One-Dimensional-Unsteady Thermal Stress in Heat-Ray Absorbing Sheet Glass: Influence of a Sudden Weather Change

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Abstract: Heat-ray absorbing sheet glass can decrease electric energy used for air-conditioning by controling the incoming heat-ray through windows into the rooms. On the other hand, the glasses increase the temperature and sometimes yield heat cracks by thermal stresses. It is important to know the state of thermal stress accurately in order to develop heat-ray absorbing sheet glasses with higher performance and without heat cracks. A conventional design manual at field site treats the steady state and the thermal boundary condition that all heat-rays are absorbed at glass surface. In this paper, it is assumed that the heat-ray is absorbed over all the plate thickness. The idea of the local absorptibity per unit length is introduced. The modeling of internal heat absorbing process is proposed. It can explain well that the total absorptivity depends on the plate thickness. The temperature and the thermal stresses are calculated and discussed. Sudden weather changes such as rain and/or wind after the glass is heated to be steady state are also discussed. Those weather changes are treated with the change of amount of absorbed heat-ray and/or the change of heat transfer coefficient between the glass surface and the outside atmosphere.

Key words: thermal stress; sheet glass; heat-ray; absorptivity; heat crack

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1 Introduction

The importance of energy saving against the global warming and for the sustainable development has become widely understood. The newly-built buildings and houses have had the higher performance of heat insulating property and air tightness than older ones as the demand for air-conditioning increases. On the other hand, the area of windows has not become smaller than older ones because of day lighting and designing against the idea of energy saving. As a result, the energy loss through windows is estimated to be 70% of total energy loss from houses. It is demanded strongly to reduce energy loss from windows with satisfying the demand for the lighting and designing.

The solar light reaching at the window glass surface reflects, is absorbed in the glass, or transmits through the glass into the room. Those optical properties of glass is defined as the transmittance, the absorptivity and reflectance, respectively. The summation of these three optical properties is constant of 1. Therefore increasing the reflectance and/or the absorptivity is needed to decrease the amount of heat-ray transmitting into the room. The heat-ray reflecting glass enhances a function of mirror at surface and is one of the effective candidates to decrease the transmitting heat-ray. But it has defects that the reflected light may blind the driver′s eye and that the mirror may raise radio disturbance for mobile phone because the glass reflects not only heat-ray but also light and radio wave. The heat-ray absorbing glass is another candidate to decrease the incoming heat-ray amount into the room. It contains dispersed metal particles which are absorbing heat-ray. But the heat-ray absorbing glass sometimes yields heat cracks by thermal stress. It is important to know the thermal stresses in

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the glass to use the heat-ray absorbing glass effectively.

We treat thermal stress in the heat-ray absorbing sheet glass in this paper. Although the design manual to avoid heat crack is open for builders and/or clients[1], it refers to steady state and assumes that the absorbing of heat-ray occurs at only the glass surface. We introduce the idea of local absorptivity per unit length and propose the new model that the heat-ray is absorbed at not only surface but also all over the thickness. We also analyze the problem as the unsteady problem.

In addition, we consider the sudden weather changes such as sudden heavy rain and/or sudden strong wind after the glass is heated by the absorbed heat-ray and is reached to be steady state. The thick cloud may decrease the amount of absorbed heat-ray. The surface of the glass is suddenly cooled by rain and wind. Such sudden temperature change may cause heat cracking in the glass. These sudden weather changes are treated with the change of amount of absorbed heat-ray and/or the change of heat transfer coefficient between the glass surface and the outside atmosphere.

2 Analysis

2.1 Heat-ray absorbing process

We define that the heat-ray passing in the glass is the energy per unit time and per unit area, i. e., it has the same units to the heat flux. We may treat the internal heat absorbing process as the steady state because the passing time in the sheet glass of the heat-ray with light speed is much smaller than the time that the heat transfers in the glass by the heat conduction. Therefore we assume that the heat-ray passing at the arbitrary position of x depends on only the position x and not on the time.

Now we consider the energy balance in a tiny element as shown in Fig. 1. The incoming heat-ray at the position x is $I(x)$; the outgoing heat flux at $x + \mathrm{d}x$ is $I(x) + \mathrm{d}I(x)$. We introduce

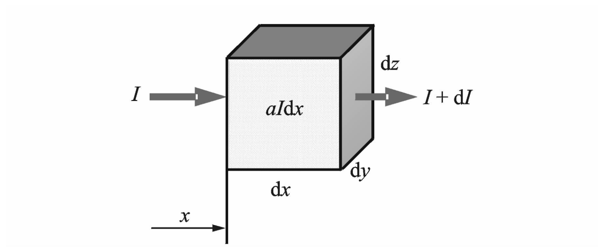


Fig. 1 Energy balance in the tiny element

the local absorptivity is constant because metal particles considered to be dispersed uniformly in sheet glass. Then the absorbed heat-ray in the tiny element is $aI\mathrm{d}x$.

The energy balance gives the following equation

$$Ia\,\mathrm{d}x\mathrm{d}y\mathrm{d}z = [I - (I + \mathrm{d}I)]\mathrm{d}y\mathrm{d}z \tag{1}$$

Integrating Eq. (1) gives Eq. (2)

$$I = C\mathrm{e}^{-ax} \quad C = \text{constant} \tag{2}$$

Boundary condition is given as follows

$$I = I_0 \quad \text{at} \quad x = 0 \tag{3}$$

where I_0 is the amount of the heat-ray reaching the glass surface. Applying Eq. (3) into Eq. (2) gives

$$I = I_0\mathrm{e}^{-ax} \tag{4}$$

We also define the total absorptivity for sheet glass with thickness b

$$AI_0 = a\int_0^b I_0\mathrm{e}^{-ax}\,\mathrm{d}x \tag{5}$$

Analyzing Eq. (5) gives the relationship between total absorptivity A and that per unit length a as follows

$$a = -\frac{\ln(1 - A)}{b} \tag{6}$$

2.2 Temperature field

We consider a heat-ray absorbing sheet glass with thickness b as shown in Fig. 2. The fine metal particles are dispersed uniformly and absorb heat-ray I_0 incomes at the surface of $x=0$. Now we assume that the quantity of heat-ray passing through the cross-section at the arbitrary position of x is $I(x)$, which is the function of position x. We treat the heat conduction and thermal stress in which the absorbed heat-ray becomes the internal heat generation as the unsteady problems. We also assume that the thermal

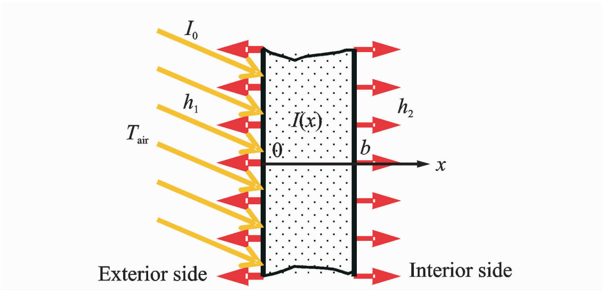


Fig. 2 Coordinate system and thermal boundary conditions

boundary conditions are that heat convention occurs at $x=0$ and $x=b$ with heat transfer coefficient h_1 and h_2 between around air, respectively.

We assume that the heat-ray absorbed at dispersed metal becomes into the heat generation. Then the governing equation, boundary conditions and the initial condition are given as follows

Basic equation

$$\frac{\partial T(t,x)}{\partial t}=\kappa\frac{\partial^2 T(t,x)}{\partial x^2}+\beta\frac{aI_0\mathrm{e}^{-ax}}{\rho C} \tag{7}$$

where ρ, C and κ is the density, the specific heat and the thermal diffusivity, respectively.

Boundary conditions

$$-\lambda\frac{\partial T(t,x)}{\partial x}=h_1[T_{air}-T(t,x)] \quad \text{at} \quad x=0 \tag{8}$$

$$\lambda\frac{\partial T(t,x)}{\partial x}=h_2[T_{room}-T(t,x)] \quad \text{at} \quad x=b \tag{9}$$

Initial condition: heating

$$T=T_0 \quad \text{at} \quad t=0 \tag{10}$$

Initial condition: sudden weather change

$$T=T_{MAX} \quad \text{at} \quad t=0 \tag{11}$$

where T_0 is the initial temperature, T_{MAX} the maximum temperature of the glass.

By solving Eq. (7) under the conditions Eqs. (8−11), we obtain the following temperature equation

$$\begin{aligned}T(t,x)=&\frac{\beta I_0}{\lambda a}(1-ax-\mathrm{e}^{-ax})+\\&\frac{1}{A_0}\Big[\frac{\beta I_0}{h_1h_2\lambda a}f_1(x)(h_1x+\lambda)+f_2(x)\Big]-\\&2\sum_{m=1}^{\infty}\frac{\lambda}{D_mp_m}\times\\&\Big\{h_2\overline{T}_{room}\Big(\frac{h_2}{\lambda}\sin p_mx+p_m\cos p_mx\Big)+\\&h_1\overline{T}_{air}\Big[\frac{h_2}{\lambda}\sin p_m(b-x)+p_m\cos p_m(b-x)\Big]+\\&\frac{E_m}{p_m}g_m(x)(h_1\sin p_mx+\lambda p_m\cos p_mx)\Big\}\mathrm{e}^{-\kappa p_m^2t}\end{aligned} \tag{12}$$

where β is coefficient of solar radiation and p_m the eigenvalues which satisfy the following equation

$$\tan p_mb=-\frac{\lambda p_m(h_1+h_2)}{\lambda^2p_m^2+h_1h_2} \quad m=1,2,3,\cdots \tag{13}$$

and

$$A_0=\frac{1}{h_1}+\frac{b}{\lambda}+\frac{1}{h_2}$$

$$f_1(x)=\Big(\frac{h_2}{\lambda}-a\Big)(\mathrm{e}^{-ax}-1)+\frac{abh_2}{\lambda}$$

$$f_2(x)=\overline{T}_{air}\Big(\frac{b-x}{\lambda}+\frac{1}{h_2}\Big)+\overline{T}_{room}\Big(\frac{x}{\lambda}+\frac{1}{h_1}\Big)$$

$$\begin{aligned}D_m=&-\lambda p_m[2\lambda+b(h_1+h_2)]\sin p_mb+\\&[\lambda(h_1+h_2)+b(h_1h_2-\lambda^2p_m^2)]\cos p_mb\end{aligned}$$

$$E_m=\beta\frac{aI_0}{\lambda}\frac{1}{a^2+p_m^2}$$

$$\begin{aligned}h_m(x)=&p_m\Big(\frac{h_2}{\lambda}-a\Big)(\mathrm{e}^{-ax}-\cos p_mb)+\\&\Big(\frac{ah_2}{\lambda}-p_m^2\Big)\sin p_mb\end{aligned}$$

2.3 Thermal stress field

Let apply the result of temperature to the thermal stress of a plate. At the time t, assuming that the distribution of temperature is $T(t, x)$, the strain and curvature at $x=0$ are ε_0 and r_0, respectively, the thermal stress $\sigma(t, x)$ is given by

$$\sigma(t,x)=2G\frac{1+\nu}{1-\nu}\Big[\varepsilon_0+\frac{x}{r_0}-\alpha T(t,x)\Big] \tag{14}$$

where G is the modulus of transverse elasticity, α the coefficient of linear thermal expansion and ν the Poisson's ratio. The strain and curvature at $x=0$, i. e., ε_0 and r_0, are determined to satisfy the mechanical boundary conditions if the elongation (E) and bending (B) are free (f) or clamped (c).

3 Analysis Result and Consideration

3.1 Absorptivity

Table 1 shows the total absorptivity of commercial heat-ray absorbing sheet glass[2] and the local absorptivity per unit length calculated by Eq. (6). The total absorptivity depends on the thickness of sheet glasses and the thicker sheet glass has the larger total absorptivity. This result suggests that our modeling of internal heat absorbing process is reasonable. We use these absorptivities for the numerical calculation and refer other physical properties needed for the calculation to Ref. [2].

Table 1 Absorptivity of commercial heat-ray absorbing sheet glass

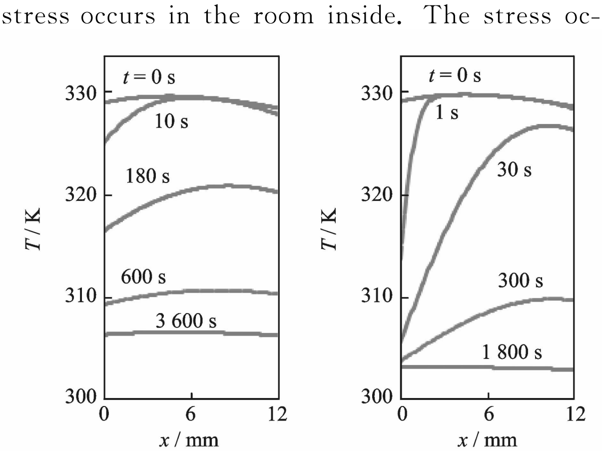
Nominal thickness b/mm	Total absorptivity A	Absorptivity per unit length a/mm^{-1}
5	0.428	0.111
6	0.488	0.111
8	0.589	0.111
12	0.726	0.107

3.2 Sudden change of weather

Fig. 3 shows unsteady temperature distribution in cases of a strong wind and rain. They are calculated for the sheet glass with 12 mm nominal thickness, which is the maximum thickness among commercial heat-ray absorbing sheet glass.

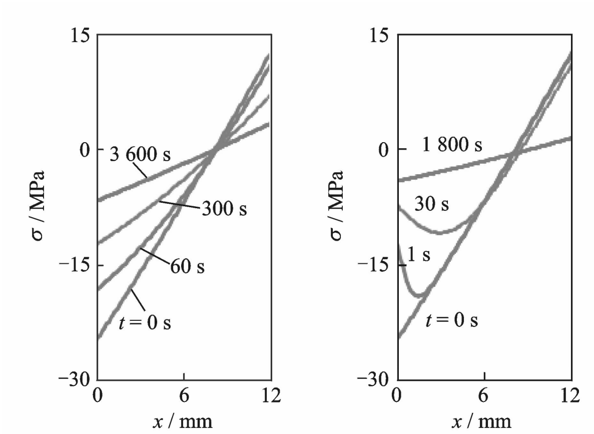
In case of a strong wind, the temperature shows distribution to gradually fall. In case of rain, the temperature shows distribution to suddenly fall. However, the room side becomes the moderate temperature change.

Fig. 4 shows the distributions of unsteady thermal stress for temperature distributions in Fig. 3. In case of a condition of EcBf, compress stress occurs in the room outside and tensile stress occurs in the room inside. The stress occurs in the tensile side because of a sudden temperature change. However, the stress becomes small gently because the room side is a moderate temperature change. In addition, it is unlikely to be the heat crack because the thermal stress is not exceeding allowable stress.



(a) Wind (b) Rain

Fig. 3 Unsteady temperature distribution

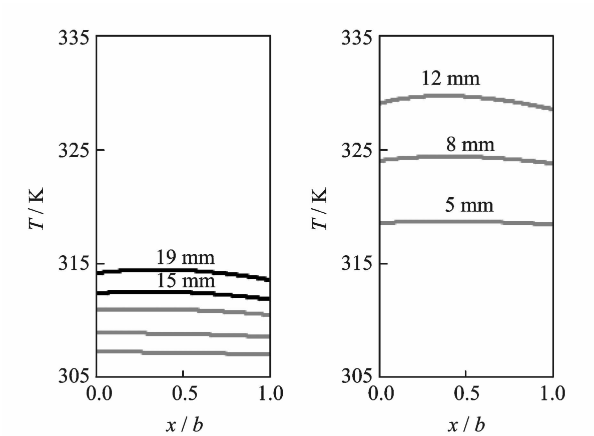


(a) Wind (EcBf) (b) Rain (EcBf)

Fig. 4 Unsteady thermal stress distribution

3.3 Soda glass and heat-ray absorbing glass

Next, we compare the common soda glass with heat-ray absorbing glass. Fig. 5 shows the distributions of maximum temperature of common soda glass and heat-ray absorbing glass. The absorption factor of common soda glass is $a = 0.018\ mm^{-1}$. The glass does not absorb much heat, and the temperature does not rise, too. Therefore the thickness of soda glass is up to $b = 19$ mm, but the thickness of heat-ray absorbing glass is up to $b = 12$ mm.



(a) Common soda glass (b) Heat-ray absorbing glass

Fig. 5 Steady temperature distributions of different glasses

Fig. 6 shows the distribution of unsteady thermal stress . Under the influence of rain , big

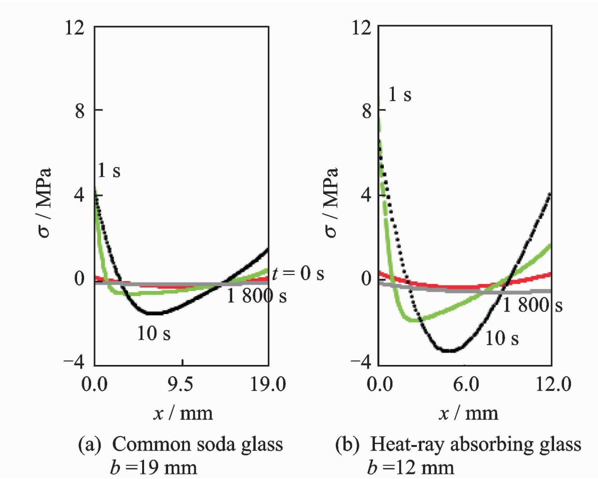


Fig. 6 Unsteady thermal stress distributions of different glasses (EfBf)

tensile stress occurs temporarily. Big stress occurs in the heat-ray absorbing glass which is higher in initial temperature than common soda glass. However, it is unlikely to be the heat crack because the thermal stress is not exceeding allowable stress with both glasses either.

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