Thermoviscoelastic Analysis of Stress in Composite Structures-Micro-to-Structural Approach

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Abstract: Manufacturing of composite materials is usually accompanied with residual stresses. These stresses should be evaluated and assessed. To this end, a micromechanical model for periodic material whose temperature dependent constituents behave as thermorheologically complex materials (TCM) has been developed. This model, referred as the high fidelity generalized method of cells (HFGMC), takes into account the detailed interaction between the fiber and resin, their volume ratios, the fibers distribution and their waviness. This model is linked, in conjunction with a special UMAT subroutine, to the ABAQUS finite element code for prediction of the response of thermoviscoelastic composite structures during cool down process. The present investigation shows the effect of the cool down rate on the residual stress developed in the composite cylindrical structures.

Key words: cool down; thermorheologically complex materials (TCM); epoxy; high fidelity generalized method of cells (HFGMC); finite element method (FEM)

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Nomenclature

T—time dependent temperature

σ_{ij}—stress tensor

S_{ij}—deviatoric stress

$v(t)$, $w(t)$—viscoelastic parts of the deviatoric strain tensor e_{ij}

e_{ij}—strain tensor

e—dilatation volumetric tensor

$K(T)$—temperature dependent bulk modulus

α—temperature independent coefficient of thermal expansion

D—creep compliance

D_0—initial value of creep compliance

ΔD—time dependent component

t_n—relaxation time

$\xi(t)=\int_0^t \frac{d\tau}{a_T[T(\tau)]}$—reduced time

a_T—the shift factor

A_{ijkl}—concentration tensor at $(\alpha\beta\gamma)$

C^*_{ijkl}—effective stiffness tensor

$\bar{\sigma}_{il}$—global stress

$\bar{\varepsilon}_{ij}$—global strain

$\bar{\varepsilon}^T_{ij}$—global thermal strain

E_A, υ_A—axial Young modulus and Poisson ratio

E_T, υ_T—transverse Young's modulus and Poisson ratio

G_A—axial shear modulus

α_A, α_T—axial and transverse coefficients of thermal expansion

J_{ij}—creep-like function

δ_{ij}—Kronecker Delta

T_0—reference temperature

1 Introduction

The manufacturing of fiber reinforced composites with polymeric matrixes, is always characterized by presence of residual stresses usually induced by the cool-down process. It should be emphasized that a part of these stresses are caused by the composite material behavior, where another part result from the tooling. The present study addresses the former type. Thermorheologically complex materials (TCMs) are temperature dependent viscoelastic materials whose characterization requires vertical and horizontal shifting in order to obtain their properties at every temperature[1-8]. With the established constitutive equation of the TCM the high fidelity generalized

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method of cells (HFGMC) micromechanical model[9] phases are governed by this equation and employed to obtain the macroscopic composite behavior at any instant of applied combined loading. Various types such as unidirectional, woven, can be considered in the framework of HFGMC. Finally, the HFGMC method is coupled via a specially developed UMAT subroutine within ABAQUS software, for prediction of composite cylindrical viscoelastic structures subjected to different cool down temperature rates.

2 3D Constitutive Law for TCM Material

In the following the multi-axial constitutive equation of TCM is briefly presented.

$$\begin{aligned}\sigma_{ij}(t)=&\frac{\sigma_{kk}}{3}\delta_{ij}+S_{ij}=\\&[K(T)\varepsilon_{kk}-3K(T)\alpha\Delta T(t)]\delta_{ij}-\\&\frac{1}{2v(t)}\Delta D[\xi(t)]g_1(T)g_2(T)S_{ij}(0)+\\&\frac{1}{v(t)}\left[\varepsilon_{ij}(t)-\frac{1}{3}\varepsilon_{kk}(t)\delta_{ij}\right]+\frac{w(t)}{v(t)}S_{ij}(t-\Delta t)-\\&\frac{1}{2v(t)}\sum_n \mathrm{e}^{-\frac{[\xi(t)-\xi(t-\Delta t)]}{t_n}}J_{ij(n)}(t-\Delta t)\end{aligned}\tag{1}$$

$$e_{ij}=\varepsilon_{ij}-\frac{1}{3}e\delta_{ij}\tag{2}$$

$$\begin{aligned}v(t)=&\frac{1}{2}D_0g_0(T)+\\&\frac{1}{2}g_1(T)g_2(T)\sum_n D_n\mathrm{e}^{-\frac{\zeta(t)-\zeta\left(t-\frac{\Delta t}{2}\right)}{t_n}}-\\&\frac{1}{2}g_1(T)g_2(T)\sum_n D_n\end{aligned}\tag{3}$$

$$w(t)=\frac{1}{2}g_1(T)g_2(T)\sum_n D_n\mathrm{e}^{-\frac{\zeta(t)-\zeta\left(t-\frac{\Delta t}{2}\right)}{t_n}}\tag{4}$$

where $g_0(T)$, $g_1(T)$, $g_2(T)$ are property functions dependent on the temperature

$$D(T_0,t)=D_0+\Delta D(t)\tag{5}$$

The time dependent component of creep compliance can be defined with Prony series

$$\Delta D(t)=\sum_{n=1}^{N}D_n(-1+\mathrm{e}^{-t/t_n})\tag{6}$$

The above constitutive equations are implemented in the HFGMC-TCM model. The time and temperature dependent material properties for epoxy Hercules 3502 are characterized using creep-recovery tests under a constant stress 10 MPa and several temperatures ranging from 30 °C to 130 °C as formulated for example in Ref. [6]. The time dependent functions are

$$g_0(T)=\exp\{0.675[(T-T_0)/T_0]^{0.782}\}\tag{7}$$

$$g_1(T)=\exp\{-5.79[(T-T_0)/T_0]^{1.11}\}\tag{8}$$

$$g_2(T)=\exp\{4.5[(T-T_0)/T_0]^{2.48}\}\tag{9}$$

$$a_T(T)=\exp\{-13.7[(T-T_0)/T_0]^{0.985}\}\tag{10}$$

The properties of the Hercules 3502 at T_0 303 K are given in Table 1.

Table 1 Epoxy Hercules 3502 properties at 303 K

K/GPa	υ	$\alpha/(10^{-6}\ \mathrm{K}^{-1})$
3.58	0.3	30

These properties of epoxy Hercules 3502 based on a one-dimensional strain stress relation, are generalized to the above multi axial constitutive relations by considering the deviatoric and volumetric parts.

In Eq. (1) $1\leqslant K\leqslant 4$[10-11] is based on a series of experiments for obtaining the time and temperature dependent bulk modules.

3 Micromechanical model HFGMC-TCM

This micromechanical model takes into account the material behavior of the fiber and the matrix, their detailed interaction, volume ratio, and the fibers distribution and waviness. The HFGMC method is based on the assumption that the microstructure of the composite is periodic, see Fig. 1 and homogenization procedure provides the macroscopic behavior of the entire composite structure. Due to the periodic characteristic of the composite it is sufficient to consider a repeating unit cell (RUC) that characterizes the entire composite. RUC can be micromechanically analyzed, see Fig. 2.

The RUC is divided to several subcells and the displacement, traction, continuity condition are imposed together with the periodic conditions see Ref. [9] for details.

For illustration we represent the resulting equation for the simple and special case of thermo elastic composite.

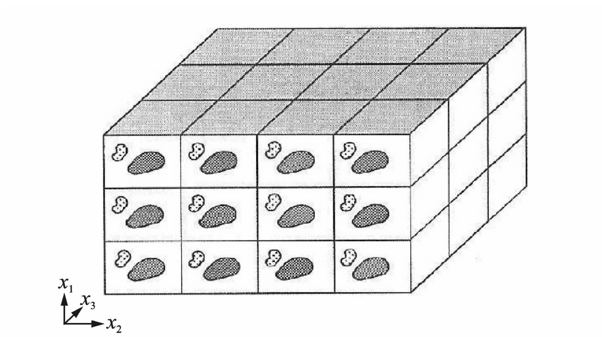

Fig. 1 Triply periodic composite

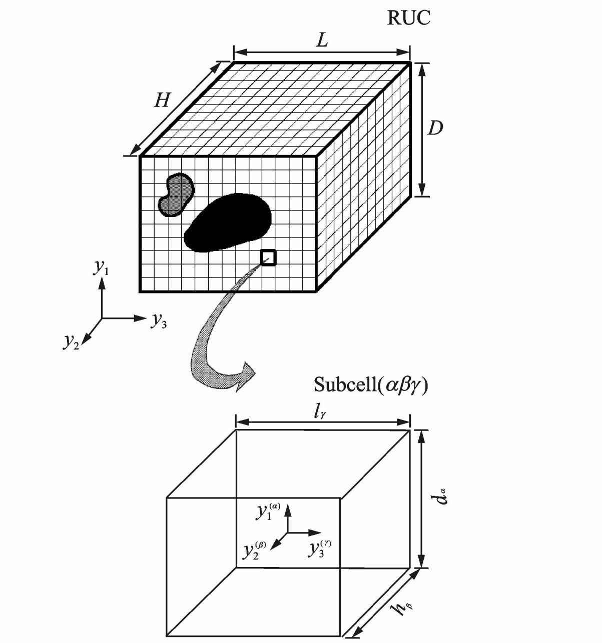

Fig. 2 RUC

The effective modulus calculated from the average stress and strain relation in the subcell are shown in Fig. 2

$$\varepsilon_{ij}^{(\alpha\beta\gamma)} = A_{ijkl}^{(\alpha\beta\gamma)} \bar{\varepsilon}_{kl} \tag{11}$$

$$\bar{\sigma}_{ij} = \frac{1}{V}\sum_{\alpha}\sum_{\beta}\sum_{\gamma} v_{\alpha\beta\gamma}\sigma_{ij}^{(\alpha\beta\gamma)} = \frac{1}{V}\sum_{\alpha}\sum_{\beta}\sum_{\gamma} v_{\alpha\beta\gamma}C_{ijkl}^{(\alpha\beta\gamma)}\varepsilon_{kl}^{(\alpha\beta\gamma)} = \frac{1}{V}\sum_{\alpha}\sum_{\beta}\sum_{\gamma} v_{\alpha\beta\gamma}C_{ijkl}^{(\alpha\beta\gamma)}A_{klpq}^{(\alpha\beta\gamma)}\bar{\varepsilon}_{pq} = C_{ijkl}^{*}\bar{\varepsilon}_{pq} \tag{12}$$

The macroscopic constitutive equation for composite material Eq. (13) is

$$\bar{\sigma}_{ij} = C_{ijkl}^{*}(\bar{\varepsilon}_{kl} - \bar{\varepsilon}_{kl}^{T}) \tag{13}$$

For thermoreologically complex composite the effect of the strain rate is caused by viscoelasticity.

4 Thermoreological Behavior of Composite FEM Structure-Analysis Result

4.1 Analysis procedure

The resulting macroscopic constitutive equations are used in a special UMAT subroutine which links the 3D HFGMC-TCM to the ABAQUS FEM code to obtain the stresses that develop as a result of the composite structure cool-down. This incremental algorithm provides the variation of the displacements, strain and stress at any step time, see Fig. 3.

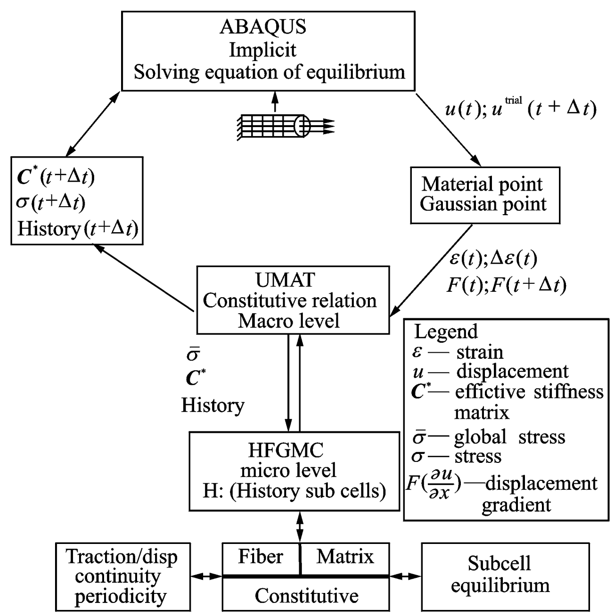

Fig. 3 UMAT

Several results of the FE analysis predict the residual stresses for different strain rates in a composite cylinder. A creep-recovery analysis has been verified by comparison with experimental results[6].

4.2 Effect of cool-down process on residual stress

A finite element analysis model of a cylinder with 1 250 3D linear brick per elements (C3D8R), is generated for 100 mm composite cylinder.

$R_{in} = 12.5$ mm, $R_{out} = 20$ mm, see Fig. 4.

The thick composite cylindrical structure is constructed from a unidirectional carbon fibers, whose properties are given in Tables 2－3. The matrix is a TCM resin which exhibits nonlinear thermoviscoelastic (TCM) as previously discussed.

Table 2 Properties of transversely isotropic carbon fibers

E_A/GPa	υ_A	E_T/GPa	υ_T	G_A/GPa
231	0.3	22.4	0.35	22.1

Table 3 Properties of transversely isotropic thermal expansion carbon fibers $10^{-6}K^{-1}$

α_A	α_T
−1.33	7.04

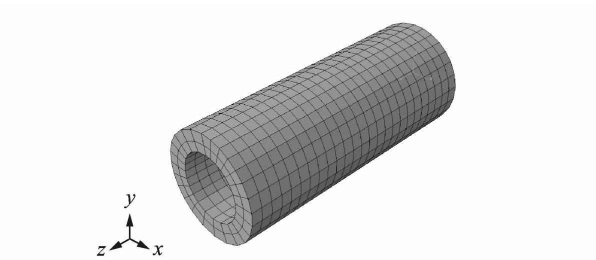

Fig. 4 FE cylinder

Clamped boundary conditions are imposed on the one end of the cylinder where the other end is kept free.

The cool-down process is simulated by a temperature decrease from 130 °C to 30 °C with different time durations.

The stresses obtained at the end of the analysis are the residual stresses of the composite structure.

In Fig. 5 the results of two procedures are shown, where in the first one the cool-down duration is 0.5 h and in the second one it is 6 h. It can be clearly observed the strong effect of the cool-down rate on the develop residual stresses.

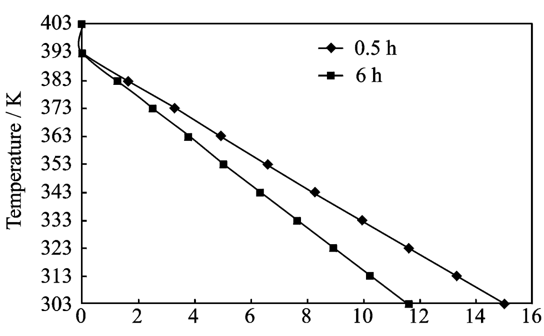

Fig. 5 Residual stress different strain rate

5 Conclusions

The described method allows to predict the contribution of the viscoelasticity effect, to the residual stresses in composites structures. The actual value of stresses may be tuned by modification of the resin properties upon request.

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