OPTIMIZED STRAPDOWN CONING CORRECTION ALGORITHM

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Abstract: Traditional coning algorithms are based on the first-order coning correction reference model. Usually they reduce the algorithm error of coning axis (z) by increasing the sample numbers in one iteration interval. But the increase of sample numbers requires the faster output rates of sensors. Therefore, the algorithms are often limited in practical use. Moreover, the noncommutivity error of rotation usually exists on all three axes and the increase of sample numbers has little positive effect on reducing the algorithm errors of orthogonal axes (x, y). Considering the errors of orthogonal axes cannot be neglected in the high-precision applications, a coning algorithm with an additional second-order coning correction term is developed to furtherimprove the performance of coning algorithm. Compared with the traditional algorithms, the new second-order coning algorithm can effectively reduce the algorithm error without increasing the sample numbers. Theoretical analyses validate that in a coning environment with low frequency, the new algorithm has the better performance than the traditional time-series and frequency-series coning algorithms, while in a maneuver environment the new algorithm has the same order accuracy as the traditional time-series and frequency-series algorithms. Finally, the practical feasibility of the new coning algorithm is demonstrated by digital simulations and practical turntable tests.

Key words: SINS; rotation vector; coning algorithm; coning correctness

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Nomenclature

- a, b Amplitudes of angular vibrations in two orthogonal axes of body
- $\boldsymbol{I}$, $\boldsymbol{J}$ Unit vectors along two body axes about which oscillations are occurring
- $\boldsymbol{K}$ Unit vector along body axis which is perpendicular to $\boldsymbol{I}$, $\boldsymbol{J}$
- Ω Frequency associated with angular oscillations
- ω Angular rate expressed with coordinates in body frame
- T Iteration interval

INTRODUCTION

From Bortz′s first-order rotation vector equation, Miller proposed the traditional three-sample coning algorithm[1]. In Ref. [1], Miller validated that the coning algorithm had the optimal performance in a coning motion environment. On the purpose of reducing algorithm error to satisfy the requirement of high-precision applications, other improved coning algorithms were proposed[2-5]. To be computationally more efficient, these algorithms generally employed simplified vector cross product terms and reduced the algorithm error of coning axis (z) by increasing the sample numbers per iteration interval. In 2006, Paul Savage made a summarization on all coning algorithms according to different mathematical models, and divided coning algorithms based on the first-order rotation vector equation

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into two categories[6]. The algorithm errors under vibration and general maneuvering motion were theoretically analyzed and validated. In 2010, Paul Savage divided traditional coning algorithms into two categories: Time-series coning algorithms and frequency-series coning algorithms depending on different time or frequency series expansion techniques[7]. And a new approach to coning algorithm design using least-squares error minimization was proposed in Ref. [7] to achieve optimal performance over a design frequency range.

In Refs. [1-7], all the coning algorithms were based on the first-order rotation vector equation. When body is undergoing vibration motion, these algorithms can only compensate the noncommutativity of coning axis which is directed along an axis z perpendicular to the orthogonal axes x, y. But the noncommutivity error of rotation usually exists on all three axes. In this paper, the solution to the second-order rotation vector rate equation in a vibration environment is calculated and the truth-value for second-order coning correction is given. From the truth-value, a second-order coning algorithm which can effectively reduce the algorithm errors on orthogonal axes (x, y) is developed.

1 SECOND-ORDER CONING CORRE- CTION UNDER VIBRATION

Assume that a coning motion defined by the angular rate vector is[5-7]

$$\omega = a\Omega\cos\Omega t\boldsymbol{I} + b\Omega\sin\Omega t\boldsymbol{J} \tag{1}$$

Over an iteration interval T ending at t_k, the first-order coning correction $\delta\Phi$ in Ref. [5] is

$$\delta\Phi = \frac{1}{2}\int\Delta\theta\times\omega\mathrm{d}t = \frac{1}{2}ab\Omega\left(T - \frac{1}{\Omega}\sin\Omega T\right)\boldsymbol{K} \tag{2}$$

From the Savage's paper[7], coning algorithms derived from the first-order coning correction model $\delta\Phi$ can be divided into two categories: Time-series coning algorithms and frequency-series coning algorithms. The typical frequency-series algorithm is the classical three-sample coning algorithm presented by Miller[1]. In the following section it is denoted as Algorithm A.

Algorithm A

$$\Phi_A = \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + 9/20\Delta\theta_1\times\Delta\theta_3 + 27/40\Delta\theta_2\times(\Delta\theta_3 - \Delta\theta_1)$$

The typical time-series algorithm is denoted as Algorithm B in this paper. The algorithm is[7]

Algorithm B

$$\Phi_B = \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + 33/80\Delta\theta_1\times\Delta\theta_3 + 57/80\Delta\theta_2\times(\Delta\theta_3 - \Delta\theta_1)$$

From Eq.(2) we can easily find that in a vibration environment the first-order coning correction term $\delta\Phi$ only exists on the coning axis z. However, the noncommutivity of rotation usually exists on all three axes. Therefore, it is necessary to study new method to compensate the algorithm errors on two orthogonal axes (x, y) for improving algorithm accuracy. To reach this goal, a higher-order rotation vector equation is employed

$$\dot{\Phi} = \omega + 1/2\Phi\times\omega + 1/12\Phi\times(\Phi\times\omega) \tag{3}$$

From the traditional coning algorithms[1-3], we can get

$$\Phi = \Delta\theta + \delta\Phi, \delta\Phi \approx 1/2\int_{t_{m-1}}^{t_m}\Delta\theta\times\omega\mathrm{d}t \tag{4}$$

Substituting Eq.(4) into Eq.(3), we have

$$\begin{aligned}\Phi =& \Delta\theta + \frac{1}{2}\int_{t_{m-1}}^{t_m}(\Delta\theta\times\omega)\mathrm{d}t + \\ & \left[\frac{1}{4}\int_{t_{m-1}}^{t_m}\left(\int_{t_{m-1}}^{t}\Delta\theta\times\omega\mathrm{d}t\right)\times w\mathrm{d}t + \right. \\ & \left.\frac{1}{12}\int_{t_{m-1}}^{t_m}\Delta\theta\times(\Delta\theta\times\omega)\mathrm{d}t\right] = \\ & \Delta\theta + \delta\Phi + \delta\delta\Phi\end{aligned} \tag{5}$$

Substituting Eq.(1) into the "$\delta\delta\Phi$" term of Eq.(5), the truth-value of $\delta\delta\Phi$ over an iteration interval T (ending at t_k) is given as

$$\begin{aligned}\delta\delta\Phi =& \left(\frac{1}{2}\Omega T\cos\frac{\Omega T}{2} - \frac{3}{4}\sin\frac{\Omega T}{2} - \right. \\ & \left.\frac{1}{12}\sin\frac{3}{2}\Omega T\right)\left[ab^2\cos\Omega\left(t_{k-1} + \right.\right. \\ & \left.\left.\frac{T}{2}\right)\boldsymbol{I} + a^2b\sin\Omega\left(t_{k-1} + \frac{T}{2}\right)\boldsymbol{J}\right]\end{aligned} \tag{6}$$

As seen from Eq.(6), the noncommutivity under vibration also exists on x-axis and y-axis. Moreover $\delta\delta\Phi_x$ and $\delta\delta\Phi_y$ have the maximum magnitude of about $(\Omega T)^5/240$ order at $t_{k-1} = (k\pi - T/2)/\Omega$ (for $\delta\delta\Phi_x$) or $t_{k-1} = k\pi/\Omega$ (for $\delta\delta\Phi_y$). Considering the z-axis error has been reduced to

$(\Omega T)^9$ order by the traditional coning algorithms[5], $\delta\delta\Phi_x$ and $\delta\delta\Phi_y$ should also be compensated to make the accuracy of coning algorithms be axisymmetric. Furthermore, $\delta\delta\Phi_x$ and $\delta\delta\Phi_y$ in Eq. (6) are periodic, thus a coning algorithm without a correction term of $\delta\delta\Phi_x$ and $\delta\delta\Phi_y$ can be considered because of a periodic noise in gyro's output (on x, y axes), and the effects on attitude determination cannot be canceled completely because attitude update equation is nonlinear. Also considering the specific force acceleration is usually variant in a dynamic environment, the effects on velocity/position determination cannot be neglected too in high-precision applications.

2 SECOND-ORDER CONING CORRECTION ALGORITHM

The gyro output is assumed as[1]

$$\Delta\theta = a\tau + b\tau^2 + c\tau^3$$
$$\tau = t - t_{k-1},\ \tau \in (0, T) \tag{7}$$

By substituting Eq. (7) into Eq. (6) and the simplifications similar to Refs. [1-4], a conclusion can be obtained that a second-order coning correction algorithm should consist of the sum of all second-order cross products of the gyro output $\Delta\theta_i$. Hence the second-order coning correction algorithm is defined as

$$\delta\delta\hat{\Phi} = \sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k=j+1}^{N} l_{ijk}\Delta\theta_i \times (\Delta\theta_j \times \Delta\theta_k) \quad N=3 \tag{8}$$

Substituting Eq. (1) into Eq. (8), we have

$$\begin{cases} \Delta\theta_i \times (\Delta\theta_j \times \Delta\theta_k) = 8\sin^3(\Omega T/6)\sin((k-j)\Omega T/3)\cdot \\ \quad (ab^2 M\boldsymbol{I} - a^2 bN\boldsymbol{J}) \\ M = \sin\Omega(t_{k-1} + (2i-1)T/6) \\ N = \cos\Omega(t_{k-1} + (2i-1)T/6) \end{cases} \tag{9}$$

The optimal coning algorithm is to minimize the difference between the truth-value $\delta\delta\Phi$ and its algorithmic approximation $\delta\delta\hat{\Phi}$, which can be accomplished by using a truncated Taylor series expansion in powers of ΩT for each expression and equating coefficients of like terms. Considering that Eq. (9) is only relevant to the value of $|k-j|$ and i, there are six different values altogether

$$l_{112}\Delta\theta_1 \times (\Delta\theta_1 \times \Delta\theta_2),\ l_{113}\Delta\theta_1 \times (\Delta\theta_1 \times \Delta\theta_3)$$
$$l_{312}\Delta\theta_3 \times (\Delta\theta_1 \times \Delta\theta_2),\ l_{313}\Delta\theta_3 \times (\Delta\theta_1 \times \Delta\theta_3)$$
$$l_{212}\Delta\theta_2 \times (\Delta\theta_1 \times \Delta\theta_2),\ l_{213}\Delta\theta_2 \times (\Delta\theta_1 \times \Delta\theta_3) \tag{10}$$

The coefficients of terms in Eqs. (6,8), up to and including the seventh power multiplied by $\cos\Omega(t_{k-1} + T/2)$ on x-axis and multiplied by $\sin\Omega(t_{k-1} + T/2)$ on y-axis, are equated. Considering that making two equations equal only needs two independent parameters, we can let $l_{212} = l_{213} = 0$ and $l_{112} = -l_{312}$, $l_{113} = -l_{313}$. These simplifications leave two independent terms in Eq. (8): $l_{112}(\Delta\theta_1 - \Delta\theta_3) \times (\Delta\theta_1 \times \Delta\theta_2)$, $l_{113}(\Delta\theta_1 - \Delta\theta_3) \times (\Delta\theta_1 \times \Delta\theta_3)$. And there is

$$\begin{bmatrix} -2/243 & -4/243 \\ 11/26\ 244 & 23/13\ 122 \end{bmatrix} \begin{bmatrix} l_{112} \\ l_{113} \end{bmatrix} = \begin{bmatrix} -1/240 \\ 11/40\ 320 \end{bmatrix} \tag{11}$$

The solutions are

$$l_{112} = 837/2\ 240 \qquad l_{113} = 297/4\ 480 \tag{12}$$

The new second-order coning algorithm using the coefficients of Eq. (12) is denoted as Algorithm C, shown as

Algorithm C

$$\Phi_C = \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 + 9/20\Delta\theta_1 \times \Delta\theta_3 + 27/40\Delta\theta_2 \times (\Delta\theta_3 - \Delta\theta_1) + 837/2\ 240[(\Delta\theta_1 - \Delta\theta_3) \times (\Delta\theta_1 \times \Delta\theta_2)] + 297/4\ 480[(\Delta\theta_1 - \Delta\theta_3) \times (\Delta\theta_1 \times \Delta\theta_3)]$$

3 ALGORITHM ERROR ANALYSIS

In practical applications, the motion of aircraft can be divided into two categories: Vibration and non-vibration[5-7]. Typical vibration is usually expressed by a classical sinusoidal coning motion defined in Eq. (1). Non-vibration is usually expressed by a maneuver profile. If a coning algorithm works satisfactorily both in a vibration environment and in a maneuver (non-vibration) environment, it will satisfy most requirements of other environments[8]. Therefore the algorithm error under vibration and maneuver is analyzed in the following section to illustrate the advantages of the new second-order coning Algorithm C.

3.1 Algorithm errors under vibration

The error of Algorithm C is calculated by the algorithm output Φ_C minus the truth-value of rotation vector generated from Eqs. (2,5,6). Under vibration the z-axis error is as same as the error of the traditional three-sample coning algorithm which is about $(\Omega T)^9$ and has higher order over an iteration interval T (Ref. [5]) because the second-order coning correction defined by Eq. (6) only affects x and y axes. As stated in Section 2, the $(\Omega T)^5$, $(\Omega T)^7$ orders of the errors on x, y axes for Algorithm C are reduced to zero. Therefore the residual errors of Algorithm C on the x and y axes are

$$e_{Cx} = 1/2\ 799\ 360\cos\Omega(t_{k-1} + T/2)[(\Omega T)^9 + \cdots]$$
$$e_{Cy} = 1/2\ 799\ 360\sin\Omega(t_{k-1} + T/2)[(\Omega T)^9 + \cdots] \tag{13}$$

By comparing errors of the x and y axes of Algorithm C given in Eq. (13) with errors of traditional coning Algorithms A, B given in Refs. [5-6], a conclusion can be drawn that under vibration the new second-order coning Algorithm C has an advantage over the traditional coning Algorithms A, B.

3.2 Algorithm errors under maneuvers

Suppose that in a maneuver environment, the angular rate of body can be approximated as

$$\omega = A + B(t - t_{k-1}) + C(t - t_{k-1})^2 \tag{14}$$

where A, B, C are the polynomial coefficients.

Substituting Eq. (14) into Eq. (5), the second-order truth-value of rotation vector in a maneuver environment is given as

$$\begin{aligned}\Phi = {} & \Delta\theta + \delta\Phi + \delta\delta\Phi = \Delta\theta + 1/2\int(\Delta\theta \times \omega)\mathrm{d}t + \\ & 1/4\int\left(\left(\int\Delta\theta \times \omega\mathrm{d}t\right) \times \omega\mathrm{d}t\right) + 1/12\int\Delta\theta \times \\ & (\Delta\theta \times \omega)\mathrm{d}t = \Delta\theta + 1/6A \times BT^3 + \\ & 1/4A \times CT^4 + 1/10B \times CT^5 - 1/60B \times \\ & (A \times B)T^5 - 1/36C \times (A \times B)T^6 + \\ & 1/120A \times (A \times C)T^5 - 1/72B \times \\ & (A \times C)T^6 - 5/168C \times (A \times C)T^7 + \\ & 1/180A \times (B \times C)T^6 - 1/420B \times \\ & (B \times C)T^7 - 1/120C \times (B \times C)T^8\end{aligned} \tag{15}$$

The errors of Algorithms A, B, C are calculated by the algorithm output minus the truth-value of rotation vector given in Eq. (15), shown as

$$\begin{aligned} e_A &= \Phi_A - \Phi = [-1/540B \times C - 1/120A \times \\ &\quad (A \times C) + 1/60B \times (A \times B)]T^5 \\ e_B &= \Phi_B - \Phi = [-1/120A \times (A \times C) + \\ &\quad 1/60B \times (A \times B)]T^5 \\ e_C &= \Phi_C - \Phi = [-1/540B \times C - \\ &\quad 1/120A \times (A \times C)]T^5 \end{aligned} \tag{16}$$

In the past, Algorithm B was usually considered that it had the highest accuracy under cubic-polynomial maneuvers[5]. While the results given by Eq. (16) reveal an interesting property that Algorithms A, B, C actually have the same order accuracy under cubic-polynomial maneuvers. This is because that in the traditional researches (Refs. [5-7]) the error analysis is usually based on the first-order coning correction truth model. If higher (second)-order coning correction truth model (Eq. (15)) is considered, the results are different. Of course, the result of error analysis based on the higher (second)-order coning correction truth model is more precise.

4 SIMULATIONS

This section describes the simulation tests to demonstrate the advantages of the new second-order coning Algorithm C both in a vibration environment and in a maneuver environment.

4.1 Attitude errors under vibration

In this test with 600 s duration, the vibration profile is given in Eq. (1) with $a = b = 1°$, $\Omega = 2\pi$ rad/s, $T = 0.1$ s. Attitude errors are calculated by the attitude outputs minus attitude truth-value. The attitude truth-value is calculated by quaternion updating algorithm using rotation vector. The truth-value of the rotation vector is generated from the formulas provided in Eqs. (1,2,5,6) with $a = b = 1°$, $\Omega = 2\pi$ rad/s, $T = 0.1$ s. To assess the performance of three algorithms, the attitude error means and attitude error standard

Table 1 Comparison of attitude errors under vibration ($T=0.1$ s, $\Omega=2\pi$)

Attitude error	Algorithm A		Algorithm B		Algorithm C	
	Mean	StDev	Mean	StDev	Mean	StDev
Roll error	-2.057×10^{-7}	1.390×10^{-7}	-4.682×10^{-4}	4.496×10^{-4}	-5.151×10^{-8}	4.948×10^{-8}
Pitch error	8.791×10^{-7}	1.386×10^{-7}	0.001 5	8.048×10^{-4}	1.612×10^{-7}	8.824×10^{-8}
Head error	-3.846×10^{-6}	2.221×10^{-6}	−0.089 6	0.051 7	-3.831×10^{-6}	1.676×10^{-6}

deviations (StDev) over an iteration interval T during the digital validation are listed in Table 1. The axes of the navigation frame (n) are set to east (x-axis), north (y-axis), and up (z-axis).

As is shown in Table 1, the roll and pitch errors of Algorithm C are much smaller than those of Algorithms A and B while the head error is little smaller than that of Algorithms A, B. The reason is, as stated in Section 3.1, that under vibration the algorithm C has higher performance only on x (pitch channel) and y (roll channel) axes, the error of Algorithm C on z (head channel) axis is as same as that of the traditional coning algorithm (note that with the navigation frame defined in the above paragraph, a body rotating around x-axis (east) will cause pitch change). Therefore, the results are similar to the theoretical analysis and provide confidence in the validity of the error analysis for the new Algorithm C.

4.2 Attitude errors under maneuver

In this test, the errors of three coning Algorithms A, B, C are calculated by the algorithm outputs minus the truth-value of rotation vector given in Eq. (15) with the iteration interval $T=0.1$ s. The tests are conducted using the following incremental angle input maneuver profile

$$\begin{aligned}\Delta\theta_x &= -11(t-t_{k-1})+9(t-t_{k-1})^2-6(t-t_{k-1})^3\\ \Delta\theta_y &= -5(t-t_{k-1})+10(t-t_{k-1})^2+3(t-t_{k-1})^3\\ \Delta\theta_z &= 3(t-t_{k-1})-12(t-t_{k-1})^2+20(t-t_{k-1})^3\end{aligned} \tag{17}$$

The errors of three coning Algorithms A, B, C over an iteration interval T are

$$\begin{aligned}e_A &= (-0.253, -0.051, -0.515)\times10^{-3}\\ e_B &= (-0.249, -0.053, -0.513)\times10^{-3}\\ e_C &= (-0.004, -0.121, -0.372)\times10^{-3}\end{aligned} \tag{18}$$

From Eq. (18), we can see that none of the three algorithms can has higher accuracy than other ones on all three axes. That means Algorithms A, B, and C have the same order accuracy under maneuvers. This result is same to the theoretical analysis in Section 3.2.

5 TURNTABLE TEST

To further illustrate the advantages of the new coning Algorithm C, a vibration test with 100 s duration is produced by a two-axis turntable. The navigation frame is as same as the preceding simulation in Section 4.1. The turntable has a work mode of "vibration" which can simultaneously produce sinusoidal/cosinoidal angular vibrations about the x (pitch channel) and z (head channel) axes (note that the orthogonal axes are x, z and the coning axis is y now). The frequency and amplitude of the vibration about x and z axes are 1 Hz and 1°. The inertial measurement unit (IMU) used in test is composed of orthogonal three-axis laser gyros and accelerometers. The bias stability of the gyro is 0.01°/h. The output rates of the gyro and the turntable are both 50 Hz. Hence the iteration interval T is 0.06 s. The experimental photo is given in Fig. 1.

Fig. 1 Turntable experimental photo

The angular changes of head channel and pitch channel are calculated by using the new second-order coning Algorithm C and the traditional coning Algorithm A. The truth-values of the angular changes are achieved by the output angles of the turntable (the turntable output is a relative angle to the initial attitude). The synchronization between the turntable output and the IMU output is achieved in two steps: (1) Before vibration, the turntable is held in a horizontal position for several minutes, and in that time both outputs of the turntable and IMU are very small (≈ 0); (2) The turntable begins to vibration, then both outputs of the turntable and IMU boom immediately. Therefore we choose the moment where both outputs of turntable and IMU begin to boom (increased by ten times in two adjacent outputs) as their common start signals (timing reference). By this method the turntable output is synchronized with the IMU output successfully. And the synchronization error is less than an output interval 0.02 s. Relative to the turntable vibration period 1 s, it is so small that can be neglected. The errors of the calculated results using two Algorithms are listed in Table 2.

Table 2 Turntable test results (°)

Algorithm	Error of angular changes in head channel		Error of angular changes in pitch channel	
	Mean	StDev	Mean	StDev
A	−0.001 9	0.025	−0.001 3	7.09×10^{-4}
B	−0.001 9	0.025	−0.001 4	7.09×10^{-4}
C	−0.001 5	0.023	-8.88×10^{-4}	7.01×10^{-4}

The turntable test results listed in Table 2 are slightly different with the simulation results listed in Table 1. This is because of the following two reaons: (1) The gyro outputs have noise; (2) in simulation (Table 1) the iteration interval T is 0.1 s, whereas in turntable test (Table 2) the iteration interval T is 0.06 s. But from Table 2 we can still see that under the same gyro condition, the navigation errors of head channel and pitch channel using the new Algorithm C are smaller than errors using traditional Algorithms A and B (the two-axis turntable cannot output the roll channel angle so that the errors of angular changes in roll channel is not calculated). These results provide further confidence in the validity of the advantages of the new second-order coning algorithm.

6 CONCLUSIONS

The key contributions of the study are:

(1) A new coning algorithm with an additional second-order coning correction term is developed. Compared with the traditional coning algorithms, the new algorithm can effectively reduce the coning algorithm error whereas sample numbers are not increased.

(2) The algorithm error analysis under maneuvers using a higher (second)-order truth model than traditional methods (Refs. [5-7]) reveals an interesting property that the new second-order coning algorithm, as well as the traditional frequency-series coning algorithm has the same order accuracy as the actual one of the traditional time-series coning algorithm. The result provides further confidence in the justification of the practical feasibility for the new coning algorithm. Simulations and practical tests are presented to illustrate the advantages of the new second-order coning algorithm.

Because a coning algorithm that works satisfactorily both in a vibration environment and in a maneuver (non-vibration) environment will satisfy most requirements of other environments[8], the new coning Algorithm C can be applied to strapdown inertial navigation systems, especially for highly dynamic (high frequency or large angle) angular motion and high-precision applications. And it can also be applied to highly maneuverable precision-pointing spacecraft or alignment calibration for maneuvering spacecraft as well as rotation inertial navigation system. Because Refs. [4,8-11] have demonstrated that in these applications coning motion has a nonnegligible effect.

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