

Dynamical Model Updating Based on Modal Tests with Changed Structure

Wang Ke(王轲)*, Yang Jian(杨健)

State Key Laboratory of Mechanics and Control of Mechanical Structures,
Nanjing University of Aeronautics & Astronautics, Nanjing, 210016, P. R. China

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Abstract: A new approach to modifying the stiffness and mass matrices of finite element models is presented to improve the calculation precision. By measuring the mode frequencies and shapes of both of the original and the new structures with changed stiffness and mass, the stiffness and mass matrices of the finite element model can be updated through matrices calculation and solving algebra equations. Taking a multi-freedom model as an example, the relation between the number of the modes and the correction precision of stiffness and mass matrix elements is researched. The facility and precision of the method are totally confirmed especially when the modeling error is known limited to a definite local range. The feasibility of the approach is proven by an effective engineering application to the model updating of a wing piece used in flutter test.

Key words: finite element; structural dynamics; building model; model updating; modal analysis

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1 Introduction

Finite element (FE) method is an important analysis approach to modern structural dynamics, which is applied to the calculation of mode and response of various kinds of complicated structures. The highly accurate model is necessary for analyzing, but the error in modeling can seriously affect calculating accurateness. If the structure to be analyzed has been produced, the model can be valued and updated by comparing its analytical modes of vibration with the results of a model test. Before 1970s, techniques of FE modeling and dynamical tests were developed independently, and the results of tests were only subjectively used in verification and validation of FEs. Then two categories of FE model updating based on modal tests developed gradually. Some methods are to update the selected parameters of the model, and some physical parameters of the structure such as Young's modulus, mass density and thickness of shell can be selected and updated af-

ter the structural sensitivities are found^[1-2]. Other methods, categorized into the 'direct' methods, are to update the whole stiffness and mass matrices provided by Berman, et al^[3-4]. However, both of the methods are not good enough because the modes recognized from a single modal test are limited. In recent years, Cha and some experts suggested a special updating method by which the FE model can be updated based on the measurement of the original mode shapes of the structure and the new mode shapes with added mass^[5-7]. But in fact, solely adding mass to the structure can hardly be achieved, because the mass block always has volume and inertia which are ignored in the course of this updating process. Another factor affecting the accuracy of this updating work is that the attachment stiffness is inevitable when masses are added.

In the paper a new approach to FE model updating is proposed, which is based on the measurement of the modes of the original and the new

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* **Corresponding author:** Wang Ke, Associate Professor, E-mail: wangk@nuaa.edu.cn.

structural modes with changed stiffness and mass. In the approach, the stiffness and mass matrixes are updated in a single solution step. If the location of modeling error or the inaccurate elements have been known, the approach will be very effective. The paper provides an example of FE model updating of a chain of coupled oscillators with 30 degrees of freedom. The results prove the facility and the precision of the approach.

2 Updating Arithmetics

The number of degrees of freedom of FE model is much bigger than that of computer aided test (CAT) model in practical engineering problems. But the degrees of freedom of the two models will be equal when the mode reduction suggested by Kammer or SEREP^[3] is applied. Therefore the assumption is adopted that the freedoms of the FE model and CAT model are uniform.

If the damping in the vibration system is not taken into account, the linear dynamical FE equation is as follow

$$\mathbf{M}_0 \ddot{\mathbf{x}} + \mathbf{K}_0 \mathbf{x} = \mathbf{F} \quad (1)$$

where \mathbf{M}_0 is the analytical mass matrix, and \mathbf{K}_0 the analytical stiffness matrix. Both of matrixes are of size $N \times N$. The eigenvalue equations of Eq. (1) is

$$\mathbf{M}_0 \boldsymbol{\Phi}_0 \boldsymbol{\Lambda}_0 = \mathbf{K}_0 \boldsymbol{\Phi}_0 \quad (2)$$

where $\boldsymbol{\Lambda}_0$ is the analytical eigenvalue matrix, and $\boldsymbol{\Phi}_0$ the analytical eigenvector matrix.

The mode results of the test of the same vibration system satisfy

$$\mathbf{M} \boldsymbol{\Phi} \boldsymbol{\Lambda} = \mathbf{K} \boldsymbol{\Phi} \quad (3)$$

where $\boldsymbol{\Lambda}$ consists of a diagonal matrix of size $N_e \times N_e$, whose elements are the testing eigenvalues of the vibration system, N_e is the number of tested modes, and $\boldsymbol{\Phi}$ is the testing eigenvector matrix of size $N \times N_e$. The mass matrix \mathbf{M} and the stiffness matrix \mathbf{K} in Eq. (3) have no relationship with \mathbf{M}_0 and \mathbf{K}_0 in FE equation, and they are not unique and cannot be used for model updating directly.

When the vibration system is changed, such as fastening some mass blocks in some local places or adding springs to connect two points of the structure, the stiffness matrix and mass ma-

trix of the FE model will be changed. The new stiffness and mass added are written as \mathbf{K}_a and \mathbf{M}_a .

When the number of freedom degrees and the measurement freedoms are not changed, the eigenvalue matrix is written as $\boldsymbol{\Lambda}_a$ of size $N_e \times N_e$ and the eigenvector matrix as $\boldsymbol{\Phi}_a$ of size $N \times N_e$ if the number of modes measured also remains N_e .

The tested mode satisfies

$$(\mathbf{M} + \mathbf{M}_a) \boldsymbol{\Phi}_a \boldsymbol{\Lambda}_a = (\mathbf{K} + \mathbf{K}_a) \boldsymbol{\Phi}_a \quad (4)$$

Take the transpose of Eq. (3) and postmultiply the resultant matrix equation by $\boldsymbol{\Phi}_a$, then we get

$$\boldsymbol{\Phi}^T \mathbf{K} \boldsymbol{\Phi}_a = \boldsymbol{\Lambda} \boldsymbol{\Phi}^T \mathbf{M} \boldsymbol{\Phi}_a \quad (5)$$

The right and left hand sides of Eq. (5) are matrix of size $N_e \times N_e$.

Premultiply Eq. (4) by $\boldsymbol{\Phi}^T$, then we have

$$\boldsymbol{\Phi}^T (\mathbf{K} + \mathbf{K}_a) \boldsymbol{\Phi}_a = \boldsymbol{\Phi}^T (\mathbf{M} + \mathbf{M}_a) \boldsymbol{\Phi}_a \boldsymbol{\Lambda}_a \quad (6)$$

Eq. (5) minus Eq. (6) equals

$$\boldsymbol{\Lambda} \boldsymbol{\Phi}^T \mathbf{M} \boldsymbol{\Phi}_a - \boldsymbol{\Phi}^T \mathbf{M} \boldsymbol{\Phi}_a \boldsymbol{\Lambda}_a = \boldsymbol{\Phi}^T \mathbf{M}_a \boldsymbol{\Phi}_a \boldsymbol{\Lambda}_a - \boldsymbol{\Phi}^T \mathbf{K}_a \boldsymbol{\Phi}_a \quad (7)$$

Since the matrix \mathbf{M} is unknown, let

$$\mathbf{P} = \boldsymbol{\Phi}^T \mathbf{M} \boldsymbol{\Phi} \quad (8a)$$

And

$$\mathbf{Q} = \boldsymbol{\Phi}^T \mathbf{M}_a \boldsymbol{\Phi}_a \boldsymbol{\Lambda}_a - \boldsymbol{\Phi}^T \mathbf{K}_a \boldsymbol{\Phi}_a \quad (8b)$$

Eq. (7) becomes

$$\boldsymbol{\Lambda} \mathbf{P} - \mathbf{P} \boldsymbol{\Lambda}_a = \mathbf{Q} \quad (9)$$

where \mathbf{Q} is a matrix of size $N_e \times N_e$. It can be obtained directly if the frequencies and modes measured in the test are known.

Since $\boldsymbol{\Lambda}$ and $\boldsymbol{\Lambda}_a$ are diagonal matrices, using straightened algorithm, the (i, j) th element of Eq. (9) satisfies

$$(\lambda_i - \lambda_{aj}) P_{ij} = Q_{ij} \quad (10)$$

where λ_i is the i th measured eigenvalue of the original structure, λ_{aj} the j th measured eigenvalue of the structure whose stiffness and mass have been changed. Since $i, j = 1, 2, \dots, N_e$, the number of equations is $N_e \times N_e$.

Assuming λ_i does not equal λ_{aj} (it can be realized by adjusting the amount of mass or stiffness attached.), the unknown elements of \mathbf{P} can be solved.

Assuming \mathbf{K} and \mathbf{M} can be expressed as

$$\mathbf{M} = \mathbf{M}_0 + \delta \mathbf{M} \quad (11a)$$

and

$$\mathbf{K} = \mathbf{K}_0 + \delta \mathbf{K} \quad (11b)$$

Substituting Eq. (11a) into Eq. (8) yields

$$\Phi^T \delta \mathbf{M} \Phi_a = \mathbf{P} - \Phi^T \mathbf{M}_0 \Phi_a = \mathbf{R} \quad (12)$$

Because Φ^T and Φ_a are both rectangular matrixes, they have no inverses. $\delta \mathbf{M}$ can not be obtained directly by Eq. (12).

However, Φ , Φ_a , \mathbf{P} , \mathbf{M}_0 all have been known, so Eq. (12) can be expanded, then $\delta \mathbf{M}$ appears as an unknown vector $\delta \mathbf{m}$ as follows

$$\mathbf{A} \delta \mathbf{m} = \mathbf{r} \quad (13)$$

where

$$\mathbf{A} = \Phi^T \otimes \Phi_a^T \quad (14a)$$

$$\delta \mathbf{m} = \{ \delta M_{11} \quad \delta M_{12} \quad \cdots \quad \delta M_{1N} \quad \delta M_{21} \quad \delta M_{22} \quad \cdots \quad \delta M_{2N} \quad \cdots \quad \delta M_{N1} \quad \delta M_{N2} \quad \cdots \quad \delta M_{NN} \}^T \quad (14b)$$

$$\mathbf{r} = \{ R_{11} \quad R_{12} \quad \cdots \quad R_{1N} \quad R_{21} \quad R_{22} \quad \cdots \quad R_{2N} \quad \cdots \quad R_{N1} \quad R_{N2} \quad \cdots \quad R_{NN} \}^T \quad (14c)$$

Matrix \mathbf{A} (size $N_e^2 \times N^2$) is the direct product. Vector $\delta \mathbf{m}$ is of length N^2 . And vector \mathbf{r} on the right hand side is of length N_e^2 .

Therefore δM_{ij} ($i, j = 1, 2, \dots, N$) can be obtained by Eq. (13).

For most physical systems, the number of measured modes is less than that of degree of freedom. Therefore $N_e \ll N$, which means the number of equations is less than the number of unknowns. Eq. (13) becomes an underdetermined problem, so it typically has infinite number of solution, in which an exact vector $\delta \mathbf{m}$ will not be found. Only the minimum Euclidean norm solution can be found when the vector $\delta \mathbf{m}$ satisfies

$$\text{Min} = \|\mathbf{A} \delta \mathbf{m} - \mathbf{r}\| \quad (15)$$

The stiffness matrix can also be updated in analogy method.

Premultiply Eq. (5) by Λ^{-1} , we have

$$\Lambda^{-1} \Phi^T \mathbf{K} \Phi_a = \Phi^T \mathbf{M} \Phi_a \quad (16)$$

Postmultiply Eq. (6) by Λ_a^{-1} , we obtain

$$\Phi^T (\mathbf{K} + \mathbf{K}_a) \Phi_a \Lambda_a^{-1} = \Phi^T (\mathbf{M} + \mathbf{M}_a) \Phi_a \quad (17)$$

Eq. (16) minus Eq. (17) yields

$$\Lambda^{-1} \Phi^T \mathbf{K} \Phi_a - \Phi^T \mathbf{K} \Phi_a \Lambda_a^{-1} = \Phi^T \mathbf{K}_a \Phi_a \Lambda_a^{-1} - \Phi^T \mathbf{M}_a \Phi_a \quad (18)$$

Define that

$$\mathbf{S} = \Phi^T \mathbf{K}_a \Phi_a \Lambda_a^{-1} - \Phi^T \mathbf{M}_a \Phi_a \quad (19a)$$

$$\mathbf{U} = \Phi^T \mathbf{K} \Phi_a \quad (19b)$$

Because Λ and Λ_a are both diagonal matrices, using straightened algorithm and substituting Eqs. (19a, 19b) into Eq. (18), we have

$$\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_{aj}} \right) U_{ij} = S_{ij} \quad (20)$$

Assuming λ_i does not equal λ_{aj} , the unknown elements of \mathbf{U} can be solved.

Substituting \mathbf{U} and Eq. (11b) into Eq. (19), we have

$$\Phi^T \delta \mathbf{K} \Phi_a = \mathbf{U} - \Phi^T \mathbf{K}_0 \Phi_a = \mathbf{T} \quad (21)$$

The right hand side of this equation is known. Using direct product and straightened algorithm, Eq. (21) becomes

$$\mathbf{A} \delta \mathbf{k} = \mathbf{t} \quad (22a)$$

where

$$\delta \mathbf{k} = \{ \delta K_{11} \quad \delta K_{12} \quad \cdots \quad \delta K_{1N} \quad \delta K_{21} \quad \delta K_{22} \quad \cdots \quad \delta K_{2N} \quad \cdots \quad \delta K_{N1} \quad \delta K_{N2} \quad \cdots \quad \delta K_{NN} \}^T \quad (22b)$$

$$\mathbf{t} = \{ T_{11} \quad T_{12} \quad \cdots \quad T_{1N} \quad T_{21} \quad T_{22} \quad \cdots \quad T_{2N} \quad \cdots \quad T_{N1} \quad T_{N2} \quad \cdots \quad T_{NN} \}^T \quad (22c)$$

Vector $\delta \mathbf{k}$ is of length N^2 , and vector \mathbf{t} is of length N_e^2 . The $\delta \mathbf{k}$ can be found by Eq. (21) or Eq. (22a).

When the number of measured mode is N (that is $N_e = N$), the coefficient matrix of the left hand side of Eq. (22a) is full rank, and the unique solution of $\delta \mathbf{K}$ can be obtained. But for the same reason as calculating $\delta \mathbf{M}$, when $N_e \ll N$, we can only get the minimum Euclidean norm solution whose error is difficult to predict and control.

Fortunately, in the physical engineering problem, not all of the elements of the mass and stiffness matrices need to be updated. On the one hand, the geometrical relationship between FEs does not change, and the zero elements of these matrixes should not be updated; on the other hand, the local range where error may exist is often known and the position of element to be updated is definite. Hence the number of elements to be updated is more less than $2N \times N$. Sometimes, $\delta \mathbf{M}$ and $\delta \mathbf{K}$ can be obtained through following steps.

Applying elementary transformation to both sides of Eqs. (13, 22a) and moving the elements that need to be updated to the upside, new Eqs. (23, 24) are deduced with block matrix algorithm, shown as

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \delta \mathbf{M} \\ 0 \end{bmatrix} = \mathbf{r} \quad (23)$$

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \delta \mathbf{K} \\ 0 \end{bmatrix} = \mathbf{s} \quad (24)$$

Then, we have

$$\mathbf{A}_1 \delta \mathbf{M} = \mathbf{r} \quad (25)$$

$$\mathbf{A}_2 \delta \mathbf{K} = \mathbf{s} \quad (26)$$

Error control is the most important requirement in updating model. But in fact, it is very difficult to compare the difference between the two sets of matrices directly. In the example of this paper, the mass and stiffness matrices of the example have been known. Therefore if the stiffness and mass matrixes are strengthened and changed into "stiffness vector" and "mass vector" (\mathbf{K}_{AV} and \mathbf{M}_{AV}), \mathbf{Q}_K , \mathbf{Q}_M can be used to calculate the error of these vectors, shown as

$$\mathbf{Q}_K = \frac{[\mathbf{K}_{AV}^T \mathbf{K}_R]^2}{[\mathbf{K}_R^T \mathbf{K}_R] \times [\mathbf{K}_{AV}^T \mathbf{K}_{AV}]} \quad (27a)$$

$$\mathbf{Q}_M = \frac{[\mathbf{M}_{AV}^T \mathbf{M}_R]}{[\mathbf{M}_R^T \mathbf{M}_R] \times [\mathbf{M}_{AV}^T \mathbf{M}_{AV}]} \quad (27b)$$

where \mathbf{M}_R and \mathbf{K}_R are the "accurate stiffness vector" and "accurate mass vectors" which are straightened from matrixes. The value of \mathbf{Q}_K or \mathbf{Q}_M equal with 1 means the stiffness matrix or mass matrix of the model is accurate. But on most occasions, the accurate stiffness and mass matrices could not be found at all.

A better method for valuing a FE model is to compare experimental mode results from prototype structure with predicted results from the corresponding FE model. The most commonly used results are mode frequencies (ω_T), because their test precision is relatively higher than that of mode shapes. The correlation extent of the calculation frequencies (ω_A) and (ω_T) can be defined as^[8]

$$\Delta\omega = (\omega_T - \omega_A) / \omega_A \quad (28)$$

In general, the smaller the value of $\Delta\omega$ is, the more accurate FE model will be. But it is difficult to estimate the corresponding relation between the two sets of modes when the closely spaced frequencies exist.

The modal assurance criteria (MAC) is also a widely used technique to compare numerical data with test results. MAC is often used to pair mode shapes derived from analytical models ob-

tained experimentally^[9]. MAC between the i -th analytical mode ($\boldsymbol{\varphi}_{iA}$) and the j -th measured mode ($\boldsymbol{\varphi}_{jT}$) is defined as

$$\text{MAC}_{ij} = \frac{[\boldsymbol{\varphi}_{iA}^T \boldsymbol{\varphi}_{jT}]}{[\boldsymbol{\varphi}_{iA}^T \boldsymbol{\varphi}_{iA}] [\boldsymbol{\varphi}_{jT}^T \boldsymbol{\varphi}_{jT}]} \quad (29)$$

The value of MAC is between zero and 1. The value of 1 means that the analytical mode shape is as same as the measured one. MAC forms a matrix. In order to use it conveniently, we defines the variable \mathbf{Q} as

$$\mathbf{Q} = \frac{\sum_{i=1}^N \left(\frac{\text{MAC}_{ii}}{\sum_{j=1}^N \text{MAC}_{ij}} \right)}{N} \quad (30)$$

If \mathbf{Q} nearly equals 1, the FE model would be accurate.

3 Example Analyses

3.1 Simple example

The method above can be illustrated with the following example. It is a simple vibration system of 30 degrees of freedom which is made of springs and mass blocks. Its mass matrix is diagonal and the stiffness matrix is symmetric and tri-diagonal. The system is showed in Fig. 1. M_3 equals 8.6 kg, M_{13} equals 20 kg and M_{20} equals 1.0 kg. Each of the other masses equals 6 kg, and the stiffness of each spring is 5 000 kN/m except K_3 which equals 3 000 kN/m. In the process of FE modeling, the mass and stiffness are 6 kg and 5 000 kN/m respectively.

The change of system is that 3 kg is added on the 29th mass block and the movement of the 30th mass block is restricted. It means that the added mass element $\mathbf{M}_{a(29 \times 29)}$ is 3 kg and the stiffness element $\mathbf{K}_{(30 \times 30)}$ is nearly infinite. The new system is shown in Fig. 2. Assuming all the vibration modes of the original and new systems can be measured, very perfect results will be obtained. In this simulating example, the measured modes are substituted by the calculated FE results which have no errors. The element errors of the updated FE model are listed in Table 1. It is obvious that it does not need to compare the vibration modes, since the updating results are perfect.

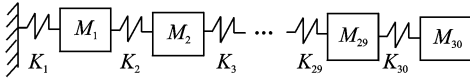


Fig. 1 Original system of mass and spring

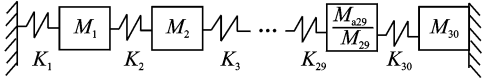


Fig. 2 New system of mass and spring after changing stiffness and mass

As we know, in fact, it is very difficult to get all the modes by test. Therefore the updating effects using incomplete modes must be evaluated. To the above example, the effects are different when the number of modes is different. After calculation and analysis, the rule can be confirmed that the more the measured modes are gotten, the more accurate the updated model is. Table 2 shows the relative error of FEs when the model is updated based on different numbers of test modes (frequency truncated). Table 3 shows the relation between the accuracy of some calculated modes and the number of measured modes. Table 4 gives the relation between the variables, Q_K, Q_M, Q , and the number of measured modes. It is obvious that both the updated elements and the calculated modes become accurate as the increase of the number of measured modes and the truncated frequency. Especially, the frequencies and shapes of calculated mode reach real results faster. In this example, when the truncated frequency is 45 Hz and the number of measured modes is 11, the relative error of an element of the mass matrix is 66%, but the maximum error of the first six modes is only 2.4%. Another em-

pirical rule can be justified in the example is that when the measured truncated frequency is low, the accuracy of the updated elements and calculated modes would be fluctuant.

Just as what is stated above, the matrices of FE model can be updated accurately by using less measured modes if the locations of errors are known in advance, which is to be illustrated by the example shown in Figs. 1–2. And the updating effects are researched based on different numbers of measured modes on the occasion of ascertaining the locations of errors. Its results are shown in Tables 5–7. Table 5 shows the relation between the relative error of updated matrix elements and the number of test modes (truncated frequency). Table 6 gives the relation between the accuracy of some calculating frequencies and the number of measured modes. Table 7 shows the relation between the variables Q_K, Q_M, Q , and the number of measured modes. From these tables, it can be found that updated elements and calculating modes are all accurate even if only first two modes are measured.

3.2 Complicated example

Below is a more complicated example of wing flutter test piece. There are 141 nodes and 222 FEs in the model, which is shown in Fig. 3. The comparison of the testing and calculating frequencies is given in Table 8.

The structure is changed by adding mass at the different point of the wing tip respectively. Firstly, a 0.2 kg steel block is fixed at the leading edge. Then, a 0.4 kg steel block is fixed at trailing

Table 1 Elements of stiffness and mass matrices updated by complete modes

Value	$M_{3,3}$ / kg	$M_{13,13}$ / kg	$M_{20,20}$ / kg	$K_{2,2}$ / (N · m ⁻¹)	$K_{3,3}$ / (N · m ⁻¹)	$K_{2,3}$ / (N · m ⁻¹)
No updating value	6.0	6.0	6.0	1×10^6	1×10^6	-5×10^5
Exact value	8.6	25.0	3.5	8×10^5	8×10^5	-3×10^5
Updated value	8.6	25.0	3.5	8×10^5	8×10^5	-3×10^5

Table 2 Relation between error of elements and number of tested modes (truncated frequency)

Truncated frequency/Hz // Number of modes	Relative error / %					
	$M_{3,3}$	$M_{13,13}$	$M_{20,20}$	$K_{2,2}$	$K_{3,3}$	$K_{2,3}$
No updating	30.232 6	76.000 0	71.428 6	25.000 0	25.000 0	66.666 7
25//6	27.341 0	68.276 1	70.654 9	24.917 6	24.815 4	66.995 9
45//11	24.770 4	51.075 6	65.538 5	23.129 7	24.499 9	64.450 5
65//16	11.785 3	26.231 5	59.938 8	19.825 6	14.566 0	48.417 1
80//21	4.070 3	8.804 3	54.945 6	2.172 0	4.820 0	9.433 9
98//26	0.253 8	0.857 4	50.842 3	0.000 0	0.000 0	0.000 0
100//29	0.000 0	0.000 1	32.469 3	0.000 0	0.000 0	0.000 0

Table 3 Relation between errors of 1st—6th frequencies and number of tested modes

Truncated frequency/Hz // Number of modes	Relative error/%					
	1st frequency	2nd frequency	3rd frequency	4th frequency	5th frequency	6th frequency
No updating	5.440 0	9.895 5	2.316 5	9.861 8	1.882 6	9.168 9
25//6	1.104 4	1.339 4	1.254 7	0.868 5	0.347 1	2.923 0
45//11	2.376 6	1.782 5	0.251 5	0.776 8	0.026 1	1.567 7
65//16	0.734 0	0.088 5	0.395 3	0.383 0	0.091 2	0.312 4
80//21	0.075 3	0.072 1	0.076 6	0.032 2	0.020 9	0.146 7
88//26	0.004 5	0.001 5	0.002 8	0.009 0	0.000 3	0.011 1
100//29	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0

Table 4 Relation between parameters (Q_K , Q_M , Q) and number of tested modes

Truncated frequency/Hz// Number of modes	Q_K	Q_M	Q
No updating	0.996 4	0.785 0	0.500 2
25//6	0.996 4	0.803 0	0.514 5
45//11	0.996 6	0.850 2	0.559 4
65//16	0.997 5	0.921 4	0.677 8
80//21	0.999 5	0.971 8	0.831 4
88//26	1.000 0	0.994 9	0.917 8
100//29	1.000 0	0.998 3	0.931 2

Table 5 Relation between errors of elements and number of tested modes

Truncated frequency/Hz // Number of modes	Relative error/%					
	$M_{3,3}$	$M_{13,13}$	$M_{20,20}$	$K_{2,2}$	$K_{3,3}$	$K_{2,3}$
No updating	30.232 6	76.000 0	71.428 6	25.000 0	25.000 0	66.666 7
5//1	20.861 4	44.190 3	30.806 3	24.095 0	21.996 8	71.079 2
8//2	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0
25//6	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0

Table 6 Relation between errors of 1st—6th frequencies and number of tested modes

Truncated frequency/Hz // Number of modes	Relative error/%					
	1st frequency	2nd frequency	3rd frequency	4th frequency	5th frequency	6th frequency
No updating	5.440 0	9.895 5	2.316 5	9.861 8	1.882 6	9.168 9
5//1	3.056 7	3.026 2	4.240 6	0.627 1	0.216 2	1.046 0
8//2	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0
25//6	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0	0.000 0

Table 7 Relation between parameters (Q_K , Q_M , Q) and number of tested modes

Truncated frequency/Hz // Number of modes	Q_K	Q_M	Q
No updating	0.996 4	0.785 0	0.500 2
5//1	0.996 5	0.856 6	0.417 9
8//2	1.000 0	1.000 0	0.969 9
25//6	1.000 0	1.000 0	0.969 9

ling edge. The first four modes of two structures are measured and can be used in model updating. In the process of updating, the freedoms U_X , U_Z , R_Y of the models are reduced. And the freedoms of reduced model are coinciding with testing system.

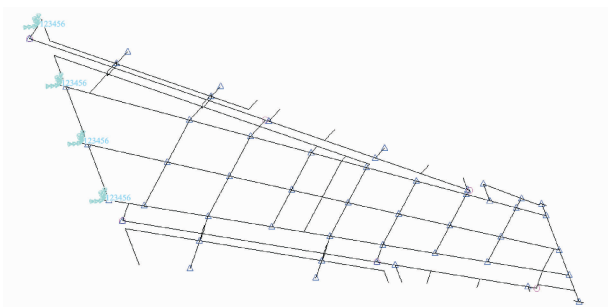


Fig. 3 FE model of wing flutter model

The comparison of the testing and calculating frequencies after updating is also given in Table 8. MAC is shown in Fig. 4. Variable Q is 0.838. Therefore the updating precision is satisfactory.

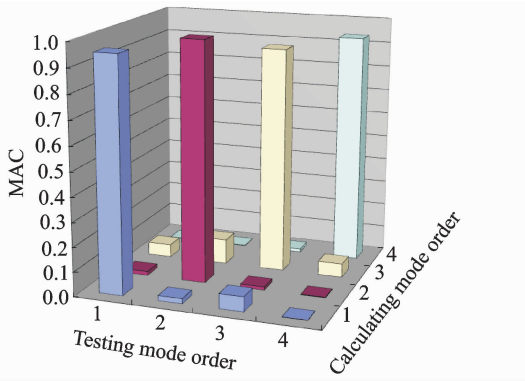


Fig. 4 Histogram of MAC

Table 8 Comparison of testing and calculating frequencies

Mode No.	1	2	3	4
Testing frequency/Hz	3.50	10.00	15.30	20.50
Calculating frequency before updating/Hz	3.38	9.73	15.24	20.10
Relative error before updating/%	3.40	2.70	0.40	2.00
Calculating frequency after updating/Hz	3.47	10.05	15.28	20.51
Relative error after updating/%	0.86	0.50	0.13	0.05

4 Conclusions

The paper presents a new approach used to update the matrices of mass and stiffness. The precondition of updating is that the vibration modes of the new system are measured, whose structural stiffness and mass have been changed. The wholly process is easy to be put into practice.

On the basis of theoretical analysis and two examples, the results of the updating method are presented as follows.

(1) When the degrees of FE model coincide with those of modal test, and if the complete modal sets of both the original and the new systems (stiffness and mass are changed) are measured, all of the elements of the stiffness and mass matrices can be updated accurately.

(2) If only some measured modes can be obtained, the amended matrices $\delta\mathbf{M}$ and $\delta\mathbf{K}$ are minimal Euclidean norm solutions. The updated stiffness and mass matrices are close to accurate solutions when the number of measured modes is increased.

But even if $\delta\mathbf{M}$ and $\delta\mathbf{K}$ are not accurate, the precision of the calculating modes can be satisfied. That is to say, the updating method is also effective and useful when the complete modal sets can not be obtained.

(3) If the modeling error can be localized to some positions, the needed number of measured modes will be decreased effectively. Correspondingly, the practicability of this updating method will be increased.

It is acknowledgeable that if a structure model is complicated and the errors are big, using the updating method will be difficult especially when the measured modes are less, which needs to be further researched.

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