Dec. 2013 Transactions of Nanjing University of Aeronautics & Astronautics Vol. 30 No. 4

HIGH CYCLE FATIGUE RELIABILITY ANALYSIS ON ROTOR HUB BASED ON APPROXIMATION TECHNIQUE

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Abstract: A high cycle fatigue reliability analysis approach to helicopter rotor hub is proposed under working load spectrum. Automatic calculation for the approach is implemented through writing the calculating programs. In the system, the modification of geometric model of rotor hub is controlled by several parameters, and finite element method and S-N curve method are then employed to solve the fatigue life by automatically assigned parameters. A database between assigned parameters and fatigue life is obtained via Latin Hypercube Sampling (LHS) on tolerance zone of rotor hub. Different data-fitting technologies are used and compared to determine a highest-precision approximation for this database. The parameters are assumed to be independent of each other and follow normal distributions. Fatigue reliability is then computed by the Monte Carlo (MC) method and the mean-value first order second moment (MFOSM) method. Results show that the approach has high efficiency and precision, and is suitable for engineering application.

Key words: helicopters; rotor hub; parameterization; high cycle fatigue; reliability; approximate model

CLC number: V215 **Document code**: A **Article ID**: 1005-1120(2013)04-0335-08

INTRODUCTION

Main rotor, one of the key components of helicopter, has a direct impact on helicopter's flight performance. Teetering rotor is a wide application for main rotor system. Two blades are connected to the spindle by a yoke which consists of blade grips and pitch change bearings. Major loads, i. e. centrifugal forces and pneumatic pressure, are transferred through the blade grips. Just like other dynamic components, the blade grip's fatigue life is greatly dominated by cyclic load.

Mechanical fatigue can be classified to high cycle fatigue and low cycle fatigue. S-N curve method and Mason-Coffin method are usually used to describe these problems. Despite the fact that the mechanical fatigue theory has been developed in the passing decades, these two methods still have wide engineering application in fatigue life's prediction[1-2].

Approximate technology is an approximate computing technology which uses mathematical functions instead of complex engineering problems by fitting or interpolation methods, and the mathematical function relationship between design variables and response must correspond to the relationship between the known sample points and response information[3]. Approximate technology can simplify the analysis and improve the computational efficiency, and then replace the large number of implicit functions of real world problems into explicit approximate math functions. A lot of research on approximate technology had been performed by many domestic and foreign scholars and researchers[4-8], who promoted

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Received date: 2011-11-11; **revision received date**: 2013-04-20

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the development and application of approximate technology.

In recent years, integrating geometric modeling and structural computingfor structure design is becoming a tendency[9]. Although lots of researches have predicted fatigue life and reliability for the helicopter dynamic components, the quantitative relation between structural dimension and fatigue life is still far away from figure out[10-14]. Therefore, a high cycle fatigue reliability compute framework is researched and constructed for helicopter dynamic components in this paper. And it is found that the approximate technology used to fit performance function has creation utilization in rotor hub of helicopter and has a high efficiency.

1 ANALYSIS METHOD FOR STRUCTURAL FATIGUE RELIABILITY

Uncertainty cannot be avoided in shape and dimensions of structural components, such as length and diameter. The dimension variation could induce the changes of stress and strain in the structure, and will eventually influence the fatigue life[15-16].

A general approach to fatigue reliability analysis is developed as follows. Probability distribution functions for shape and dimension parameters are assumed and the quantification function between parameters and fatigue life is obtained. The performance function and the computed reliability are obtained.

1.1 Reliability requirement

In an aircraft′s design, the structural reliability is usually required over a value P_r, meanwhile the working life is no less than a fixed time, denoted as T. Using the method in Section 1.2, working life could be equivalent to the fatigue failure cycle N. Then, the structural reliability requirement can be converted as follows: Under the structural load spectrum, its fatigue life must be larger than cycle N, while its reliability reaches to P_r.

1.2 Relationship between fixed hour *T* and fatigue cycle *N*

Determination of the relationship between fixed hour T and fatigue cycle N is based on following three assumptions:

(1) Material fatigue failure cycle is independent on frequency under the same mean load and load ratio.

(2) Damage caused by fatigue circle cannot be recovered by itself.

(3) Fatigue damages under different loads and load ratios can be linearly cumulated.

Based on these assumptions, each cyclic damage of material is the same and the each damage ratio is $1/N_i$. Thus, damages can be cumulated linearly. When the sum of damage indicator reaches 1, the structure is considered to be failed. The following steps give the relationship between fatigue life N_f and fixed hours T.

(1) Different loads F_m, load ratios R and used cycles N_i are counted out from load spectrum during a flight of this aircraft. Subscript i refers to the number of loads and load ratios.

(2) Total damage D is computed as follows by used cycles N_i and fatigue life N_f.

$$D=\sum\frac{N_i}{N_f} \tag{1}$$

(3) The relationship between fixed hour T and fatigue life N_f is shown as

$$T=\frac{1}{D}t \tag{2}$$

where t refers to the flight time.

1.3 Fatigue life computation method

Assurance coefficient of rotor hub is so high that its deformation belongs to elastic deformation. Therefore, S-N curve method is a convenient method for its fatigue life prediction. In order to efficiently get quantification function between parameters and fatigue life, the process from the structural parameters to structural fatigue life must be automatic, which includes three sections as follows.

(1) Geometric model parameterization

Extract some parameters (x_1, x_2, …, x_n) to control structural geometric feature. Visual Basic language is used to compile a program to read values for control parameters, modify geometric feature in software CATIA for the structure, and export a new geometric model for this structure.

(2) Structural computation

Finite element method is taken to analyze structural in software ANSYS. Extract a peak value of load $F_{\max}$ from structural load spectrum (F_m, R). Generate a mesh for this structure. Then apply boundary condition to it. Solve its stress and strain, and get the most possible failure node in this structure. All these steps are automated in ANSYS APDL language and integrated in a batch processing system.

(3) High circle fatigue life computation

According to Eq. (3), stress level σ_m can be solved by the stress of most possible failure node $\sigma_{\max}$ and load ratio R. Then least square method is used to fit the S-N data shown in Fig. 1. And a program is generated by using FORTRAN to compute the fatigue life N_{f} of most possible failure node.

$$R=\frac{2\sigma_m-\sigma_{\max}}{\sigma_{\max}} \tag{3}$$

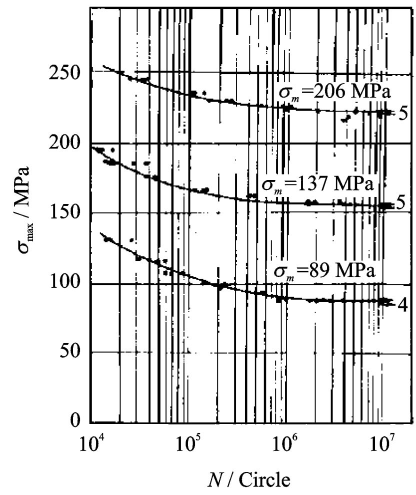

Fig. 1 S-N curve of T6 7A04 aluminum with gap

Hence, a relationship between structural feature and this fatigue life is obtained. It is quantified as

$$N_{\mathrm{f}}=f(x_1,x_2,\cdots,x_n) \tag{4}$$

where x_1, x_2, …, x_n represent the structural feature parameters, i. e. length, width, diameter, distance between holes, etc.

1.4 Failure judgment

R-S model is taken for reliability computation, as shown in Eq. (5). This judgment is summarized in Table 1.

$$P_{\mathrm{r}}=P\{N_{\mathrm{f}}>N_0\} \tag{5}$$

where N_0 represents the fatigue life of some load spectrum.

Table 1 Failure judgment

Case	Judgment
$N_{\mathrm{f}}>N_0$	Acceptance
$N_{\mathrm{f}}=N_0$	Critical fatigue life
$N_{\mathrm{f}}<N_0$	Failure

Meanwhile the performance function, shown in Eq. (6), can be got.

$$g(N_{\mathrm{f}})=N_{\mathrm{f}}-N_0 \tag{6}$$

1.5 Reliability computation

1.5.1 Monte-Carlo (MC) method

In order to guarantee enough sample points fall in the failure field, an ample sample must be taken. Eq. (7) gives a determination rule for the sample number. Usually, the structural failure probability P_{f} is very low, for example 0.001. And according to Eq. (7), the sample number N_{sample} must be very large, achieving 100 000 samples. The samples trend to be state distribution, which is validated by the strong law of large numbers (SLLN)[17].

$$N_{\text{sample}}=\frac{100}{P_{\mathrm{f}}} \tag{7}$$

1.5.2 Mean-value first order second moment (MFOSM) method

MFOSM is a classical reliability compute method. The performance function is expanded in Taylor series at the mean-value point and given by

$$g(N_{\mathrm{f}})=N_{\mathrm{f}}-N_0=g(x_1,x_2,\cdots,x_n)=$$
$$g(\mu_{x_1},\cdots,\mu_{x_n})+\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x}\right)_{\mu_{x_i}}(x_i-\mu_{x_i}) \tag{8}$$

If design variables x_1, x_2, …, x_n are independent, the performance function's mean value and variance can be given by

$$\mu = g(\mu_1, \mu_2, \cdots, \mu_n) \tag{9}$$

$$\sigma^2 = \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x}\right)^2_{\mu_{x_i}} \sigma_{x_i}^2 \tag{10}$$

Reliability index β can be solved by using Eq. (11). Then the reliability can be got by using Eq. (12).

$$\beta = \frac{\mu}{\sigma} \tag{11}$$

$$R = \Phi(\beta) \tag{12}$$

2 APPROXIMATION OF STRUCTURAL FATIGUE COMPUTATION

The reliability problem of rotor hub is a small probable failure issue. According to Eq. (6), the MC method needs to take a large number of samples. And the MFOSM method needs to know the expression of performance function (Eq. (6)). Thus, it is important to obtain an efficient and exact function for Eq. (4). Hence an approximate technology is introduced to fit Eq. (4).

2.1 Experiment design

The objective of experiment design is to obtain a database for the approximation. Fig. 2 gives the feature of sample points of the hyper Latin sample method. It can be seen that the method has a good description of the whole design space. Therefore, this method is taken for the construction of approximation database.

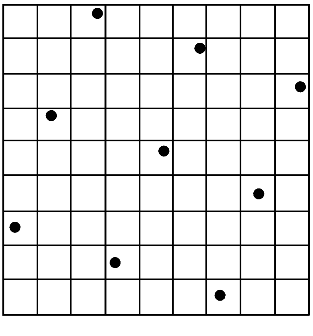

Fig. 2 Feature of sample points of hyper Latin sample method

2.2 Approximation

2.2.1 Closest interpolation (CI) method

Taking use of the database, a program is compiled to compare database points with the values of structural featural parameters. Thus, a closest database point is found, and its fatigue life is given as the fatigue life of structural feature.

2.2.2 Response method (R method)

Different from the above-mentioned method, $(n^2+3n+2)/2$ closest points are chosen during the program comparison. Then the constant in Eq. (13) is solved by $(n^2+3n+2)/2$ closest points. At the end, the value of structural feature parameter is substituted into Eq. (13) to get its fatigue life[18].

$$f(X) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_{ii} x_i^2 + \sum_{i=1,j=1}^{n} a_{ij} x_i x_j \tag{13}$$

2.2.3 Kriging method (K method)

Taking use of the database, a polynomial expression $f(X)$ and probability function are built and given by

$$y(X) = f(X) + z(X) \tag{14}$$

The detailed description on the K method can be found in Ref. [19].

2.2.4 Precision judgment

The sub-class of sampling points is taken to compute their fatigue life by fatigue life compute method and approximation, respectively. As shown in Eq. (15), n_s represents the point numbers in sub-class, y_i the fatigue life got by fatigue life compute method, and $\bar{y}_i$ the fatigue life got by the approximation. RMSE gives a judgment of approximation precision. The lower the RMSE is, the higher the precision is.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n_s} (y_i - \bar{y}_i)^2}{n_s}} \tag{15}$$

3 STRUCTURAL FATIGUE RELIABILITY ANALYSIS

Based on the discussions previously, a flow chart shown in Fig. 3 can be drawn for structural fatigue reliability analysis.

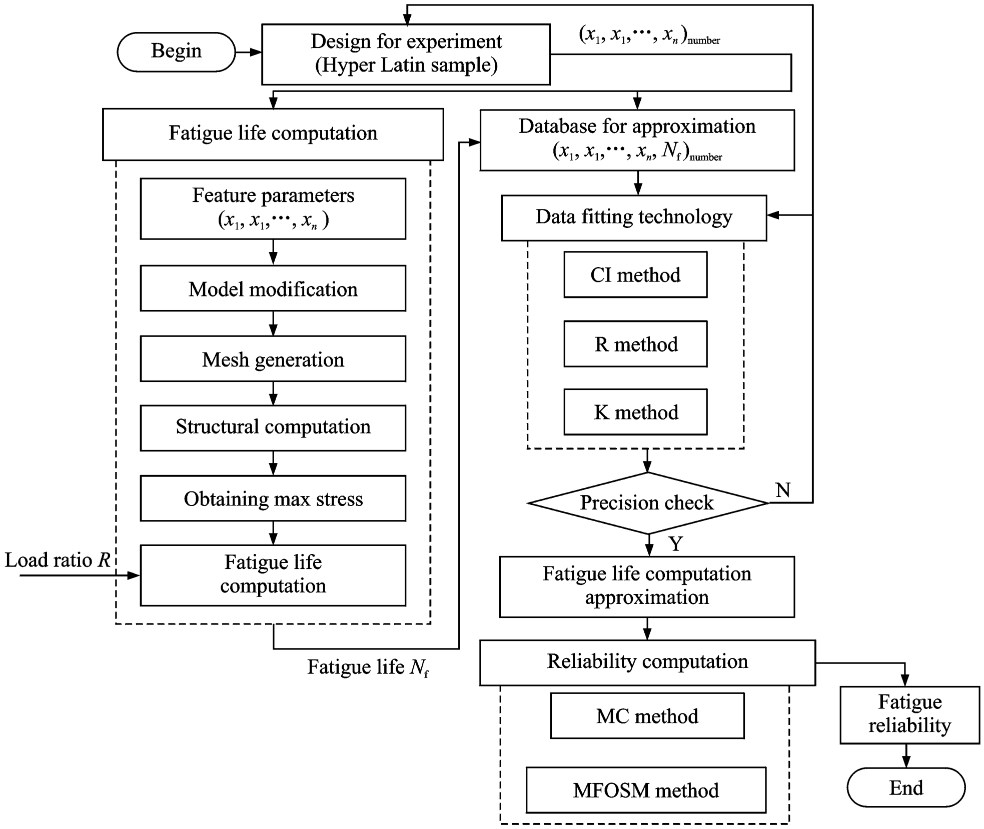

Fig. 3 Flow-chart for fatigue reliability computation

4 EXAMPLE AND DISCUSSION

A rotor hub of helicopter shown in Fig. 4 is taken as an example. The material property is listed in Table 2. A load spectrum (F_m, R) is extracted from helicopter flight. The load F_m makes up of centrifugal force of blade 45 200 N and lifting moment 2 000 N * m at rotational speed 520 rad/min. Let R be 0.667. The fatigue life of the helicopter rotor hub is required not to be lower than 65 000 cycles. The fatigue reliability is solved using the framework shown in Fig. 3.

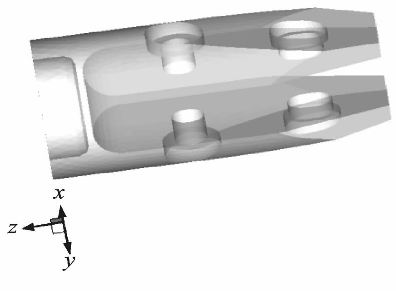

Fig. 4 Rotor hub model

Table 2 LC4 (7A04) material property

Material	Yield strength σ_S/MPa	Tensile strength σ_b/MPa	Young's modulus E/MPa	Poisson's ratio μ	Density/ (g · cm^{-3})
LC4CS	571	620	73×10^3	0.3	2.76

The shape parameters for model Ⅰ and model Ⅱ are shown in Figs. 5, 6. Six parameters are taken as the design variables to control the changing of round hole and elliptical hole via a visual basic compiler. Probability nature of parameters is listed in Table 3.

By using batch processing and APDL languages, the tetrahedral mesh is generated (Fig. 7), the boundary condition is applied (Figs. 8, 9), and the stress distribution is solved step by step (Fig. 10). Then FORTRAN compiler is used to extract the maximum stress from structural compute results, and solve its fatigue life by integrating S-N curve method. Result is shown in Table 4.

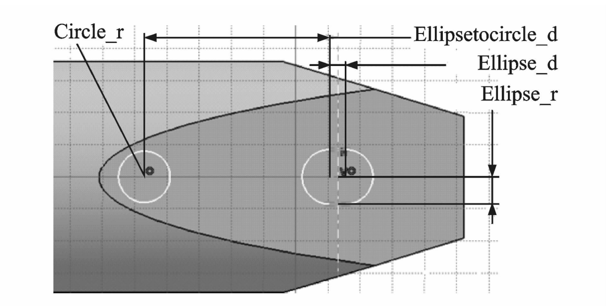

Fig. 5 Shape parameters for model Ⅰ

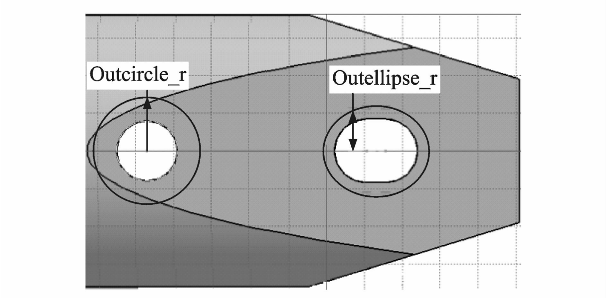

Fig. 6 Shape parameters for model Ⅱ

Table 3 Probability nature of parameters mm

Design variable	Mean value	Variation
Circle_r	16.0	0.006 0
Outcircle_r	28.2	0.010 0
Ellipse_r	8.5	0.006 0
Ellipse_d	5.0	0.004 0
Outellipse_r	11.5	0.017 3
Ellipsetocircle_d	60.0	0.005 0

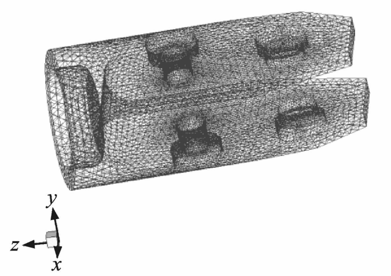

Fig. 7 Tetrahedral mesh

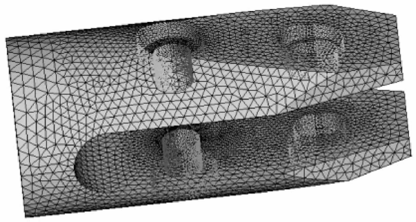

Fig. 8 Area for applied load

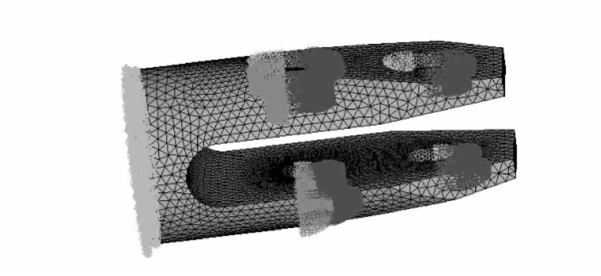

Fig. 9 Boundary condition

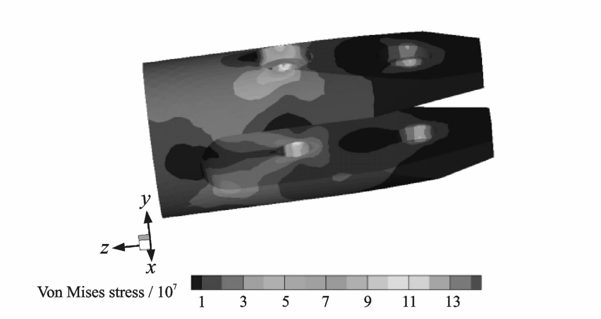

Fig. 10 Mises stress distribution

Table 4 Fatigue computation result

Parameter	Result
$\sigma_{\max}$/MPa	$0.143\,08\times10^9$
σ_m/MPa	119.23
N/Circle	242 457.0

Based on the hyper Latin sampling method, the fatigue life distribution of rotor hub is obtained. And in the simulation, only 350 times are sampling between tolerance zones of geometric variables of rotor hub. The rule of life distribution about rotor hub is shown in Fig. 11. From Fig. 11, two conclusions can be easily drawn:

(1) Fatigue life is distributed between 60 000

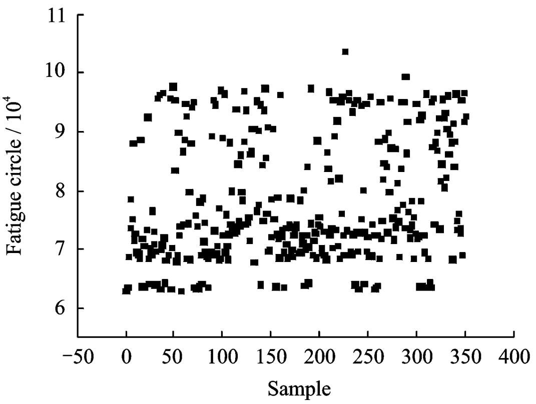

Fig. 11 Fatigue life distribution in parameter variation

cycle and 110 000 cycle, when rotor hub is designed to fit the shape tolerance. And most fatigue life is close to 72 000 cycle.

(2) The distribution of fatigue life has a strong nonlinearity in the whole shape tolerance.

In order to quantify the rule of fatigue life distribution, three approximations are constructed according to Section 2. 2. And their precession is compared by Eq. (15), listed in Table 5. It is shown that the closet interpolation method has the highest precession for the strong nonlinearity problem.

Table 5 Comparison of approximation precession

Data-fitting method	RMSE
Fatigue computation program	0. 000
CI method	0. 003
R method	0. 200
K method	0. 200

Finally, based on the approximation which has the highest degree of accuracy, the reliability is computed by both the MC method and the MFOSM method with the assumption that the design variables follow normal distribution. The mean value and variation are given in Table 6. The number of samples is set to be 2×10^6 in MC method. Refers to the comparisons shown in Table 6, it can be found that the reliability given by the MC method is 0. 88, while the reliability given by the MFOSM method is 1. 00.

Table 6 Comparisons of reliability

Method	Failure judgment	Fatigue reliability	Precession
MFOSM method	>65 000	1. 00	Lower
MC method	>65 000	0. 88	Higher

5 CONCLUSION

A framework for fatigue reliability computation of helicopter rotor hub is proposed. In the framework, a program is generated by several programming languages to automate the process from structure remodeling to fatigue life computation. Hyper Latin sample method is used to build a database to describe the distribution rule of fatigue life. In order to find a highest accurate approximation, different data-fitting methods are introduced and compared for the rule fitting. Then the MC method and the MFOSM method are taken to compute the reliability of fatigue life. Results show that, the MFOSM method has a poor performance for disposing nonlinear problem. The CI method is superior in the multi-dimension and non-linear problem, when the ample data are commanded. In a word, this framework, bringing efficiency and precession together to form a unit, is valuable for engineering applications.

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