Thermoelastic Stability of Closed Cylindrical Shell in Supersonic Gas Flow

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Abstract: In a linear framework, the problem of stability of closed cylindrical shell is briefly discussed. The cylindrical shell is immersed in a supersonic gas flow and under the influence of temperature field varying along the thickness. An unperturbed uniform velocity flow field, directed along the short edges of the shell, is applied. Due to the inhomogeneity of the temperature field distribution across the thickness shell buckling instability occurs. This instability accounts for the deformed shape of the shell, to be referred as the unperturbed state. Stability conditions and boundary for the unperturbed state of the system under consideration are presented following the basic theory of aero-thermo-elasticity. The stability boundary depends on the variables characterizing the flow speed, the temperature at the middle plane of the shell and the temperature gradient in the direction normal to that plane. It is shown that the combined effect of the temperature field and flowing stream regulates the process of stability, and the temperature field can significantly change the flutter critical speed.

Key words: cylindrical shell; flutter; thermal field; supersonic gas flow; aero-thermo-elastic stability; stability region

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1 Introduction

The issue of stability of plates and shells in supersonic gas flow has many practical applications in the aerospace field. As a result it has been studied extensively. The interested reader is referred to the monographs[1-2] and the review paper[3]. The influence of constant temperature field on the stability of deformable thin bodies in supersonic gas flow was also investigated in Refs. [4-8]. In a linear framework, the problem of stability of long rectangular plate is briefly discussed. The long rectangular is immersed in a supersonic gas flow and under the influence of temperature field varying along the thickness. An unperturbed uniform velocity flow field, directed along the short edges of the plate, is applied. Due to the inhomogeneity of the temperature field distribution across the thickness plate buckling instability occurs. This instability accounts for the deformed shape of the plate, to be referred as the unperturbed state. Stability conditions and boundary for the unperturbed state of the system under consideration are presented following the basic theory of aero-thermo-elasticity. The stability boundary depends on the variables characterizing the flow speed, the temperature at the middle plane of the plate and the temperature gradient in the direction normal to that plane. It is shown that the combined effect of the temperature field and flowing stream regulates the process of stability; and the temperature field can significantly change the flutter critical speed.

2 Problem Formulation and Main Results

Let us consider a thin isotropic closed circu-

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lar cylindrical shell with the constant thickness h, the radius R and the length l. The shell is located in a orthogonal system of coordinate α,β,γ. The coordinate axes α and β coincide with the curvature lines of the middle surface of the shell (α-along the generatrix, β-along the arc of the cross-section), γ-the distance between the points (α,β, 0) and (α,β,γ) of the shell. Let the shell flows around by the external supersonic stream of gas with the undisturbed velocity U, directed along the axis 0α and located in a stationary temperature field. The issues of stability of considered aeroelastic system are investigated in case of axisymmetric deformation. To investigate the aero-thermo-elastic stability of the panel the following assumptions are considered:

(1) The Kirchhoff-Love hypothesis on non-deformable normal[9];

(2) Deformations in both unperturbed and perturbed states are so small that one can use the linear theory of aero-thermo-elasticity;

(3) The Piston theory aerodynamics (PTA) is used when calculating the aerodynamic pressure[10-11]

$$p=p_\infty\left(1+\frac{æ-1}{2}\frac{v_3}{a_\infty}\right)^{\frac{2æ}{æ-1}} \tag{1}$$

where p is the gas pressure at the shell's surface, $a_\infty^2=æp_\infty\rho_\infty^{-1}$ the sound velocity for the undisturbed gas, $æ$ the isentropic gas coefficient, and v_3 the normal component of the speed of points of shell's surface. p_∞ and ρ_∞ are the pressure and gas density in undisturbed state.

(4) A linear law of temperature field $T(\alpha,\beta,\gamma)$ through the thickness of the plate is assumed[1]

$$T=T_0(\alpha,\beta)+\gamma\Theta(\alpha,\beta) \tag{2}$$

(5) The Neumann hypothesis on absence of shears due to changes of temperature field[12].

For the sake of simplicity and clarity in the future it is assumed also that on the facial surfaces of the shell ($\gamma=\pm h/2$) and its surroundings, heat exchange occurs according to the Newton-Richman law, therefore on the shell surfaces the temperature is constant with the values T^+ and T^-, respectively, while the surface sides $\alpha=0$ and $\alpha=l$ are thermally insulated. Then the stationary temperature field in the shell can be obtained from the solution of the following equation of thermal conductivity

$$\Delta T=0 \tag{3}$$

at the area occupied by the shell, with the following surface conditions

$$\lambda\frac{\partial T}{\partial n}=k(T-T^\pm)\quad \gamma=\pm\frac{h}{2} \tag{4}$$

$$\frac{\partial T}{\partial n}=0\qquad \alpha=0,1 \tag{5}$$

The solution of the addressed problem of thermal conductivity within the assumption $kh\ll 1$ can be presented in the form (2), where

$$T_0=\frac{T^++T^-}{2},\Theta=\frac{k(T^+-T^-)}{kh-2\lambda} \tag{6}$$

In Eqs. (3—6) Δ is the two-dimensional Laplace operator, λ the coefficient of thermal conductivity, k the heat-transfer coefficient, and n the external normal to the shell's surface.

Under the influence of non-uniform, along the thickness, stationary temperature field ($\Theta\neq 0$), the shell is subjected to buckling, thus an out-of-plane deflection $w_T(\alpha)$ and a corresponding aerodynamic pressure appears which, according to the assumption (3), can be determined using Eq. (1). Using a linear approximation, the expression $æp_\infty Ma\,\mathrm{d}w_T\,\mathrm{d}\alpha$ ($Ma=Ua_\infty^{-1}$ the Mach number) is obtained for the pressure. The noted buckled state is taken as an unperturbed[8] and the stability of this state is investigated under the influence of the temperature field and the pressure of gas flow.

On the basis of the above-brought assumptions and the theory of thermoelasticity of isotropic body, as in Refs. [1,8], the following linear differential equation of stability of the considered thermo-gas-elastic system is obtained

$$D\frac{\partial^4 w}{\partial\alpha^4}-\delta\left[N_1^T\frac{\partial^2 w}{\partial\alpha^2}+N_1\left(\frac{1}{R}+\frac{\partial^2 w}{\partial\alpha^2}+\frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha^2}\right)\right]+\rho h\frac{\partial^2 w}{\partial t^2}+\left(\rho h\varepsilon+\frac{æp_\infty}{a_\infty}\right)\frac{\partial w}{\partial t}+æp_\infty Ma\frac{\partial w}{\partial\alpha}+\frac{æ(æ+1)}{2}p_\infty Ma^2\frac{\mathrm{d}w_T}{\mathrm{d}\alpha}\frac{\partial w}{\partial\alpha}=0 \tag{7}$$

where the following notations are done

$$N_1^T = -\frac{Eh}{1-\mu}\alpha T_0$$

$$N_1 = \frac{Eh}{a(1-\mu^2)}\int_0^a \frac{\mathrm{d}w_T}{\mathrm{d}\alpha}\frac{\partial w}{\partial \alpha}\mathrm{d}\alpha$$

$$\delta = \begin{cases} 0 & \text{when the edges of the plate are move freely} \\ 1 & \text{when the edges of the plate are fixed} \end{cases} \tag{8}$$

where $w(\alpha, t)$ is the perturbation of transverse displacement of the shell, $D = Eh^3/12(1-\mu^2)$, E the elastic modulus, μ the Poisson′s ratio, α the coefficient of thermal expansion, ρ the density of shell material, and ε the coefficient of linear damping.

To determine the characteristics of unperturbed state which are included in Eqs. (7—8), based on the accepted assumptions, the current boundary conditions, the main equations and relations of the thermoelasticity theory of thin shells, one can obtain:

the following relations

$$u_0^{(3)} = w_T(\alpha); u_0^{(1)} = u_T(\alpha) - \gamma\frac{\mathrm{d}w_T}{\mathrm{d}\alpha} \tag{9}$$

according to the Kirchhoff hypothesis;

the following equations

$$\frac{\mathrm{d}^2 u_T}{\mathrm{d}\alpha^2} = 0 \tag{10}$$

$$D\frac{\mathrm{d}^4 w_T}{\mathrm{d}\alpha^4} + \frac{3}{Rh^2}\left(\mu\frac{\mathrm{d}u_T}{\mathrm{d}\alpha} + \frac{w_T}{R}\right) + \text{æ}p_\infty Ma\frac{\mathrm{d}w_T}{\mathrm{d}\alpha} = 0 \tag{11}$$

with respect to u_T and w_T according to the theory of thin shells;

the following expression for stress σ_{11}^0 of the unperturbed state

$$\sigma_{11}^0 = \frac{E}{1-\mu^2}\left[\frac{\mathrm{d}u_T}{\mathrm{d}\alpha} - \mu\frac{w_T}{R} - \alpha_1(1+\mu)T_0 - \gamma\left(\frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha} + \alpha_1(1+\mu)\Theta\right)\right] \tag{12}$$

according to generalized Hooke′s law.

Based on Eq. (12) the axial force T_{11}^0 and bending moment M_{11}^0 can be readily obtained

$$T_{11}^0 = \int_{-h/2}^{h/2}\sigma_{11}^0\mathrm{d}\gamma = \frac{Eh}{1-\mu^2}\left[\frac{\mathrm{d}u_T}{\mathrm{d}\alpha} - \alpha_1(1+\mu)T_0\right] \tag{13}$$

$$M_{11}^0 = \int_{-h/2}^{h/2}\gamma\sigma_{11}^0\mathrm{d}\gamma = -D\left[\frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha^2} + \alpha_1(1+\mu)\Theta\right] \tag{14}$$

where T_0 and Θ are determined according to Eq. (6).

The solutions of Eqs. (10—11) must satisfy the fixing conditions of the edges $\alpha=0$ and $\alpha=l$ of the shell. The following boundary conditions will be considered:

The edges are hinge supported and can freely move along axis 0α;

The edges are hinge supported and fixed.

According to Eqs. (13—14), the proposed boundary conditions can be represented as follows:

The edges are hinge supported and can freely move along the axis 0α

$$w_T = 0$$
$$\frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha^2} + \alpha_1(1+\mu)\Theta = 0 \qquad \alpha = 0, l \tag{15}$$

$$\frac{\mathrm{d}u_T}{\mathrm{d}\alpha} - \alpha_1(1+\mu)T_0 = 0 \qquad \alpha = 0, l \tag{16}$$

The edges are hinge supported and fixed

$$w_T = 0$$
$$\frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha^2} + \alpha_1(1+\mu)\Theta = 0 \qquad \alpha = 0, l \tag{17}$$

$$u_T = 0 \qquad \alpha = 0, l \tag{18}$$

Included in Eqs. (7—8) the deflection of the shell w_T is a solution of the following equation of unperturbed state

$$D\frac{\mathrm{d}^4 w_T}{\mathrm{d}\alpha^4} + \frac{3}{Rh^2}\left(\frac{\delta\mu}{Rl}\int_0^l w(\alpha)\mathrm{d}\alpha + \frac{1-\mu}{R}w_T\right) + \text{æ}p_\infty Ma\frac{\mathrm{d}w_T}{\mathrm{d}\alpha} = 0 \tag{19}$$

and satisfies the boundary conditions

$$w_T = 0, \frac{\mathrm{d}^2 w_T}{\mathrm{d}\alpha^2} + \alpha_1(1+\mu)\Theta_0 = 0$$
$$\text{при} \qquad \alpha = 0, \alpha = a \tag{20}$$

Eqs. (19—20) is solved and its solution is used when the stability problem with respect to w was solved.

Thus, the investigation of the disturbances behavior of the shell, located in a non-uniform temperature field and supersonic gas flow is reduced to the solution of Eq. (7), under the corresponding boundary conditions along the edges of the shell. I. e. the solutions of Eq. (7) must satisfy the following boundary conditions

$$w=0,\frac{\partial^2 w}{\partial\alpha^2}=0 \quad \alpha=0,l \tag{21}$$

which follow from the simple supported conditions in the disturbed state.

The addressed boundary-value problem is solved via the Fourier method and the issues of stability of considered aeroelastic system is reduced to the investigation of the sign of real parts of the roots of characteristic equation, depending on the flow velocity and the characteristics of temperature field.

On this basis, using the Hurwitz theorem, the stability regions are constructed on the plane (U, T_0) (the case of a uniform temperature field $(\Theta=0)$) and on the plane (U,Θ) (the case of a non-uniform temperature field ($T_0=0$)). In Figs. 1—2 these regions are constructed having taken the following initial data: $\alpha=23.8\times10^{-6}\,°\mathrm{C}^{-1}$; $k=1\ 200\ \mathrm{W/(m^2\cdot°C)}$; $\lambda=210\ \mathrm{W/(m\cdot°C)}$; $\mu=0.34$; $h/a=1/100$, $R=\infty$. In these figures the following notations are done

$$\overline{T}=RT_0\left(\frac{a}{h}\right)^2\left(\frac{12}{\pi^2}\right),\overline{\Theta}=R\Theta_0 h\left(\frac{a}{h}\right)^2\frac{1}{36+K^2\nu^2}$$

$$K=\frac{4\ae p_\infty}{\rho\omega_1^2h^2},\nu=M\frac{h}{a},\omega_i=\sqrt{\frac{D}{\rho h}}\left(\frac{i\pi}{a}\right)^2\ (i=1,2)$$

$$\gamma=\frac{\omega_2}{\omega_1},\ \chi=\frac{1}{\omega_1}\left(\varepsilon+\frac{a_\infty\rho_\infty}{\rho h}\right);\ \nu_*=\frac{3}{4}\frac{\gamma^2-1}{K}\cdot\sqrt{1+\frac{2\chi^2(\gamma^2+1)}{(\gamma^2-1)^2}}$$

where ν_* is the critical flow speed in the absence of temperature field (or in the case of a shell with free edges in the presence of the considered temperature field).

These figures show that:

(1) If $T_0>0$, then the temperature field substantially narrows the stability region (half-strip $(0<\nu<\nu_*^0, T_0>0)$ converted into the region A).

(2) If $T_0<0$, then the temperature field significantly increases the stability region, instead of B the stability region is $B\cup C$.

(3) If $\Theta>0$, then the temperature field noticeably narrows stability region.

(4) If $\Theta<0$, then till a certain negative value of Θ_* the stability region noticeably increases; and after this particular value, with the increasing $|\Theta|$ the stability area considerably decreases.

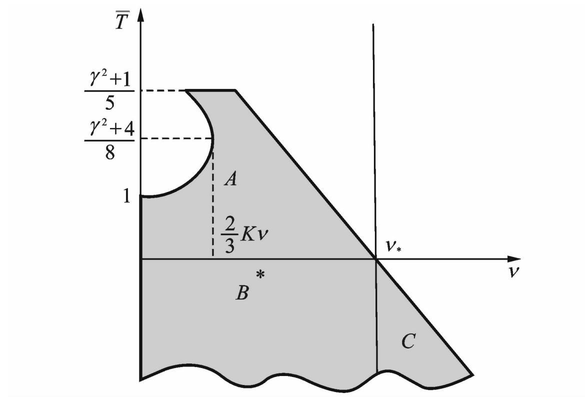

Fig. 1 Stability boundary for constant temperature field

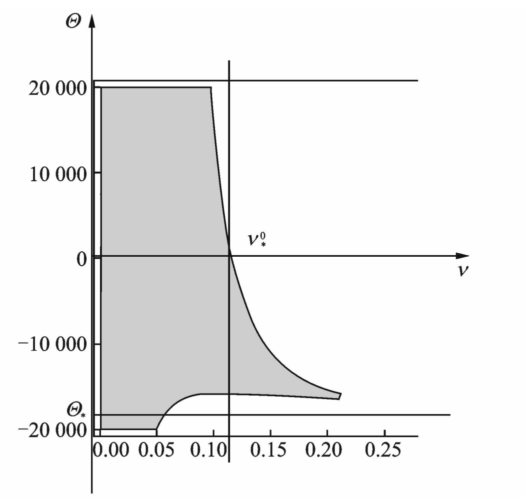

Fig. 2 Stability boundary for $T_0=0$

3 Conclusions

Conditions of stability of unperturbed state of examined termogasoelastic system are obtained and on its base the stability region is constructed in the space of variables characterizing the value of the flown speed, the temperature at the middle plane of the shell and the temperature gradient along the normal direction of this plane. It is shown that via the combined action of the temperature field and the flowing stream one can regulate the process of stability and with the help of the temperature field one can significantly change the value of the flutter critical speed. It is also shown, that:

(1) If $\Theta>0$, then the temperature field significantly reduces the stability region.

(2) If $\Theta<0$, then up to a certain negative value Θ_0 the stability region increases significantly, then, with the increasing $|\Theta|$ the stability region is decreases essentially.

(3) The effect of temperature gradient on the critical speed is significant in the case of relatively thick shells.

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