Multi-objective Optimization of Differential Steering System of Electric Vehicle with Motorized Wheels

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Abstract: A differential steering system is presented for electric vehicle with motorized wheels and a dynamic model of three-freedom car is built. Based on these models, the quantitative expressions of the road feel, sensitivity, and operation stability of the steering are derived. Then, according to the features of multi-constrained optimization of multi-objective function, a multi-island genetic algorithm (MIGA) is designed. Taking the road feel and the sensitivity of the steering as optimization objectives and the operation stability of the steering as a constraint, the system parameters are optimized. The simulation results show that the system optimized with MIGA can improve the steering road feel, and guarantee the operation stability and steering sensibility.

Key words: electric vehicle with motorized wheels; differential steering; multi-island genetic algorithm; multi-objective

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1 Introduction

Based on the electric vehicle steering technology and the traditional power steering technology, the differential steering system (DSS) of electric vehicle with motorized wheels can be developed, which has very broad applicable prospect. As an ideal steering technology of vehicle, DSS of electric vehicle with motorized wheels can combine the steering probability and the steering road feel perfectly, and unite the safety and flexibility as well[1-2].

Up to now, there are hardly any researches on the optimization of the DSS system parameters both at home and abroad, and the optimization of traditional power steering system still focuses on the genetic algorithm (GA)[3-4]. Reviewing the related literatures, the system parameters are generally optimized with steering road feel as optimization target and steering stability as constraint. Such kinds of optimizations do have some shortcomings. On the one hand, it is easy to fall into local optimal solution with GA[5-6]; on the other hand, the system is optimized without taking the steering portability into consideration[7-8].

The multi-island genetic algorithm (MIGA) is a new optimization algorithm based on GA. It can not only keep the diversity of optimal solutions, but also increase the chances to get global optimal solutions, thus avoid getting the local optimal solutions as much as possible and restrain the precocious phenomena. In this paper, a dynamic model of the DSS system for electric vehicle with motorized wheels is built. The DSS sys-

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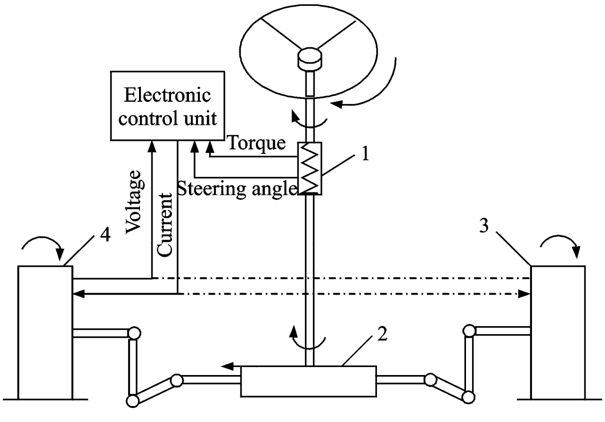
tem parameters are optimized based on MIGA, with steering road feel and steering portability as optimizing target, and steering stability as constraint. It can offer a theoretical basis for the model selection and the design for the DSS system of electric vehicle with motorized wheels.

2 Vehicle Dynamic Model

The DSS structure of the electric vehicle with motorized wheels is shown in Fig. 1. Based on the traditional mechanical front-wheel-drive steering system, it has a torque sensor and a steering angle sensor in the steering shaft to measure the steering torque and the angle given by the driver, which is motivated by two in-wheel motors in the front shafts. With force and displacement coupled control of the left and right in-wheel motors, DSS of the electric vehicle with motorized wheels can coordinate the active steering safety and the steering road feel perfectly.



1—Torque sensor; 2— Rack and pinion; 3, 4— In-wheel motors.

Fig. 1 DSS of electric vehicle with motorized wheels

2.1 Three-degree-of-freedom model of vehicle

The three-degree-of-freedom dynamic differential equations of the vehicle can be expressed as

$$\begin{cases} I_Z\dot{\omega}_{\mathrm{r}} - I_{XZ}\ddot{\phi} = 2ak_1\alpha_1 - 2bk_2\alpha_2 + \Delta T_{\mathrm{m}}l/d \\ mu(\omega_{\mathrm{r}} + \dot{\beta}) - m_{\mathrm{s}}h\ddot{\phi} = 2k_1\alpha_1 + 2k_2\alpha_2 \\ I_X\ddot{\phi} - m_{\mathrm{s}}u(\omega_{\mathrm{r}} + \dot{\beta})h - I_{XZ}\dot{\omega}_{\mathrm{r}} = \\ \qquad -(D_1 + D_2)\dot{\phi} - (C_{\phi1} + C_{\phi2} - m_{\mathrm{s}}gh)\phi \end{cases} \tag{1}$$

where g is the acceleration due to gravity; u, ω_{r}, ϕ, and β are velocity, yaw rate, roll angle, sideslip angle of the vehicle, respectively; l is the distance between the two front wheels; ΔT_{m} is the driving torque difference of the two front wheels; δ is the front-wheel steer angle; α_1, α_2, a, b, and h are front-wheel sideslip angle, rear-wheel sideslip angle, distance between the front axle and the center of mass, distance between the rear axle and the center of mass, and the rolling moment arm of the vehicle, respectively; m, m_{s}, I_X, and I_Z are mass, sprung mass, moment of inertia of spring mass about X axis, moment of inertia of mass about Z axis of the vehicle, respectively; I_{XZ}, k_1, k_2 are product of inertia of sprung mass about X and Z axes, front-wheel cornering stiffness, rear-wheel cornering stiffness, respectively; $C_{\phi1}$, $C_{\phi2}$, D_1, and D_2 are roll angle stiffness of front suspension, roll angle stiffness of rear suspension, roll angle damping of front suspension, roll angle damping of rear suspension of the vehicle, respectively.

2.2 Steering system model

By the mathematical analyses of the steering system, the dynamic and electromagnetic equations can be obtained as follows

$$\begin{cases} J_{\mathrm{h}}\ddot{\theta}_{\mathrm{h}} + B_{\mathrm{h}}\dot{\theta}_{\mathrm{h}} = T_{\mathrm{h}} - K_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{e}}) \\ T_{\mathrm{m}} = K_{\mathrm{a}}i_A \\ J_{\mathrm{e}}\ddot{\theta}_{\mathrm{e}} + B_{\mathrm{e}}\dot{\theta}_{\mathrm{e}} = K_{\mathrm{s}}(\theta_{\mathrm{h}} - \theta_{\mathrm{e}}) + \Delta T_{\mathrm{m}}/n_1 - T_{\mathrm{r}} \\ \Delta T_{\mathrm{m}} = \dfrac{d}{r_{\mathrm{w}}}(T_1 - T_2) \end{cases} \tag{2}$$

where J_{h} is the moment of inertia of the steering input shaft and steering wheel; T_{h} is the steering torque acting on the steering wheel; B_{h} is the damping coefficient; θ_{h} is the angle of rotation; K_{s} is the stiffness of the input shaft; θ_{e} is the angle of the output shaft; K_{a}, J_{e}, and B_{e} are the torque coefficient, the moment of inertia of steering output shaft, and the damping coefficient of steering output shaft, respectively; n_1 is the transmission ratio of the steering screw to the front wheel; T_{r} is the anti-torque exerted on the steering output shaft; d is the offset distance of master pin of the left and right front wheels; r_{w} is the rolling radius of the wheel; T_{m} is the electromagnetic torque of motor, T_1 the electromagnetic torque of the left motorized wheel, and T_2 the electromagnetic torque of the right motorized wheel; i_A is the motor current.

3 Optimization Model

3.1 Design variables

The effects of some parameters on the steering operation performance are unchangeable in a real situation or just determined by experience, so some powerful and practically changeable parameters are chosen to be taken into consideration when designing variables. Those variables are K_m, K_s, J_e, and B_e which stand for the torque gain coefficients of in-wheel motors, the stiffness of the input shaft, the moment of inertia of steering output shaft, and the damping coefficient of steering output shaft, respectively.

3.2 Objective function

(1) Steering road feel

In order to transfer the information from the road to the driver completely, the average frequency power of the steering road feel should be as big as possible in a certain frequency range[9-10]. The objective function $f(K_m, K_s, J_e, B_e)$ is the average frequency power of the steering road feel in the $(0, \omega_0)$ frequency range, and $\omega_0 = 40$ Hz. The bigger the objective function $f(K_m, K_s, J_e, B_e)$ is, the better transfer characteristics of the steering road feel it has. Namely

$$f(K_m, K_s, J_e, B_e) = \frac{1}{2\pi\omega_0}\int_0^{\omega_0} |E(j\omega)|^2 d\omega = \frac{1}{2\pi\omega_0} \cdot \int_0^{\omega_0} \frac{K_s^2}{\left(K_s + \frac{2d}{r_\omega n}K_a K_m K_s - J_e\omega^2\right)^2 + B_e^2\omega^2} d\omega \tag{3}$$

(2) Steering portability

In order to get good steering portability for the vehicle, the average frequency power of the steering portability should be in an appropriate range[10]. The objective function f_2 is the average frequency power of steering portability in the $(0, \omega_0)$ frequency range and $\omega_0 = 40$ Hz, shown as

$$g(K_m, K_s, J_e, B_e) = \frac{1}{2\pi\omega_0}\int_0^{\omega_0} \left|\frac{\omega_r(j\omega)}{\theta_c(j\omega)}\right|^2 d\omega =$$

$$\frac{1}{2\pi\omega_0}\int_0^{\omega_0} \left|\frac{(F_3(j\omega)^3 + F_2(j\omega)^2 + F_1(j\omega) + F_0)\left(K_s + \frac{2d}{r_\omega}K_a K_m K_s\right)}{Q_6(j\omega)^6 + Q_5(j\omega)^5 + Q_4(j\omega)^4 + Q_3(j\omega)^3 + Q_2(j\omega)^2 + Q_1(j\omega) + Q_0}\right|^2 d\omega \tag{4}$$

In order to guarantee the steering portability, the average frequency power of the steering portability should be in an appropriate range. The steering portability f_2 is set in a range $[a_0, b_0]$, namely

$$g(X) \in [a_0, b_0] \tag{5}$$

3.3 Constraints

The requirement of the steering stability of the electric vehicle with motorized wheels is that the numbers of the first column of the steering stability Routh table should be positive[10], namely

$$Q_6 > 0;\ Q_5 > 0;\ a_1 > 0;\ b_1 > 0;\ c_1 > 0;$$
$$d_1 > 0;\ Q_0 > 0 \tag{6}$$

4 Optimization Algorithm

4.1 Multi-island genetic algorithm

GA belongs to unclassical optimization algorithm. As an adaptive probability, it is developed by simulating the genetics and evolution procedure of biology in the natural environment, which includes robust searching algorithm to deal with the complicated, large-scale and multivariable non-linear inversion problems.

Based on the traditional genetic algorithm (TGA), the new characteristic of MIGA is that every individual in each population is divided into several subgroups which are called the "islands". The whole operations of TGA such as selecting, crossing and mutation happen in every island respectively, and each island chooses some individuals to migrate to other islands at regular interval. Then, the operation repeats.

The migration process is controlled by two parameters including migration interval and migration rate. The former one stands for the generation between two adjacent migrations. The latter one means the percentage of migrated individuals in each migration. The migration operation in MIGA keeps the diversity of optimal solution and increases the chances of getting global optimal solutions. The optimization procedure of MIGA is as follows: (1) Optimize the initial value; (2) when it achieves preliminary convergence, a new initial value begins to operate due to mutation and migration; (3) as the operation repeats, it can avoid the local optimal solution as much as possible and restrain the precocious phenomena.

4.2 Design optimization algorithm

The issue of selecting the migration rate is complex. As the migrants are generally the optimal individuals in each subgroup, they can not only benefit the spread of excellent individuals in the whole group, but also improve the convergence speed, if the migration rate is in a high level. Meanwhile, high rate can increase the communication overhead, reduce the speed-up ratio, lead to a decline in population diversity, and even go against the feature of the algorithm that can search in multiple directions at the same time. Hence, an appropriate migration rate should be selected by experience. A small migration interval is conducive to the fusion of subgroup and allows the excellent individuals to spread throughout the subgroups timely, thus providing a favorable guide to the revolution direction of the group as well as improving the accuracy of solutions and the convergent velocity of the group. However, it will increase the communication and synchronization overhead obviously and go against the enhancement of the speed-up ratio. Additionally, the dominant position of some excellent individuals may have a negative impact on the diversity of the group and even make the revolution of the group fall into the local minimum point. On the contrary, if the migration interval is over-sized, the effect will be reversed. The migration operation of MIGA not only keeps the diversity of optimal solutions, but also improves the chance of the global optimal solution. In conclusion, the chosen parameters are as follows.

The initial values of K_m, K_s, J_e, B_e are 10, 139.82, 0.002 9, 2.3, respectively and the initial ranges of them are (1, 50), (50, 250), (0.0001, 0.0100), (1.1, 10), respectively. Meanwhile, in order to ensure $x_{j1}-x_{j2}<0$, $J>0$ is set. The number of subgroups and the individual number of each subgroup are set as 3 and 15. The probability of replication is 0.8. The probability of crossover, which generates progeny by using the weighted average of parent, is 0.8. The mutation probability is 0.01 by using the uniform method. The probability of migration, the migration interval and the maximum algebra are set as 0.3, 4, and 1 000, respectively.

4.3 Optimization results

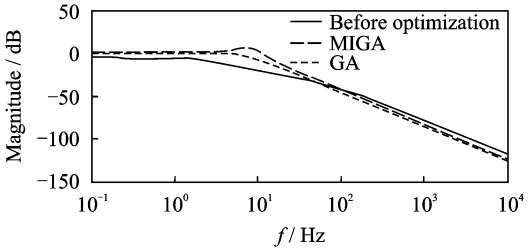
The optimization results are shown in Table 1. According to Table 1, the energy of steering road feel after optimization is 0.095 011, and it is 5.150 times bigger than the one before optimization. The steering sensitivity is 0.007 320, which meets the demand. Meanwhile, all the parameters in the first column of the Routh table are bigger than zero, which meet the demand of stability. In addition, compared with GA, although the energy of steering road feel with MIGA is only increased by 0.48%, the steering sensitivity is improved by 21.01%. Therefore, the optimization result of MIGA is more advantageous when optimizing the steering system.

Table 1 Two kinds of optimization results

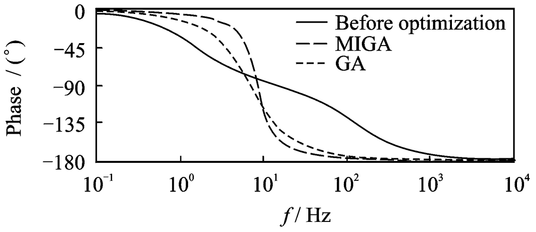
Parameter	GA	MIGA
$K_m/(\mathrm{A\cdot N^{-1}\cdot m^{-1}})$	4.146 9	3.363 7
$K_s/(\mathrm{N\cdot m\cdot rad^{-1}})$	103.859 9	135.466 2
$J_e/(\mathrm{kg\cdot m^2})$	0.055 36	0.078 77
$B_e/(\mathrm{N\cdot m\cdot rad^{-1}\cdot s})$	3.422 7	5.045 6
Road feel	0.094 556	0.095 011
Sensitivity	0.006 049	0.007 320

The bode diagram of steering road feel is shown in Fig. 2. As shown in Fig. 2, the bandwidth and amplitude increase, and the phase delay decreases after optimization. Additionally, compared with GA, the bandwidth and amplitude increase further with MIGA.

The bode diagram of steering sensibility is shown in Fig. 3. According to Fig. 3, the steering sensibility only changes a little after optimization. The differential assisted steering system optimized with MIGA can improve the steering road feel, while guaranteeing the steering sensibility and stability. Additionally, MIGA is more advantageous than GA in terms of optimization.

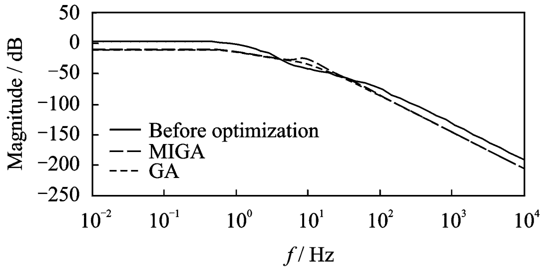


(a) Amplitude-frequency characteristic

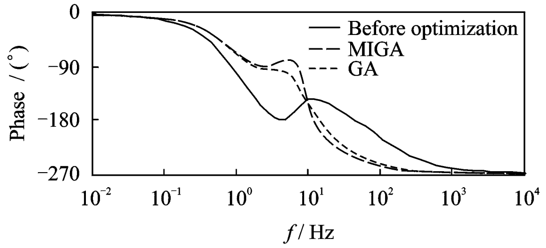


(b) Phase-frequency characteristic

Fig. 2 Bode diagram of steering road feel



(a) Amplitude-frequency characteristic



(b) Phase-frequency characteristic

Fig. 3 Bode diagram of steering sensibility

5 Conclusions

In this paper, the DSS system with force and displacement coupled control for electric vehicle with motorized wheels and the model of the three-freedom car are built. Then, according to the features of multi-constrained optimization of multi-objective function, MIGA is designed and the parameters of system are devised with multi-objective optimization. The optimization results show that the system optimized with MIGA can improve the steering road feel, reduce the steering energy consumption efficiently while guaranteeing the operation stability and steering sensibility. Additionally, MIGA is more advantageous than GA in terms of optimization, thus getting a more satisfactory result.

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