FAST SIMULATION OF FLOWS IN SHOCK TUBE WITH AREA CHANGE

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Abstract: The shock tubes with area change are used in the free piston shock tunnels, owing to its higher driver effect. For optimized operation of this kind of shock tube, a computer program for fast simulation of transient hypersonic flow is presented. The numerical modeling embodied within this code is based on a quasi-one-dimensional Lagrangian description of the gas dynamics. In this code, a mass-loss model is also applied by using Mirels' theory of shock attenuation. The simulation of particular condition for T4 free piston shock tunnel is conducted and compared with experimental measurements and numerical simulation. The results provide good estimate for shock speed and pressure obtained after shock reflection.

Key words: shock tube with area change; Lagrangian description; Riemann solver; mass-loss model

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INTRODUCTION

At present, the free piston shock tunnel is one of the most successful ground facilities to achieve high enthalpy flow and duplicate real gas effect. The free piston shock tunnel has a compressor tube with its diameter above three times than that of the shock tube. Essentially, this structure is the area-changed shock tube. To design and operate this facility, more information of the transient flow in this tunnel should be obtained. For T4 free piston shock tunnel, Queensland University has developed a quasi-one-dimensional simulation program, which is widely used in operation of the other wind tunnels[1-2].

The code presented in this paper is still based on a quasi one-dimensional Lagrangian description. To upgrade the accuracy of calculation and the extensibility of program, the precise Riemann resolver and HLLC approximation Riemann resolver are used in the code. Furthermore, a model of mass loss in Ref. [3] is introduced into the code, which improves the accuracy of calculation about the velocity of shock wave and the contact surface as well.

1 GOVERNING EQUATIONS

In the shock tube with area change, the driving and driven gas are respectively divided into a number of control-mass cells and the area of the shock tuber section is assumed to change continuously. Here, the j-th cell has two interfaces (labeled $j-1/2$ and $j+1/2$) between itself and its two adjacent cells. Based on the Lagrangian framework, the locations of the cells can be defined by motion of these interfaces. The equation of those interfaces is defined as

$$\frac{\mathrm{d}x_{j\pm1/2}}{\mathrm{d}t}=u_{j\pm1/2} \tag{1}$$

where $u_{j\pm1/2}$ on the interface is solved by Riemann resolver and it would be interpreted in detail later. The average density of each cell can be written as

$$\bar{\rho}_j=\frac{m_j}{\bar{A}_j(x_{j+1/2}-x_{j-1/2})} \tag{2}$$

where m_j is the mass of the j-th cell and $\bar{A}_j$ the average area of the section of the j-th cell. The momentum equation of the j-th cell can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}(m_j\bar{u}_j)=[P_{j-1/2}A_{j-1/2}-P_{j+1/2}A_{j+1/2}+\bar{P}_j(A_{j+1/2}-A_{j-1/2})-\bar{F}_{\mathrm{wall}}-\bar{F}_{\mathrm{loss}}] \tag{3}$$

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where P is the pressure, $\overline{F}_{\text{wall}}$ the wall shear force due to viscous effect, and $\overline{F}_{\text{loss}}$ the effective force due to pipe-fitting loss. The effect of the power of pressure and the conduction of heat on the interface should be mainly considered in the energy equation of the cell, described as

$$\frac{\mathrm{d}}{\mathrm{d}t}(m_j\overline{E}_j)=[P_{j-1/2}A_{j-1/2}u_{j-1/2}-P_{j+1/2}A_{j+1/2}u_{j+1/2}+\bar{q}_j] \tag{4}$$

where $E_j=\mathrm{e}_j+\frac{1}{2}u_j^2$, e the specific internal energy, and $\bar{q}_j$ the heat conduction rate. For perfect driver gas, the equation of state is

$$P=\rho RT=\rho(\gamma-1)e \tag{5}$$

For the air test gas, which is assumed to be in chemical equilibrium, the curve given in Ref. [4] is used to obtain parameters P,T,a and γ as functions of ρ,e for temperature up to 25 000 K.

The detailed expression of the viscous shearing force of the cell is

$$\overline{F}_{\text{wall}}=\tau_{\text{wall}}\pi\overline{D}(x_{j+1/2}-x_{j-1/2}) \tag{6}$$

where $\overline{D}=2\left(\frac{\overline{A}}{\pi}\right)^{1/2}$, $\tau_{\text{wall}}=\frac{-\rho f|u|u}{8}$, f is the friction factor and relative to the Reynolds number and the recovery factor with its detailed expression shown in Refs. [5-6]. The abrupt change of the shock tube section produces a part of pressure loss, expressed as

$$\overline{F}_{\text{loss}}=\frac{\Delta P_{\text{loss}}}{L_{\text{loss}}}\overline{A}(x_{j+1/2}-x_{j-1/2}) \tag{7}$$

where $\Delta P_{\text{loss}}=-\frac{1}{2}K_L\rho|u|u$, L_{loss} is the pressure loss length from the area change of the section, and the viscosity loss coefficient is $K_{\text{L}}=0.25$. The heat conduction rate of the cells can be formed as the following in Ref. [7]

$$q=\pi h\overline{D}(x_{j+1/2}-x_{j-1/2})(T_{\text{w}}-T_{\text{aw}}) \tag{8}$$

where the heat conduction coefficient $h=\rho C_P|u|St$, the Stanton number $St=\frac{f}{8}Pr^{-2/3}$, T_{w} is the wall temperature, and T_{aw} the adiabatic wall temperature.

2 RIEMANN RESOLVER

The Riemann resolver in the programs is mainly used to obtain the velocity and pressure of interfaces. For accuracy of the programs, the design of the Riemann resolver is divided into two stages. In the first stage, the Harten-Lax-van Leer-Contact (HLLC) approximation Riemann resolver is used. In the second stage, if the larger intermediate state pressure leap is found, the precise Riemann resolver would be used.

The solving process of the HLLC approximation Riemann resolver is described as:

(1) The mid variable can be calculated as the following form

$$P_*=\frac{1}{2}(P_{\text{L}}+P_{\text{R}})+\frac{1}{2}(u_{\text{L}}-u_{\text{R}})(\hat{\rho}\hat{a})$$

$$u_*=\frac{1}{2}(u_{\text{L}}+u_{\text{R}})+\frac{1}{2}\frac{(P_{\text{L}}-P_{\text{R}})}{(\hat{\rho}\hat{a})}$$

where the subscripts "L" and "R" represent the left and right side of the cell interface, the subscript "$*$" represents the intermediate state, the average value $\hat{\rho}$ and $\hat{a}$ can be selected as $\hat{\rho}=\frac{1}{2}(\rho_{\text{L}}+\rho_{\text{R}})$ and $\hat{a}=\frac{1}{2}(a_{\text{L}}+a_{\text{R}})$, or can be calculated by Roe average.

(2) The wave speed can be calculated as the following form

$$S_{\text{L}}=u_{\text{L}}-a_{\text{L}}Q_{\text{L}},\ S_*=u_*,\ S_{\text{R}}=u_{\text{R}}-a_{\text{R}}Q_{\text{R}} \tag{9}$$

where

$$Q_K=\begin{cases}1 & P_*\leqslant P_K\\ \left[1+\frac{\gamma+1}{2\gamma}\left(\frac{P_*}{P_K}-1\right)\right]^{1/2} & P_*>P_K\end{cases}$$

(3) Due to the approximate Godunov theory, the intermediate variables (are mainly velocity and pressure) can be expressed as

$$F_{j+\frac{1}{2}}^{\text{HLLC}}=\begin{cases}F_{\text{L}} & 0\leqslant S_{\text{L}}\\ F_{*\text{L}}=F_{\text{L}}+S_{\text{L}}(U_{*\text{L}}-U_{\text{L}}) & S_{\text{L}}\leqslant 0\leqslant S_*\\ F_{*\text{R}}=F_{\text{L}}+S_{\text{L}}(U_{*\text{L}}-U_{\text{L}}) & S_*\leqslant 0\leqslant S_{\text{R}}\\ F_{\text{R}} & 0\geqslant S_{\text{R}}\end{cases} \tag{10}$$

In the second stage, if P_* in the first stage is greater than P_{L} and P_{R} (for example, $P_*>10P_{\text{R}}$, $P_*>10P_{\text{L}}$), P_* would be improved by precise Riemann resolver. The more details about the precise Riemann resolver can be seen in Ref. [8]. It is required to emphasize that the precise Riemann resolver should be always used in the iterative process of the contact surface, because the

drivering gas and the driven gas have the difference composition.

3 MASS LOSS MODEL

For the quasi-one-dimensional numerical simulation of the shock tube, the speed of contact surface is usually incorrect because typical boundary layer models do not redistribute the mass from the core flow to the walls[3]. Tani, Sharma and Wilson respectively proposed different mass loss models[9-10], but the computational effort of these models is too large. Thus, a simple mass loss model is introduced by using Mirels' result.

It is assumed that the thickness of the boundary layer in the shock tube is much smaller than the radius of the tube. Then, the mass change rate in the boundary layer can be expressed as

$$\dot{m} = \pi D \int_0^{\delta} \rho u \, \mathrm{d}y \tag{11}$$

where D is the diameter of the shock tube, and δ the thickness of the boundary layer. Using Howarth transformation[11], the above formula becomes

$$\dot{m} = \pi D \rho^* u_e \delta_i \int_0^1 \frac{u}{u_e} \mathrm{d}\left(\frac{y}{\delta_i}\right) \tag{12}$$

where u_e is the velocity of the core flow region, and δ_i the thickness of the incompressible boundary layer. In the model, the 1/7 power law velocity profile is used to describe the turbulent boundary layer velocity profile[11]. According to Refs. [12-13], we have

$$\dot{m} = \frac{7}{8} \pi D \rho^* u_e k_{\mathrm{turb}} x_l Re_{xl}^{-1/5} \tag{13}$$

where $k_{\mathrm{turb}} = 0.375$, x_l represents the distance from the present location to the initial location of the cell in the shock tube, Re_{xl} is the Reynolds number changed on x_l. For the convenience of the iteration, Eq. (13) can be rewritten as

$$\mathrm{d}\dot{m} = \frac{7}{8} \pi D \rho^* u_e k_{\mathrm{turb}} Re_{xl}^{-1/5} \mathrm{d}x_l \tag{14}$$

Eq. (14) shows that in every time step $\mathrm{d}t$ and for the cell with the length of $\mathrm{d}x_l$, the mass captured by the boundary layer is $\mathrm{d}\dot{m} \times \mathrm{d}t$. Moreover, the turbulent heat and energy loss item can be expressed as

$$\mathrm{d}\dot{q}_{\mathrm{loss}} = 0.889 u_e \mathrm{d}\dot{m}, \quad \mathrm{d}\dot{E}_{\mathrm{loss}} = h^* \mathrm{d}\dot{m} \tag{15}$$

where h^* represents the Eckert reference enthalpy in the above formula.

4 EXAMPLE

There are 1 800 cells selected in the simulation, including 300 cells of high pressure driving gases and 1 500 cells of low pressure test gases. During the every iteration, the quantities on the either side of each interface are reconstructed by limiter function and the predictor-corrector scheme is used. In addition, boundary conditions are applied by setting up two layers of ghost cells at each of the boundaries

To check the validation of the program, the experimental results in the T4 free piston shock tunnel and Jacobs' numerical results in Ref. [1] are compared with the present simulation. According to the size of the T4 wind tunnel, the size of the area change shock tube in the programs is assumed as the following: Length and diameter of the high pressure section are 0.433, 0.229 m respectively; length and diameter of the low pressure section are 10.0, 0.076 m respectively. Table 1 shows the incident shock speeds for the simulations and experiment, where X is the displacement along the shock tube with the diaphragm station as an initial point. From comparison, the simulation in this paper is closer to the experimental result.

After the reflection of the shock wave, the calculated pressure value in the nozzle reservoir is close to 80 MPa and a little larger than Jacobs' results.

Table 1 Result comparison (shock speeds)

X/m	Experimental result/ (km · s^{-1})	Jacobs' result/ (km · s^{-1})	Present result/ (km · s^{-1})
0		5.45	5.46
5	4.36	4.35	4.35
7	4.03	4.08	4.05
9	3.79	4.00	3.82
10		3.75	3.74

5 CONCLUSION

A computer program for fast simulation of

transient hypersonic flow is presented in this paper. Though this code still needs further validation and modification, what the most important is, the initial demonstration and analysis show that the speed and accuracy of the calculation of this code are both satisfied with the requirement. The programs can be extended and developed to simulate the entire running process of free piston shock tunnel.

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