V-BLAST BASED LDPC-CODED RELAY COOPERATION

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Abstract: An efficient LDPC-coded multi-relay cooperation architecture is proposed based on virtual vertical Bell Labs layered space-time (V-BLAST) processing for uplink communication, where minimum-mean-square-error (MMSE) and BP-based joint iterative decoding based on the introduced multi-layer Tanner graph are effectively designed to detect and decode the corrupted received sequence at the destination. By introducing V-BLAST transmission to the coded multi-relay cooperation, relays send their streams of symbols simultaneously, which increases the data rate and significantly reduces the transmission delay. The theoretical analysis and numerical results show that the new LDPC coded cooperation scheme outperforms the coded non-cooperation under the same code rate, and it also achieves a good trade-off among the performance, signal delay, and the encoding complexity associated with the number of relays. The performance gain can be credited to the proposed V-BLAST processing architecture and BP-based joint iterative decoding by the introduced multi-layer Tanner graph at a receiver-side.

Key words: coded multi-relay cooperation; vertical Bell Labs layered space-time processing (V-BLAST); minimum-mean-square-error detector; joint decoding; multi-layer Tanner graph

CLC number: TP911.2 **Document code**: A **Article ID**: 1005-1120(2013)04-0374-09

INTRODUCTION

Multiple-input-multiple-output (MIMO) has long been recognized as an effective technique for combating fading and improving channel capacity by offering diversity[1]. However, physical implementation of multiple antennas on a small, energy-limited mobile station (MS) might not be realistic. Cooperative diversity techniques[2-3] by implementing MIMO techniques via virtual antennas array may be one of the most effective ways to solve this crucial problem.

Coded cooperation[4-6] is a mechanism where cooperation is combined with channel coding. The basic idea is that each user, instead of repeating the received bits either via amplification or decoding, tries to transmit incremental redundancy for its partner. The current studies[7-9] have exploited LDPC codes into relay channels and achieved excellent performance improvement on various channels. In Ref. [10], two optimized Bilayer-LDPC code structures are designed for coded cooperation. However, to the best known of the author, spatial multiplexing architectures[11] which can significantly increase data rate are rarely mentioned in LDPC coded cooperation.

In this paper, a new kind of simple-encoding irregular systematic LDPC codes is applied to the uplink LDPC-coded multi-relay cooperation based on V-BLAST processing, and a new joint iterative decoding is introduced based on joint Tanner graph that can fully characterize the LDPC codes designed for the source and relays. This paper considers a scenario for uplink where source and relays are mobile stations (MSs), each equipped with single antenna, and they transmit cooperatively to a destination, which is a base station (BS) equipped with multiple antennas. Relays'

Foundation item: Supported by the Science and Technology on Avionics Integration Laboratory and National Aeronautical Science Foundation of China (20105552).

Received date: 2012-07-25; **revision received date**: 2012-11-08

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synchronous transmission forms a virtual V-BLAST transmission scheme. Hence, a concatenated system of V-BLAST detection and joint iterative decoding in BS is proposed, which consequently leads to a significant performance improvement over non-cooperation systems under the same conditions.

1 CODED MULTI-RELAY COOPERATION BASED ON VIRTUAL V-BLAST PROCESSING

The structure of the proposed coded multi-relay cooperation based on virtual V-BLAST processing is shown in Fig. 1, where the source and relay nodes transmit their messages in two consecutive time slots over a half-duplex relay channel. At the first time slot, the codeword bits encoded by the first encoder, denoted by "Enc-0", in the source (S), are sent over a broadcast channel (S-R_i, $i=1,\cdots,N_t$) to N_t ($N_t \geqslant 2$) relays ($R_1,\cdots,R_{N_t}$) and destination (D) with N_r receiver antennas, respectively. At the second time slot, the decoder, denote by "Dec-i", in the i-th relay firstly decodes the received signal corrupted by noise in S-R_i channel; then the encoder, denoted by "Enc-i", in i-th relay encodes the estimated message bits and re-transmits them to the destination over the R_i-D_j ($i=1,\cdots,N_t$; $j=1,\cdots,N_r$) channel simultaneously.

The coded sequences generated by the encoders in the relays and source are correlated since both parity-check bits from Enc-i and Enc-0 are dependent on their common original information bits. The resulted check sequences are completely correlated if the decoded bits by Dec-i are error-free over S-R_i channel; otherwise, they are partially correlated if there exists any error in the decoded bits by Dec-i. The aforementioned two cases are called ideal and non-ideal coded relay cooperation, respectively. For the rest of this paper, unless otherwise stated, all the statements about coded relay cooperation are referring to the ideal coded relay cooperation for uplink, where source and relays are MSs each equipped with single antenna and transmit cooperatively to a BS with multiple antennas. Using multiple receive antennas in the destination, the potential cooperative diversity gain of a relay cooperation is realized by a virtual MIMO, where the N_t+1 incoming signals from source and relays are the multiple inputs to the relay channel. Moreover, the partial cooperative diversity gain contributed by channel coding arises from the decoder in destination, which estimates the N_t+1 received signals, suffered from independent noises, by a joint decoding means than by a separate approach with the assumption of independence for all the incoming signals. Note that the performance gain also arises from the relatively larger SNR of partial received signal from relays over a shorten distance compared with that from the remote source.

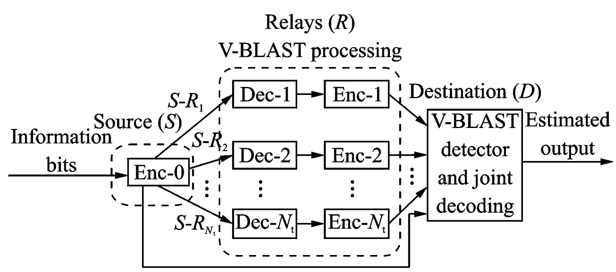

Fig. 1 Structure of the proposed coded multi-relay cooperation based on virtual V-BLAST processing

2 IRREGULAR SYSTEMATIC LDPC CODES FOR CODED MULTI-RELAY COOPERATION

In this section, simple-encoding systematic irregular LDPC codes are constructed for a coded multi-relay cooperation based on virtual V-BLAST processing.

2.1 Encoding of irregular systematic LDPC codes

Let the sparse parity-check matrix $\boldsymbol{H}_{M\times N}$ of a binary irregular LDPC code be

$$\boldsymbol{H}_{M\times N} = [\widetilde{\boldsymbol{H}}_{M\times(N-M)}\ \boldsymbol{I}_{M\times M}] \tag{1}$$

where the element of $\widetilde{\boldsymbol{H}}_{M\times(N-M)}$ is $h_{i,j}\in\{0,1\}$ ($1\leqslant i\leqslant M$, $1\leqslant j\leqslant N-M$, $M<N$) and $\boldsymbol{I}_{M\times M}$ an identity matrix. Let the numbers of "1" in each row and column of $\widetilde{\boldsymbol{H}}_{M\times(N-M)}$ be $\widetilde{d}_c$ and $\widetilde{d}_v$, which are proportional to and also comparably small in

contrast to $N-M$ and M, respectively. Thus, the number of "1" in each row of $\boldsymbol{H}_{M\times N}$ is $d_c=\tilde{d}_c+1$, while the numbers of "1" in each of the first $N-M$ columns and of the last M columns are $d_v=\tilde{d}_v$ and $d_v=1$, respectively. Consequently, the generator matrix for the above mentioned LDPC codes is

$$\boldsymbol{G}_{(N-M)\times N}=[\boldsymbol{I}_{(N-M)\times(N-M)}(\widetilde{\boldsymbol{H}}_{(N-M)\times M})^{\mathrm{T}}] \quad (2)$$

Hence, it is obviously that a linear block code defined by the sparse parity-check matrix in Eq. (1) is an irregular systematic LDPC code.

The task of an encoder is to generate the row vector $\boldsymbol{c}$ from the generator matrix as

$$\boldsymbol{c}=\boldsymbol{s}\times\boldsymbol{G}_{(N-M)\times N} \quad (3)$$

where $\boldsymbol{s}$ is a $(N-M)$-tuple message bit. Note that this linear encoding is quite suitable for the design of multiple LDPC codes applied in coded relay cooperation based on virtual V-BLAST processing. In this paper, the notation $C(N, M, d_c, \tilde{d}_v)$ is used to denote the ensemble of these irregular systematic LDPC codes.

2.2 Multiple irregular systematic LDPC codes for coded multi-relay cooperation

To illustrate the proposed multi-relay LDPC-coded cooperation, we take a two-relay cooperation corresponding to $N_t=2$ as a special example of the general case depicted in Fig. 1, where three different irregular systematic LDPC codes $C^{(1)}(N, M, d_c^{(1)}, \tilde{d}_v^{(1)})$, $C^{(2)}(N-M+M_1, M_1, d_c^{(2)}, \tilde{d}_v^{(2)})$ and $C^{(3)}(N-M+M_2, M_2, d_c^{(3)}, \tilde{d}_v^{(3)})$ are employed for the source , the first and second relays, respectively. The encoder in the source generates the codewords including the message and parity-check bits by the first LDPC code and sends them to two relays and destination over a broadcast channel. The decoder in each relay first decodes the codewords from the received signal and then produces different codewords by a distinct LDPC code, where only the check bits are retransmitted to the destination since the information bits have been already sent to the destination by the source.

According to the retransmission strategy, the general joint Tanner graph, fully characterizing component irregular systematic LDPC codes $C^{(1)}(N, M, d_c^{(1)}, \tilde{d}_v^{(1)})$, $C^{(2)}(N-M+M_1, M_1, d_c^{(2)}, \tilde{d}_v^{(2)})$ and $C^{(3)}(N-M+M_2, M_2, d_c^{(3)}, \tilde{d}_v^{(3)})$ employed in the proposed virtual V-BLAST based coded two-relay cooperation, can be described as Fig. 2. In this scenario, the 1st, 3rd and 4th layer of the joint Tanner graph are associated with the check nodes $c_i^{(1)}(1\leqslant i\leqslant M)$, $c_i^{(2)}(1\leqslant i\leqslant M_1)$ and $c_i^{(3)}(1\leqslant i\leqslant M_2)$ corresponding to the LDPC codes $C^{(1)}$, $C^{(2)}$ and $C^{(3)}$, respectively. And each check node of the 1st, 3rd and 4th layers involves with $d_c^{(1)}$, $d_c^{(2)}$ and $d_c^{(3)}$ variable nodes, respectively. The 2nd layer consists of two distinct groups of total $N+M_1+M_2$ variable nodes, $N-M$ variable nodes $v_i^{(1)}(1\leqslant i\leqslant N-M)$ for the common message bits that each participates in $\tilde{d}_v^{(1)}+\tilde{d}_v^{(2)}+\tilde{d}_v^{(3)}$ check equations and $M+M_1+M_2$ variable nodes $v_i^{(1)}(N-M+1\leqslant i\leqslant N)$, $v_i^{(2)}(N-M+1\leqslant i\leqslant N-M+M_1)$, and $v_i^{(3)}(N-M+1\leqslant i\leqslant N-M+M_2)$ each attends only one check equation.

From the view of destination, the overall code rate of three irregular systematic LDPC codes adopted by a two-relay coded cooperation is

$$R_{\text{coop}}^{(2)}=\frac{N-M}{N+M_1+M_2} \quad (4)$$

Correspondingly, the parity-check matrix for the entire resultant LDPC codes is given as

$$\boldsymbol{H}=\begin{bmatrix}\widetilde{\boldsymbol{H}}_{M\times(N-M)}^{(1)} & \boldsymbol{I}_{M\times M} & \boldsymbol{0}_{M\times M_1} & \boldsymbol{0}_{M\times M_2}\\ \widetilde{\boldsymbol{H}}_{M_1\times(N-M)}^{(2)} & \boldsymbol{0}_{M_1\times M} & \boldsymbol{I}_{M_1\times M_1} & \boldsymbol{0}_{M_1\times M_2}\\ \widetilde{\boldsymbol{H}}_{M_2\times(N-M)}^{(3)} & \boldsymbol{0}_{M_2\times M} & \boldsymbol{0}_{M_2\times M_1} & \boldsymbol{I}_{M_2\times M_2}\end{bmatrix} \quad (5)$$

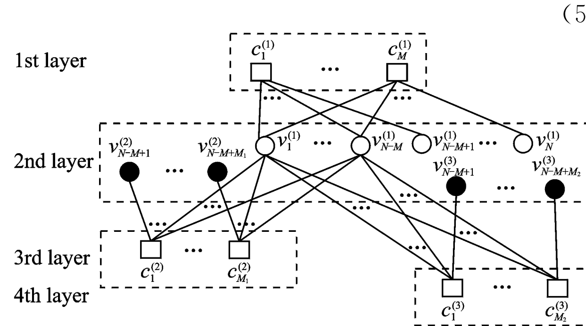

Fig. 2 Joint Tanner graph for three irregular systematic LDPC codes adopted by two-relay coded cooperation

3 V-BLAST DETECTOR FOR CODED MULTI-RELAY COOPERATION

In the proposed coded relay cooperation scheme, relays send their signals to the destination simultaneously to reduce transmission delay, which forms a virtual spatial multiplexing system as V-BLAST processing. Consequently, a V-BLAST receiver is necessary in the destination.

In the approach, we use MMSE detector[12], which achieves an optimal balance of noise enhancement and interference suppression, in BS. The aforementioned process is depicted in Fig. 3, where two MMSEs, denoted by "MMSE-1" and "MMSE-2", handle the incoming signals from source and relays in two alternative time slots, respectively. The outputs of the multiplexer are further processed by a joint iterative decoder which will be explicitly analyzed in Section 4.

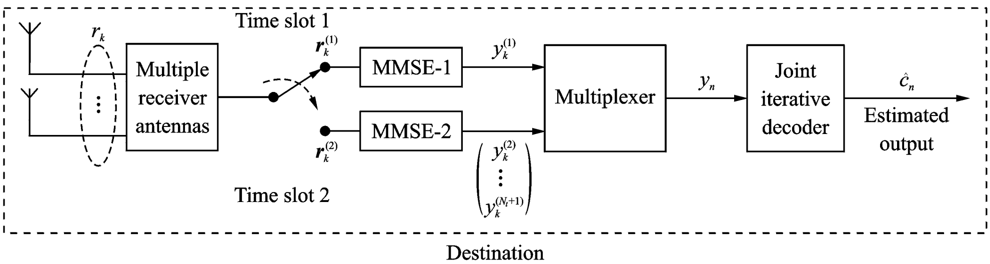

Fig. 3 Schematic diagram of V-BLAST detection and joint decoding

For the sake of simple presentation, we consider a two-relay ($N_t = 2$) cooperation assisted case in this section, which can be easily extended to the cases of N_t-relay ($N_t \geqslant 3$) cooperations. Suppose all the transmitted signals over the constituent channels, S-D and R_i-D, in a two-relay cooperation suffer from flat fading and additive Gaussian noise, simultaneously. Let the k-th received signal obtained by N_r ($N_r \geqslant N_t$) receiver antennas during the i-th ($i=1,2$) time slot in the destination be

$$\boldsymbol{r}_k^{(i)} = [r_{k,1}^{(i)}, \cdots, r_{k,N_r}^{(i)}]^{\mathrm{T}} = \boldsymbol{H}_k^{(i)} \boldsymbol{d}_k^{(i)} + \boldsymbol{n}_k^{(i)} \quad (6)$$

where $\boldsymbol{d}_k^{(1)} = d_k^{(1)} \in \boldsymbol{C}^{1\times 1}$ and $\boldsymbol{d}_k^{(2)} = [d_k^{(2)}, d_k^{(3)}]^{\mathrm{T}} \in \boldsymbol{C}^{2\times 1}$ are the transmitted symbols sent by source and relays, respectively and $\boldsymbol{n}_k^{(i)} \in \boldsymbol{C}^{N_r \times 1}$ is an Gaussian noise vector with independent and identically distributed (i. i. d.) entries with zero mean and variance of $(\sigma^{(i)})^2$. Moreover, $\boldsymbol{H}_k^{(1)} \in \boldsymbol{C}^{N_r \times 1}$ and $\boldsymbol{H}_k^{(2)} \in \boldsymbol{C}^{N_r \times 2}$ are the MIMO channel matrices for S-D channel and R-D link, respectively, while the first and second columns of $\boldsymbol{H}_k^{(2)}$ are corresponding to R_1-D and R_2-D link, respectively. Note that although the symbol sent from source is denoted as $\boldsymbol{d}_k^{(1)}$ for convenience, it is a scalar exactly.

Then MMSE-i ($i=1, 2$) will filter the output from the receiver antenna in i-th time slot, thus the decision statistic is

$$y_k^{(t)} = \boldsymbol{w}_k^{(t)} \boldsymbol{r}_k^{(i)} \begin{cases} t=1 & i=1 \\ t=2,3 & i=2 \end{cases} \quad (7)$$

where $y_k^{(1)}$, $y_k^{(2)}$ and $y_k^{(3)}$ are corresponding for a symbol from the source and each of the two relays, respectively. According to MMSE detection algorithm in Ref. [12], $\boldsymbol{w}_k^{(1)} = (\boldsymbol{H}_k^{(1)})^* (\boldsymbol{H}_k^{(1)} (\boldsymbol{H}_k^{(1)})^* + (\sigma^{(1)})^2 \boldsymbol{I})^{-1}$, $\boldsymbol{w}_k^{(2)}$ and $\boldsymbol{w}_k^{(3)}$ are the first and second rows of $(\boldsymbol{H}_k^{(2)})^* (\boldsymbol{H}_k^{(2)} (\boldsymbol{H}_k^{(2)})^* + (\sigma^{(2)})^2 \boldsymbol{I})^{-1}$, respectively.

Finally, $y_k^{(t)}$ is sent to an iterative decoder analyzed in next section for further processing.

4 JOINT ITERATIVE DECODING OF IRREGULAR SYSTEMATIC LDPC-CODED MULTI-RELAY COOPERATION

In this section, a BP-based joint iterative decoding algorithm by the proposed joint Tanner graph as shown in Fig. 2 is proposed for irregular systematic LDPC-coded two-relay cooperation with virtual V-BLAST processing. This decoding algorithm can be naturally extended to a N_t-relay ($N_t > 2$) LDPC-coded cooperation.

4.1 General description

As shown in Fig. 3, the k-th output of the multiplexer after parallel-to-serial conversion is $\boldsymbol{R}_k=(y_k^{(1)},\cdots,y_{k+N-1}^{(1)},y_k^{(2)},\cdots,y_{k+M_1-1}^{(2)},y_k^{(3)},\cdots,y_{k+M_2-1}^{(3)})$ with the first N, middle M_1 and last M_2 symbols from the source and two relays, respectively. For the sake of concise, let $M=M_1=M_2$, then $\boldsymbol{R}_k$ can be simplified as $\boldsymbol{R}=(y_1,\cdots,y_N,y_{N+1},\cdots,y_{N+M},y_{N+M+1},\cdots,y_{N+2\times M})$. Note that $y_n(1\leqslant n\leqslant N)$ are corresponding to codeword bits generated by irregular systematic LDPC codes $C^{(1)}(N,M,d_c^{(1)},\tilde{d}_v^{(1)})$, while $y_n(N+1\leqslant n\leqslant N+2\times M)$ are corresponding to check bits generated by irregular systematic LDPC codes $C^{(2)}(N,M,d_c^{(2)},\tilde{d}_v^{(2)})$ and $C^{(3)}(N,M,d_c^{(3)},\tilde{d}_v^{(3)})$, respectively.

Using the binary antipodal modulation, let $\tilde{r}_n=\mathrm{Re}(y_n)$ $(1\leqslant n\leqslant N+2\times M)$ and these sequences of soft values are directly applied to the joint iterative decoding.

4.2 Joint BP-based iterative decoding based on multi-layer Tanner graph

The steps of the proposed BP-based joint iterative decoding by a unified multi-layer Tanner graph can be realized as follows.

Step 1 (Initialization): The transmitted signal d_n takes $+1$ or -1 with equal likelihood for the codeword bit $c_n=0$ or 1 generated by irregular systematic LDPC codes $C^{(1)}(N,M,d_c^{(1)},\tilde{d}_v^{(1)})$, $C^{(2)}(N,M,d_c^{(2)},\tilde{d}_v^{(2)})$ and $C^{(3)}(N,M,d_c^{(3)},\tilde{d}_v^{(3)})$ used by source and two relays, respectively. Let two conditional probabilities for d_n on the filtered sequence $\{\tilde{r}_n\}$ be

$$f_n=\Pr(c_n=0\mid(\tilde{r}_l\mid_{l=1}^{N+2\times M}))$$
$$\bar{f}_n=\Pr(c_n=1\mid(\tilde{r}_l\mid_{l=1}^{N+2\times M}))=1-f_n \quad (8)$$

Initially, the decoder only obtains the filtered symbols from the component MMSE detectors and has no prior information from the parity checks. Thus with the known channel state information (CSI), two conditional probabilities in Eq. (8) can be further formulated according to the Appendix.

Then, let two aposteriori probabilities concerning the filtered sequences $\{\tilde{r}_n\}$ for the decoding iteration be

$$q_{m,n}^{(i)}=\Pr(c_n=0\mid(\tilde{r}_l\mid_{l=1}^{N+2\times M}),S(C(v_n)/c_m^{(i)})=1) \quad (9a)$$

$$\bar{q}_{m,n}^{(i)}=\Pr(c_n=1\mid(\tilde{r}_l\mid_{l=1}^{N+2\times M}),\ S(C(v_n)/c_m^{(i)})=1)=1-q_{m,n}^{(i)} \quad (9b)$$

The function $S(C(v_n)/c_m^{(i)})$ is given the value 1 if all the check nodes except $c_m^{(i)}$ in the set $C(v_n)$, associated with variable node v_n, are satisfied simultaneously, where $m=1,\cdots,M$ for $i=1,3,4$ is corresponding to check nodes in the 1st, 3rd and 4th layer of the joint Tanner graph, respectively. Note that v_n is the variable node in the second layer, i.e.

$$v_n=\begin{cases}v_n^{(1)} & 1\leqslant n\leqslant N\\ v_{n-M}^{(2)} & N+1\leqslant n\leqslant N+M\\ v_{n-2M}^{(3)} & N+M+1\leqslant n\leqslant N+2M\end{cases} \quad (10)$$

Before commencing the iterative decoding, $(q_{m,n}^{(i)},\bar{q}_{m,n}^{(i)})$ can be initialized as $(f_n,\bar{f}_n)$ according to the Appendix.

Step 2 (Horizontal process): In this step the extrinsic information $r_{m,n}^{(i)}$, sent from the m-th check node $c_m^{(i)}$ in each layer to a variable node v_n, is evaluated as

$$r_{m,n}^{(i)}=\frac{\Pr(S(\{c_m^{(i)}\})=1\mid c_n=0,(\tilde{r}_l\mid_{l=1}^{N+2\times M}))}{\Pr(S(\{c_m^{(i)}\})=1\mid c_n=1,(\tilde{r}_l\mid_{l=1}^{N+2\times M}))}=\frac{1+\Delta_m^{(i)}/(q_{m,n}^{(i)}-\bar{q}_{m,n}^{(i)})}{1-\Delta_m^{(i)}/(q_{m,n}^{(i)}-\bar{q}_{m,n}^{(i)})} \quad (11a)$$

where

$$\Delta_m^{(i)}=\prod_{v_n\in V(c_m^{(i)})}(q_{m,n}^{(i)}-\bar{q}_{m,n}^{(i)}) \quad (11b)$$

Then, a check node in any layer sends its extrinsic information to a variable node v_n in the second layer of the joint Tanner graph.

Step 3 (Vertical process): Let $\bar{R}_n$ for the variable node $v_n(1\leqslant n\leqslant N-M)$ regarding the n-th message bit be

$$\bar{R}_n=\prod_{c_k^{(1)}\in C(v_n)}r_{k,n}^{(1)}\times\prod_{c_l^{(2)}\in C(v_n)}r_{l,n}^{(2)}\times\prod_{c_p^{(3)}\in C(v_n)}r_{p,n}^{(3)} \quad (12)$$

Hence, in this step the extrinsic information $(q_{m,n}^{(1)},\bar{q}_{m,n}^{(1)})$, sent from a variable node $v_n(1\leqslant n\leqslant N-M)$ to a check node $c_m^{(1)}$ in the first layer of joint Tanner graph, is calculated as

$$s_{m,n}^{(1)}=\frac{\Pr(c_n=0\mid\{\tilde{r}_l\mid_{l=1}^{N+2\times M}\},S(C^{(1)}(v_n)/c_m^{(1)})=1,S(C^{(2)}(v_n))=1,S(C^{(3)}(v_n))=1)}{\Pr(c_n=1\mid\{\tilde{r}_l\mid_{l=1}^{N+2\times M}\},S(C^{(1)}(v_n)/c_m^{(1)})=1,S(C^{(2)}(v_n))=1,S(C^{(3)}(v_n))=1)}=\frac{f_n}{\bar{f}_n}\times\frac{\bar{R}_n}{r_{m,n}^{(1)}}=\frac{q_{m,n}^{(1)}}{\bar{q}_{m,n}^{(1)}} \quad (13)$$

Then, the extrinsic information ($q_{m,n}^{(1)},\bar{q}_{m,n}^{(1)}$) is updated as

$$q_{m,n}^{(1)}=s_{m,n}^{(1)}/(1+s_{m,n}^{(1)}),\bar{q}_{m,n}^{(1)}=1-q_{m,n}^{(1)}=1/(1+s_{m,n}^{(1)}) \quad (14)$$

Similarly, the extrinsic information ($q_{m,n}^{(2)}$, $\bar{q}_{m,n}^{(2)}$), sent from a variable node v_n ($1\leqslant n\leqslant N-M$) to a check node $c_m^{(2)}$ in the 3rd layer of the joint Tanner graph, is obtained as

$$s_{m,n}^{(2)}=\frac{f_n}{\bar{f}_n}\times\frac{\bar{R}_n}{r_{m,n}^{(2)}}=\frac{q_{m,n}^{(2)}}{\bar{q}_{m,n}^{(2)}} \quad (15)$$

Therefore the extrinsic information ($q_{m,n}^{(2)},\bar{q}_{m,n}^{(2)}$) is updated as

$$q_{m,n}^{(2)}=s_{m,n}^{(2)}/(1+s_{m,n}^{(2)}),\bar{q}_{m,n}^{(2)}=1-q_{m,n}^{(2)}=1/(1+s_{m,n}^{(2)}) \quad (16)$$

Moreover, the extrinsic information ($q_{m,n}^{(3)},\bar{q}_{m,n}^{(3)}$), sent from a variable node v_n ($1\leqslant n\leqslant N-M$) to a check node $c_m^{(3)}$ in the fourth layer of the joint Tanner graph, is obtained as

$$s_{m,n}^{(3)}=\frac{f_n}{\bar{f}_n}\times\frac{\bar{R}_n}{r_{m,n}^{(3)}}=\frac{q_{m,n}^{(3)}}{\bar{q}_{m,n}^{(3)}} \quad (17)$$

Consequently, the extrinsic information ($q_{m,n}^{(3)}$, $\bar{q}_{m,n}^{(3)}$) is updated as

$$q_{m,n}^{(3)}=s_{m,n}^{(3)}/(1+s_{m,n}^{(3)}),\bar{q}_{m,n}^{(3)}=1-q_{m,n}^{(3)}=1/(1+s_{m,n}^{(3)}) \quad (18)$$

Step 4 (Final decision): Repeat Step 2 and Step 3 until up to the maximum number of iterative decoding, then the aposteriori likelihood is evaluated as

$$R_n=\frac{f_n}{\bar{f}_n}\times\bar{R}_n \qquad n=1,\cdots,N-M \quad (19)$$

Finally, the estimated message bit is acquired by the following rule

$$\hat{c}_n=\begin{cases}0 & R_n\geqslant 1\\ 1 & R_n<1\end{cases} \qquad n=1,\cdots,N-M \quad (20)$$

5 SIMULATION RESULTS

In this section, we investigate the performance of proposed uplink LDPC-coded multi-relay cooperation based on virtual V-BLAST processing with known CSI over both Rayleigh fading channels and Suzuki fading channels. In the simulation, we assume that SNR of the received signal from each relay is 1.0 dB[13], higher than that of the received signal from the source in accordance with the situation of shorter distance between any relay and destination as compared with that between the source and destination.

5.1 Virtual V-BLAST based LDPC-coded multi-relay cooperation with various decoding iterations

Fig. 4 gives BER results of the proposed LDPC-coded two-relay cooperation with two receiver antennas and 1, 2, 3 decoding iterations by the proposed cascade process of V-BLAST detector and joint BP-based decoder, where three different irregular systematic LDPC codes $C^{(1)}$ (7 200, 2 400, 8, 4), $C^{(2)}$ (8 000, 3 200, 6, 4) and $C^{(3)}$ (8 000, 3 200, 6, 4) are employed for source and two relays, respectively. Note that the overall code rate from the destination is 6/17 based on the coded cooperation strategy introduced in Section 2. Simulation results show that BER over Rayleigh fading channel can be significantly reduced by using the joint BP-based iterative decoder with increasing decoding iterations. For example, at a BER of 10^{-3}, approximate 2.3 dB gain can be achieved for the 2nd iteration over the 1st iteration, while the gain is about 1.2 dB for the 3rd iteration as compared with the 2nd iteration at the same BER.

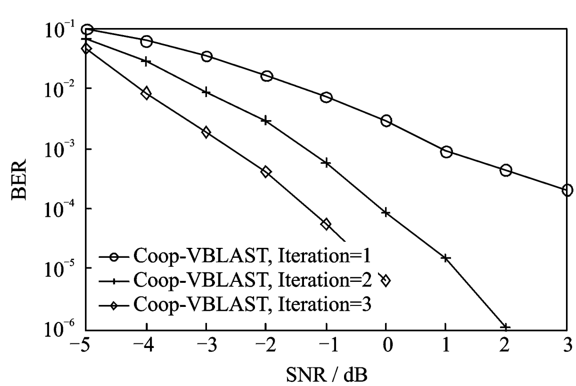

Fig. 4 BER-SNR curve with various decoding iterations in destination

5.2 Virtual V-BLAST based LDPC-coded multi-relay cooperation with different numbers of receiver antennas in destination

BER results of proposed LDPC-coded two-relay cooperation over Rayleigh fading channels based on virtual V-BLAST processing are shown in Fig. 5, with the same LDPC codes used in Fig. 4, where the decoding iteration is 3 and the number of receiver antennas in the destination is 2 and 3, respectively. Significant diversity gains can be observed by increasing the number of receiver antennas. For instance, at a SNR of -2.5 dB, BER reaches 10^{-6} for the proposed LDPC-coded multi-relay cooperation with three receiver antennas, while it is 10^{-3} in case of two receiver antennas. This merit can be credited to diversity order achieved by MMSE detectors and efficient joint BP-based iterative decoding based on the proposed multi-layer Tanner graph.

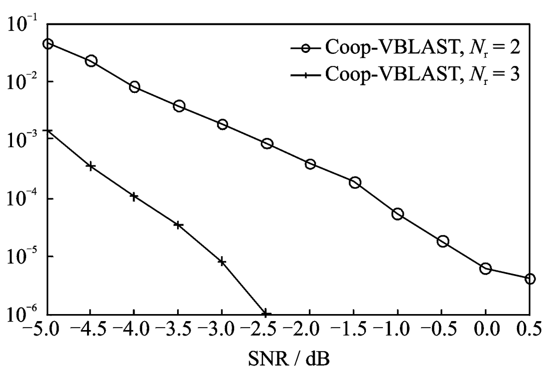

Fig. 5 BER-SNR curve in the 3rd decoding iteration with 2 and 3 receiver antennas in the destination

5.3 LDPC-coded two-relay cooperation based on virtual V-BALST processing and LDPC-coded non-cooperation

Fig. 6 compares BER of LDPC-coded two-relay cooperation based on virtual V-BLAST processing with the same irregular systematic LDPC codes employed in Fig. 4 and coded non-cooperation under the identical conditions of two receiver antennas and decoding iterations as 2 and 3, respectively. The rate-6/17 irregular systematic LDPC code $C(3\ 400, 2\ 200, 6, 11)$ is adopted in the non-cooperation as a reference. The results show that the performance of the coded cooperation based on virtual V-BLAST processing clearly outperforms the coded non-cooperation with the same number of receiver antennas in the destination, in all range of SNR under the same decoding iteration over Rayleigh fading channels.

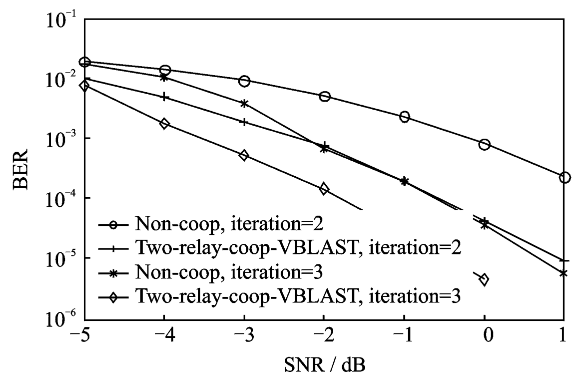

Fig. 6 BER of LDPC-coded two-relay cooperation based on virtual V-BLAST processing and LDPC-coded non-cooperation

5.4 Virtual V-BLAST based LDPC-coded multi-relay cooperation

BER results of proposed LDPC-coded two-relay cooperation based on virtual V-BLAST processing over Suzuki fading channels are shown in Fig. 7 with the same LDPC codes used in Fig. 4, where the decoding iterations are 2 and 3 while the number of receiver antennas in the destination is 2 and 3, respectively. Furthermore, the parameters of the Suzuki fading are $(s, m, \sigma_{\mu_i}^2)$, where $s=0.161$, $m=-s^2=0.025\ 9$, $\sigma_{\mu_1}^2=\sigma_{\mu_2}^2=1/2$ and $\sigma_{\mu_3}^2=1$[14]. Significant gains can be observed by increasing either the iterations or the number of receiver antennas, which is consistent with the results in Rayleigh fading channels. Fig. 7 also demonstrates the performance of proposed approach over Rayleigh fading channel is better than that over a Suzuki fading channel under the same condition, which is reasonable for the previous one with a better channel condition compared with the later one.

6 CONCLUSION

This paper introduces an efficient uplink LDPC-coded multi-relay cooperation based on virtual V-BLAST processing over a flat fading channel,

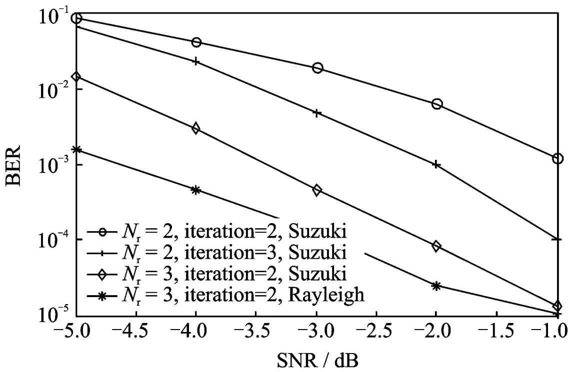

Fig. 7 BER of virtual V-BLAST based LDPC-coded two-relay cooperation

which can combine spatial multiplexing, coding gain into one perfect entity by using V-BLAST transmission and proposed multi-layer BP-based joint iterative decoding in the destination. Theoretical analysis and simulation results demonstrate that the proposed coded cooperation scheme clearly outperforms code non-cooperation at the same code rate, which provides an efficient solution for practical communications.

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Appendix

In this appendix, we derive the aposterior probabilities (f_n, $\bar{f}_n$) used in the initialization step of the joint BP-based iterative decoding for proposed V-BLAST based LDPC coded cooperation.

1. A posteriori probabilities for filtered output of symbols sent from source (f_n, $\bar{f}_n$) when $1\leqslant n\leqslant N$

Base on the V-BLAST detection algorithm addressed in Section 3, we rewrite the vector of combining coefficients for the symbols sent from source as $\boldsymbol{w}_n = \boldsymbol{H}_n^* / (\|\boldsymbol{H}_n\|^2 + (\sigma^{(1)})^2)$. Hence, the filtered output y_n is

$$y_n = \boldsymbol{w}_n(\boldsymbol{H}_n d_n + \boldsymbol{n}_n) = \frac{\|\boldsymbol{H}_n\|^2}{\|\boldsymbol{H}_n\|^2 + (\sigma^{(1)})^2} d_n + \tilde{n}_n \tag{A1}$$

where d_n take values -1 or $+1$ with equal likelihood for the codeword bit $c_n=1$ or 0 generated by irregular systematic LDPC codes $C^{(1)}(N, M, d_c^{(1)}, \tilde{d}_v^{(1)})$, used by source.

With the known channel state information (CSI), $\tilde{n}_n=\boldsymbol{w}_n\boldsymbol{n}_n$ is Gaussian RV with variance ${\sigma_n}^2$ as

$$ {\sigma_n}^2 = E((\boldsymbol{w}_n\boldsymbol{n}_n)^*\boldsymbol{w}_n\boldsymbol{n}_n) = \frac{\|\boldsymbol{H}_n\|^2(\sigma^{(1)})^2}{(\|\boldsymbol{H}_n\|^2+(\sigma^{(1)})^2)^2} \tag{A2} $$

Let $\tilde{r}_n=\mathrm{Re}(y_n)=a_nd_n+\mathrm{Re}(\tilde{n}_n)$, where $a_n=\frac{\|\boldsymbol{H}_n\|^2(\sigma^{(1)})^2}{\|\boldsymbol{H}_n\|^2+(\sigma^{(1)})^2}$. Therefore, f_n and $\bar{f}_n$ can be further formulated as

$$ f_n = \Pr(c_n=0 \mid \{\tilde{r}_n|_{n=1}^{N+2M}\}) = 1/(1+\exp(-4a_n\tilde{r}_n/{\sigma_n}^2)) \tag{A3a} $$

$$ \bar{f}_n = \Pr(c_n=1 \mid \{\tilde{r}_n|_{n=1}^{N+2M}\}) = 1-f_n = 1/(1+\exp(4a_n\tilde{r}_n/{\sigma_n}^2)) \tag{A3b} $$

2. A posteriori probabilities for filtered output of symbols sent from the first relay: (f_n, $\bar{f}_n$) when $N+1\leqslant n\leqslant N+M$

According to the V-BLAST detection algorithm addressed in Section 3, the vector of combining coefficients for the symbols sent from the first relay is rewritten as $\boldsymbol{w}_n$ corresponding to the first row of $\boldsymbol{H}_n^*(\boldsymbol{H}_n\boldsymbol{H}_n)^*+(\sigma^{(2)})^2\boldsymbol{I})^{-1}$. Hence, the filtered output y_n is

$$ y_n = \boldsymbol{w}_n(\boldsymbol{H}_n\boldsymbol{d}_n+\boldsymbol{n}_n) = \boldsymbol{w}_n\boldsymbol{H}_n\boldsymbol{d}_n+\tilde{\boldsymbol{n}}_n \tag{A4} $$

where $\boldsymbol{d}_n=\begin{pmatrix} d_n^{(2)} \\ d_n^{(3)} \end{pmatrix}$ while $d_n^{(2)}$ and $d_n^{(3)}$ take values -1 or $+1$ with equal likelihood for the codeword bit $c_n^{(i)}=1$ or 0 ($i=2,3$) generated by irregular systematic LDPC codes $C^{(2)}(N, M, d_c^{(2)}, \tilde{d}_v^{(2)})$ and $C^{(3)}(N, M, d_c^{(3)}, \tilde{d}_v^{(3)})$, used by two relays, respectively.

With the known CSI, $\tilde{n}_n=\boldsymbol{w}_n\boldsymbol{n}_n$ is Gaussian RV with variance ${\sigma_n}^2$ as

$$ {\sigma_n}^2 = (\sigma^{(2)})^2E(\boldsymbol{w}_n(\boldsymbol{w}_n)^*) = \|\boldsymbol{w}_n\|^2(\sigma^{(2)})^2 \tag{A5} $$

Let $\tilde{r}_n=\mathrm{Re}(y_n)=a_n^{(2)}d_n^{(2)}+a_n^{(3)}d_n^{(3)}+\mathrm{Re}(\tilde{n}_n)$, where $a_n^{(2)}=\mathrm{Re}(w_nH_n)_{11}$, $a_n^{(3)}=\mathrm{Re}(w_nH_n)_{12}$ ($Z_{i,j}$ denotes the element of $\boldsymbol{Z}$ in i-th row and j-th column). Therefore, f_n and $\bar{f}_n$ can be further formulated as

$$ f_n = \Pr(c_n=0 \mid \{\tilde{r}_n|_{l=1}^{N+2M}\}) = \frac{1}{1+\left(\frac{\exp(-(\tilde{r}_n+a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n+a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}{\exp(-(\tilde{r}_n-a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n-a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}\right)} \tag{A6a} $$

$$ \bar{f}_n = \Pr(c_n=1 \mid \{\tilde{r}_n|_{l=1}^{N+2M}\}) = \frac{1}{1+\left(\frac{\exp(-(\tilde{r}_n-a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n-a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}{\exp(-(\tilde{r}_n+a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n+a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}\right)} \tag{A6b} $$

3. A posteriori probabilities for filtered output of symbols sent from the second relay: (f_n, $\bar{f}_n$) when $N+M+1\leqslant n\leqslant N+2M$

According to the V-BLAST detection algorithm addressed in Section 3, the vector of combining coefficients for the symbols sent from the second relay is rewritten as $\boldsymbol{w}_n$ ($N+M+1\leqslant n\leqslant N+2M$) corresponding to the second row of $\boldsymbol{H}_{n-M}^*((\boldsymbol{H}_{n-M}\boldsymbol{H}_{n-M})^*+(\sigma^{(2)})^2\boldsymbol{I})^{-1}$. Hence, the filtered output y_n is

$$ y_n = \boldsymbol{w}_n(\boldsymbol{H}_{n-M}\boldsymbol{d}_{n-M}+\boldsymbol{n}_{n-M}) = \boldsymbol{w}_n\boldsymbol{H}_{n-M}\boldsymbol{d}_{n-M}+\tilde{n}_n \tag{A7} $$

where $\boldsymbol{d}_{n-M}=\begin{pmatrix} d_{n-M}^{(2)} \\ d_{n-M}^{(3)} \end{pmatrix}$ while $d_{n-M}^{(2)}$ and $d_{n-M}^{(3)}$ take values -1 or $+1$ with equal likelihood for the codeword bit $c_{n-M}^{(i)}=1$ or 0 ($i=2,3$) generated by irregular systematic LDPC codes $C^{(2)}(N, M, d_c^{(2)}, \tilde{d}_v^{(2)})$ and $C^{(3)}(N, M, d_c^{(3)}, \tilde{d}_v^{(3)})$, used by two relays, respectively.

With the known CSI, $\tilde{n}_n=w_nn_{n-M}$ is Gaussian RV with variance ${\sigma_n}^2$ as

$$ {\sigma_n}^2 = (\sigma^{(2)})^2E(w_nw_n^*) = \|w_n\|^2(\sigma^{(2)})^2 \tag{A8} $$

Let $\tilde{r}_n=Re(y_n)=a_n^{(2)}d_{n-M}^{(2)}+a_n^{(3)}d_{n-M}^{(3)}+\mathrm{Re}(\tilde{n}_n)$, where $a_n^{(2)}=\mathrm{Re}(w_nH_{n-M})_{21}$, $a_n^{(3)}=\mathrm{Re}(w_nH_{n-M})_{22}$ ($Z_{i,j}$ denotes the element of Z in i-th row j-th column). Therefore, f_n and $\bar{f}_n$ can be further formulated as

$$ f_n = \Pr(c_n=0 \mid \{\tilde{r}_n|_{l=1}^{N+2M}\}) = \frac{1}{1+\left(\frac{\exp(-(\tilde{r}_n-a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n+a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}{\exp(-(\tilde{r}_n-a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n+a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)}\right)} \tag{A9a} $$

$$ \bar{f}_n = \Pr(c_n=1 \mid \{\tilde{r}_n|_{l=1}^{N+2M}\}) = \frac{1}{1+\left(\frac{\exp(-(\tilde{r}_n+a_n^{(2)}-a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n-\tilde{a}_n^{(2)}-\tilde{a}_n^{(3)})^2/{\sigma_n}^2)}{\exp(-(\tilde{r}_n+a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)+\exp(-(\tilde{r}_n-a_n^{(2)}+a_n^{(3)})^2/{\sigma_n}^2)}\right)} \tag{A9b} $$