Thermoelastic Damping in Auxetic Plate

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(Received 10 October 2013; revised 10 February 2014; accepted 20 February 2014)

Abstract: The paper deals with the thermoelastic damping in a rectangular auxetic plate during its free and forced vibrations. Contrary to existing descriptions the relaxation properties of the thermal field as well as the negative material (auxetic-material of negative Poisson′s ratio) properties are taken into considerations.

Key words: thermoelastic damping; auxetics; thermal relaxation

CLC number: O34 **Document code**: A **Article ID**: 1005-1120(2014)02-0133-04

1 Introduction

The reciprocal interactions of the elastic and thermal fields in bodies of finite extent have been considered by many researchers[1-4]. The definite geometry of a body can be an origin of certain unusual phenomena. One of them is so-called the thermoelastic damping which has nothing to do with eventual viscous features of the body. The result from that phenomenon of energy dissipation (not observed in the pure elastic body) in the case, for instance, of the thermoelastic plate during its vibrations, comes from an additional heat flux normal to the boundaries of the plate. The origin of that flux is the alternate compression and extension of the upper and lower fibres of that body. This way, in the case of the plate the problem is 2D (plate)-3D (additional dimension resulting from it thickness). Zener[5] first pointed out that one of the mechanisms of thermoelastic damping is based on the stress heterogeneities giving rise to fluctuations of temperature. That idea was, among others, developed by Alblas[6-7] and then by Maruszewski[8]. Ignaczak and Ostoja-Starzewski[9] have proposed considerations dealing with the thermoelastic damping based on the extended thermodynamical model[10-11]. That phenomenon is crucial in microscience, nanoscience and engineering.

In the recent years materials with so-called negative properties have been accurately investigated for their very interesting and unexpected physical features and behavior. Such properties, among others, are characterized by negative Poisson′s ratio [12-18].

The paper deals with the free and forced bending vibrations of a thermoelastic rectangular plate, in which the thermal field is described also by one relaxation time. The particular analysis has been made both for the classic and auxetic materials.

2 Basic Equations

The subject of our considerations is a rectangular thermoelastic plate: $0\leqslant x_1\leqslant a$, $0\leqslant x_2\leqslant b$, $-\frac{h}{2}\leqslant x_3\leqslant\frac{h}{2}$. The equations which govern thermoelastic processes in that plates with the relaxation of the thermal field have the following forms[2, 7].

$$D_0 w_{\alpha\alpha\beta\beta}+\rho h\ddot{w}+\frac{1}{1-\nu_T}M_{T,\alpha\alpha}=p \tag{1}$$

$$\theta_{ii}-\left(\tau\frac{\partial}{\partial t}+1\right)\left(\frac{\rho c_v}{k}\dot{\theta}+\frac{m}{k}T_0\dot{e}\right)=0 \tag{2}$$

where $\alpha,\beta=1,2$, $i=1,2,3$, $D_0=\frac{E_T h^3}{12(1-\nu_T^2)}$, $m=\frac{E_T\alpha_T}{1-2\nu_T}$, $\theta=T-T_0$, $\left|\frac{\theta}{T_0}\right|\ll 1$, α_T is the heat ex-

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pansion coefficient, T_0 the reference temperature, k the heat conductivity coefficient, and c_v the specific heat. The mass forces and heat sources have been neglected. τ denotes the thermal relaxation time. Coefficients E_T and ν_T in Eqs. (1,2) have to be taken in a constant temperature. The moment M_T, due to the temperature distribution, is given by

$$M_T = \alpha_T E_T \int_{-\frac{h}{2}}^{\frac{h}{2}} \theta(x_1, x_2, x_3)\, x_3 \mathrm{d}x_3 \quad (3)$$

and w is the deflection. In the sequel we confine only to a simplified form of e, i. e.

$$e = -\frac{1-2\nu}{1-\nu} x_3 w_{\alpha\alpha} + \alpha_T \frac{1+\nu}{1-\nu}\theta \quad (4)$$

In the considered case of a pure small bending, Poisson′s ratio in Eq. (4) has the effective value dependent on the vibrational mode but it does not much differ from ν_T. So, in the sequel we assume that $\nu=\nu_T$. For the model of interactions taken in the paper, we assume that the changes of temperature come only from vibrations of the plate and there is no thermal influence from surrounding.

The boundary conditions for Eqs. (1,2) are as follows (the plate is simply supported on all edges)[3]

$$w(0, x_2) = w(a, x_2) = w(x_1, 0) = w(x_1, b) = 0 \quad (5)$$

$$w_{11} + \frac{1}{D_0(1-\nu_T)} M_T = 0 \text{ at } x_1 = 0, a \quad (6)$$

$$w_{22} + \frac{1}{D_0(1-\nu_T)} M_T = 0 \text{ at } x_2 = 0, b \quad (7)$$

$$\theta = 0 \text{ at } x_1 = 0, a \quad (8)$$

$$\theta = 0 \text{ at } x_2 = 0, b \quad (9)$$

$$\frac{\partial \theta}{\partial x_3} + \eta\left(\tau \frac{\partial}{\partial t} + 1\right)\theta = 0 \text{ at } x_3 = \pm\frac{h}{2} \quad (10)$$

The conditions (8—10) indicate that the temperature at lateral surfaces has been determined. At the remaining boundaries temperature varies because of alternate extension and compression of upper and lower fibres of the plate during vibrations. At those boundaries the free heat exchange has been assumed, so there is no temperature jump across upper and lower surfaces. In Eq. (10), η is the surface heat exchange coefficient.

3 Free Vibrations of Thermoelastic Plate

Since we are interested in description of the thermoelastic damping of the rectangular thermoelastic plate with Poisson′s ratio $\nu = \nu_T \in (-1; 0.5)$ during free bending vibrations, solutions of Eqs. (1, 2) with conditions (5—10) and $p=0$ are looked in the forms.

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{00mn} \sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}x_2\right) \mathrm{e}^{\mathrm{i}\omega t} \quad (11)$$

$$\theta = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{00mn}(x_3) \sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}x_2\right) \mathrm{e}^{\mathrm{i}\omega t} \quad (12)$$

For the sake of simplicity, only first terms of expansions (11,12) are taken into considerations and denoted in the sequel that $w_{0011} = w_{00}$ and $\theta_{0011} = \theta_{00}$.

Let′s take care of a material with the following properties: $E_T = 10^{11}$ N/m^2, $\alpha_T = 3 \times 10^{-6}$ K^{-1}, $\rho = 7\ 860$ kg/m^3, $k=58$ J/smK, $c_v = 460$ J/kgK, $\tau = 10^{-10}$ s, $h=0.005$ m, $a=1$ m, $b=1$ m, $T_0 = 100$ K.

From Fig. 1 it results that the first self-frequency achieves the highest values if the plate is in the auxetic state reaching minimum for $\nu=0$ and being almost constant for the natural state.

Fig. 2 indicates that the thermoelastic damping is the least in the auxetic state if Poisson′s ratio also increases depending not much on the plate thickness.

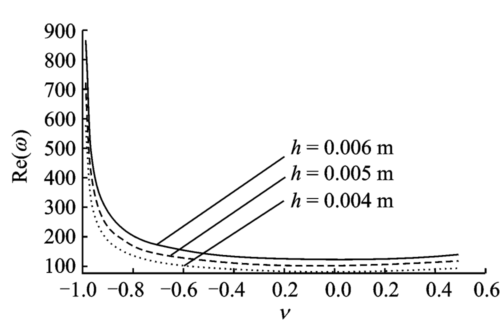

Fig. 1 First self-frequency vs. Poisson′s ratio for different plate thickness

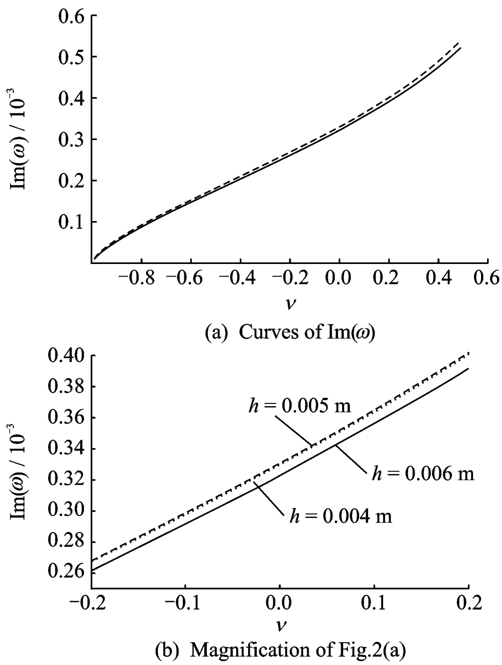

Fig. 2 Thermoelastic damping vs. Poisson's ratio for different plate thickness

The distribution of the temperature amplitude along the plate thickness is shown in Fig. 3 both for the auxetic and natural material states.

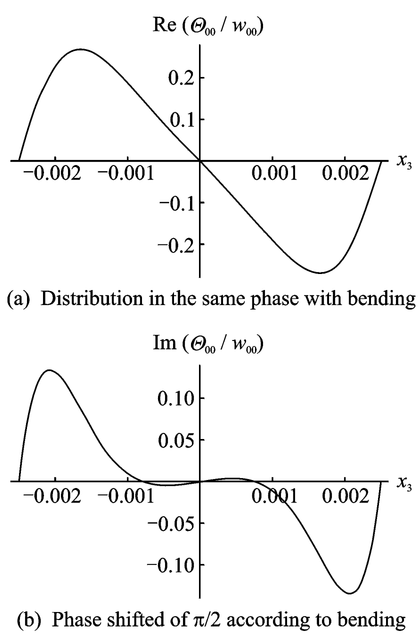

Fig. 3 Distribution of temperature amplitude along the plate thickness

It can be seen from Fig. 3(b) that if the temperature amplitude and bending are phase shifted of $\pi/2$, the thermal field in the reasonable big central region of the plate practically vanishes because of the thermoelastic damping. Then the auxetic state becomes visible if $\nu \to -1$.

From Fig. 4 it results that the first self-frequency does not practically depend on the reference temperature (independent of the surrounding temperature). But the thermoelastic damping is strongly dependent on the surrounding temperature (Fig. 5).

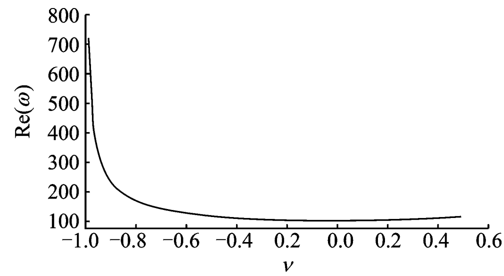

Fig. 4 First self-frequency vs. Poisson's ratio for three reference temperatures ($T_0 = 10$, 100, 273 K)

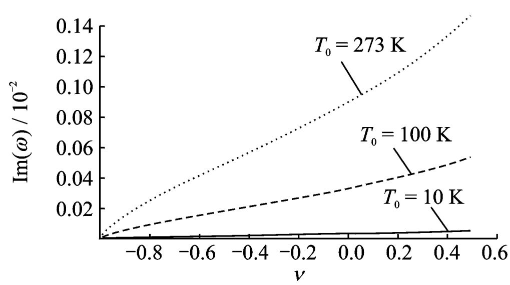

Fig. 5 Thermoelastic damping vs. Poison's ratio for different reference temperatures

And the thermoelastic damping almost linearly depends on the reference temperature T_0 for different values of Poisson's ratio (Fig. 6).

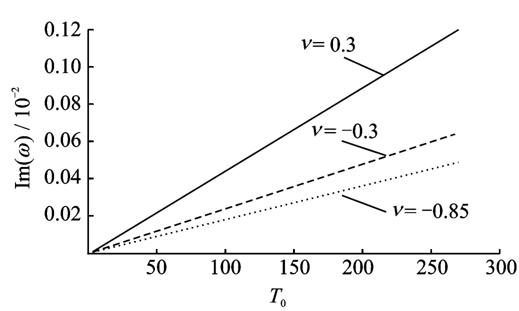

Fig. 6 Thermoelastic damping vs. reference temperature for different Poisson's ratios

4 Forced Vibrations

Now we are interested in description of a rectangular plate forced vibrations accompanied by the thermoelastic damping. The general solutions of Eqs. (1, 2) with the boundary conditions Eqs. (6−10) are looked in the Eqs. (11,12)

Those solutions concern situation that our problem is 2D-3D, as it was mentioned before. For the case that the plate vibrations have forced character, we assume that the upper surface $x_3 = \frac{h}{2}$ is loaded by (see Eq. (1))

$$p = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_{00mn} \sin\left(\frac{m\pi}{a}x_1\right) \sin\left(\frac{n\pi}{b}x_2\right) e^{i\omega t} \tag{13}$$

For the sake of simplicity, only first term of Eq. (13) is denoted by the sequel that $p_{0011} = p_{00}$.

Acknowledgements

The paper has been partially supported by MNSzW 2363/B/T02/2010/39 and 21-418/2013/DS grants.

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(Executive editor: Zhang Huangqun)