cartesian grid around RAE2822 airfoil

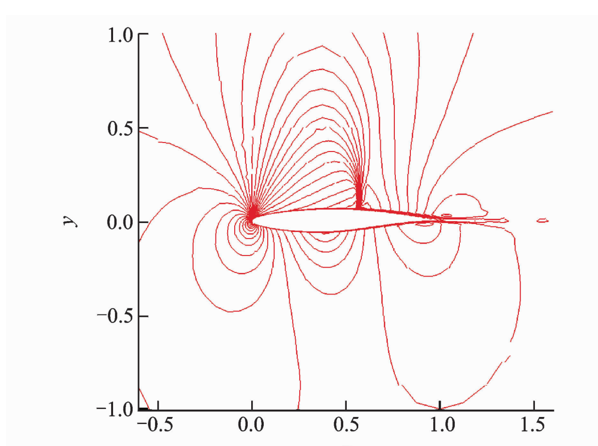

Fig. 4 Density contours around RAE2822 airfoil

of the flow field obtained with $Ma=0.21$, $\alpha=10°$, $Re=2.3\times10^6$ are shown in Fig. 7. The c_p distributions on the airfoil surface are compared with the experimental data in Fig. 8, which indicates a good agreement between the numerical result and the experimental data again.

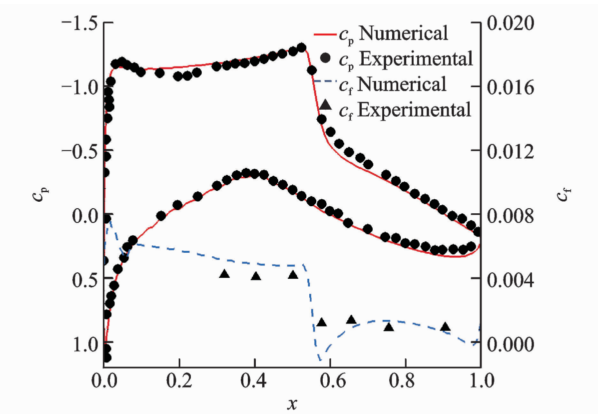

Fig. 5 c_p and c_f distributions around RAE2822 airfoil

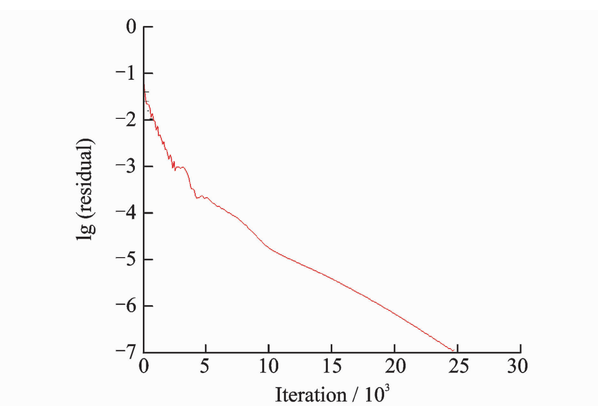

Fig. 6 Convergence history

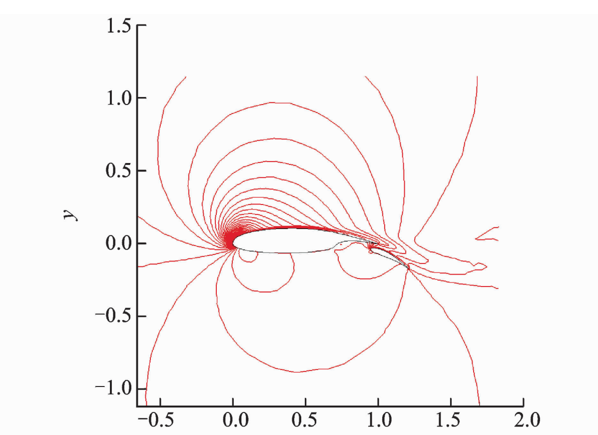

Fig. 7 Density contours around GA(W)-1 airfoil

5 CONCLUSION

The hybrid Cartesian grid/gridless method is successfully developed to simulate viscous flows over single and multi-element airfoils. The unstructured Cartesian grid can decompose the computational domain easily and efficiently, and the cloud of anisotropic points implemented in the area adjacent to the body can capture the boundary layer as accurately as the shock wave close.

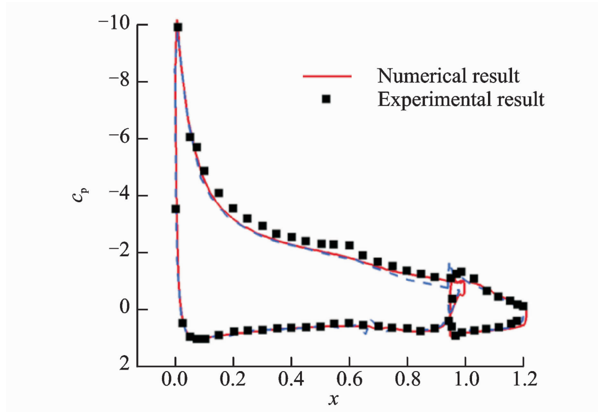

Fig. 8 c_p distribution around GA(W)-1 airfoil

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