Tunable Electromagnetic Cloaking by External Field

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Abstract: Electromagnetic cloaking based on the scattering cancellation method have been reviewed. The possibility of designing the tunable electromagnetic cloaking is analytically suggested with a single cloak composed of homogeneous materials, including semiconductor, superconductor, ferrite and ferroelectrics by using Mie scattering theory. The simulated results demonstrate that the cloaks with these homogeneous materials can drastically reduce the total scattering cross sections of the cloaked system by using the finite element method. These cloaking frequencies can be controlled by external field through tuning the permittivity or permeability of these materials by the applied field, such as temperature, magnetic field and electric field. These may provide some potential ways to design tunable cloaking with considerable flexibility.

Key words: scattering cancellation; tunable electromagnetic cloaking; scattering cross section

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1 Introduction

Cloaking has been gradually realized in the laboratory by many research groups using two theoretical methods, transformation optics[1-6] and scattering cancellation[7-15]. Transformation optics based on form-invariant coordinate transformation of Maxwell′s equations offers an unconventional approach for tailoring devices with unprecedented electromagnetic behaviors[1-2]. This coordinate transformation-based cloaking technique offers the possibility to bend the incoming electromagnetic wave around a specific region, theoretically suppressing any scattering from whatever is placed inside the cloak. Although the cloaking approach is mathematically elegant and would allow perfect cloaking in principle, it requires inhomogeneous and anisotropic material parameters for curving the isotropic space, rerouting the impinging electromagnetic rays as if the system is completely invisible to any outside observer. This technique is usually designed to work at a single operating frequency, and its cloaking frequency (CF) remains unaltered once the cloak has been fabricated.

In 2005, Alù and Engheta suggested a different way to design cloaking with the technique called scattering cancellation, which was based on the well-known Mie scattering theory[7]. A single object will produce scattering by polarizing it in the electromagnetic field, and it is covered by another object with reversed polarization or scattering. In some special cases, the total scattering of the two objects may be minimized and the two objects can be invisible for observer. Compared with the transformation optics approach, the scattering cancellation method shows the following advantages: first, it can be achieved with homogeneous and isotropic materials and provide the convenience to fabricate the cloaking structures; second, the optical properties of the homogeneous covered layer can be tuned by the electro-optic and magnetic-optic effect, and therefore the cloaking frequency can be adjusted by external field.

Our researches on electromagnetic cloaking is reviewed based on the scattering cancellation

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method. We have developed tunable electromagnetic cloaking in two-dimensional system by external field[16-18], such as temperature, magnetic field and electric field using Mie scattering theory. The analytical results numerically demonstrate that such cloak can drastically reduce the total scattering cross section at these cloaking frequencies by using the finite element method. Further, actively changing the dielectric permittivity and permeability of the cloaking shell can lead to external control of the cloaking frequency.

2 Basic Cloaking Theory

Consider a time harmonic (the dependence $e^{-i\omega t}$ is supposed) transverse magnetic (TM) monochromatic plane wave with unit amplitude, which is normally incident from free space onto the structure along the x axis positive direction on the structure, as shown in Fig. 1. We assume that the infinite long cylindrical inner circle radius is a, with relative dielectric constant ε and permeability μ, which is covered by a single homogeneous shell with the outer radius is a_c, and the relative permittivity and permeability of the material is ε_c, μ_c, respectively. According to Mie scattering theory, various regions of the electric field: the incident field (E_z^{i} ($r \geqslant a_c$)), the scattering field ($E_z^{\mathrm{s}}(r \geqslant a_c)$), the field in the shell ($E_z^{\mathrm{c}}(a \leqslant r \leqslant a_c)$), and the core ($E_z^{\mathrm{o}}$ ($r \leqslant a$)) can be expressed as

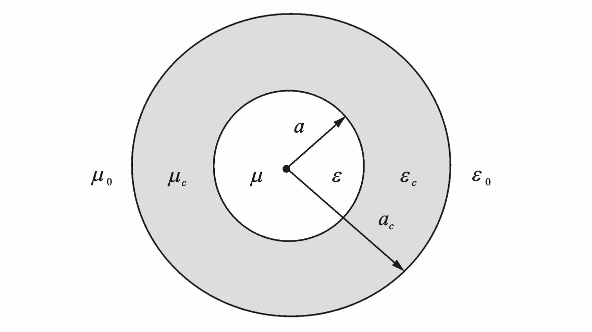


Fig. 1 Cross section of two-dimensional cylindrical invisibility cloak

$$E_z^{\mathrm{i}} = \sum_{n=0}^{\infty} J_n(k_0 r)\cos(n\theta) \tag{1}$$

$$E_z^{\mathrm{s}} = \sum_{n=0}^{\infty} C_n^{\mathrm{TM}} H_n^{(1)}(k_0 r)\cos(n\theta) \tag{2}$$

$$E_z^{\mathrm{c}} = \sum_{n=0}^{\infty} [b_n^{\mathrm{TM}} J_n(k_c r) + d_n^{\mathrm{TM}} Y_n(k_c r)]\cos(n\theta) \tag{3}$$

$$E_z^{\mathrm{o}} = \sum_{n=0}^{\infty} e_n^{\mathrm{TM}} J_n(k_r)\cos(n\theta) \tag{4}$$

where $b_n^{\mathrm{TM}}, C_n^{\mathrm{TM}}, d_n^{\mathrm{TM}}$, and e_n^{TM} are the unknown expansion coefficients, $k = k_0\sqrt{\varepsilon\mu}$, $k_c = k_0\sqrt{\varepsilon_c\mu_c}$, $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the wave number in three spaces, J_n, Y_n and $H_n^{(1)}$ are the nth order of Bessel, Neumann and Hankel cylindrical function of the first kind, respectively. Using the boundary condition of the electric field and magnetic field are continuous, the scattering coefficients C_n^{TM} can be determined

$$C_n^{\mathrm{TM}} = -\frac{U_n^{\mathrm{TM}}}{U_n^{\mathrm{TM}} + \mathrm{i}V_n^{\mathrm{TM}}} \tag{5}$$

where U_n^{TM} and V_n^{TM} are obtained in Ref. [19]. The total scattering cross section (SCS) Q_s of the structure is given by[19]

$$Q_{\mathrm{s}} = \frac{4}{k_0}\sum_{n=0}^{\infty}(2-\delta_{n,0})\left|C_n^{\mathrm{TM}}\right|^2 \tag{6}$$

where δ is the Kronecker delta.

In the case of a small cylindrical object coated by a thin shell (i. e. the outer radius is much smaller than the wavelength of the incoming electromagnetic wave), the $n=0$ order is the dominant in Ref. [19] and the scattering coefficient C_0^{TM} is minimum, which correspond to the minimum value of SCS Q_{s}. If the impinging wave is transverse electric (TE) monochromatic plane wave, the scattering coefficients C_n^{TE} can be readily obtained by electromagnetic duality.

3 Tunable Optical Properties of Shell by Applied Field

3.1 Temperature tunability of permittivity for semiconductor and superconductor

3.1.1 Semiconductor

As well known, the plasma frequency is connected with the free carrier concentration n, the relatively effective mass m and the static permittivity ε_∞ can be expressed as [20]

$$\omega_p^2 = \frac{4\pi n e^2}{m\omega_\infty} \tag{7}$$

We chose semiconductor InSb, whose free carrier density is more sensitive to temperature for its narrow band gap. When the frequency is less than the phonon resonance frequency of the semiconductor InSb, whose dielectric constant can be altered by the temperature is given by[16]

$$\varepsilon(\omega,T)=\varepsilon_{\infty}\left(1-\frac{\omega_p^2(T)}{\omega^2+\frac{\mathrm{i}\omega}{\tau}}\right) \tag{8}$$

where τ is the scattering time. Using this model for the semiconductor InSb, obviously by adjusting the temperature T, we can change its plasma frequency and then change its dielectric constant Re(ε), as shown in Fig. 2 [16].

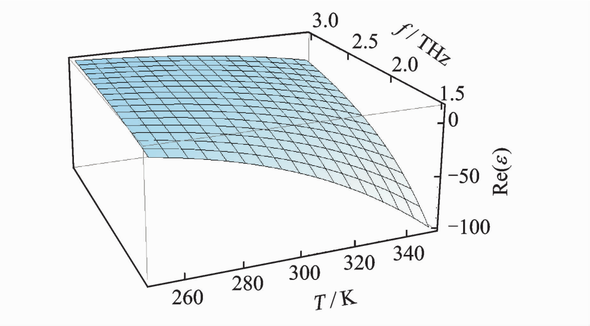


Fig. 2 Variation of Re(ε) of intrinsic InSb with T and f

3.1.2 Superconductor

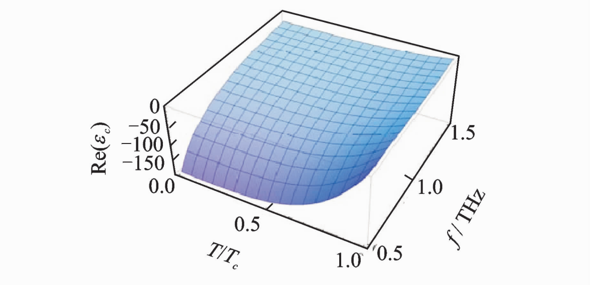
According to the two-Fluid model, the complex conductivity of a superconductor for nonzero frequency can be denoted as [21]

$$\sigma(\omega)=\left(\frac{e^2}{m}\right)\left[n_{\mathrm{n}}\tau(1-\mathrm{i}\tau\omega)/(1+\tau^2\omega^2)-\mathrm{i}n_{\mathrm{s}}/\omega\right] \tag{9}$$

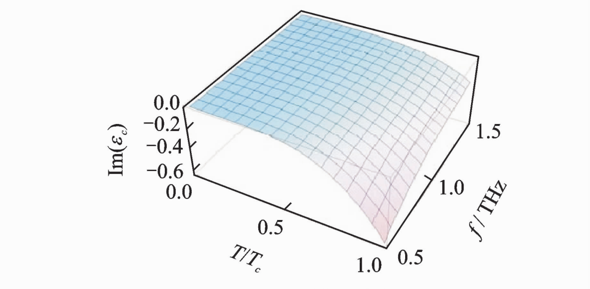
where τ is the relaxation time of the ordinary state electrons, e and m are the charge and mass of the electron, n_{n} and n_{s} are the electron number density of the unpaired normal state and the matching of the superconducting state, respectively. By using the Hagen-Rubens limit and applying the London penetration depth, the complex conductivity of a superconductor can be further modified as[17]

$$\sigma(\omega)=[\varepsilon_0 c^2/\lambda_{\mathrm{L}}^2(0)](\tau-\mathrm{i}\tau^2\omega)(T/T_c)^4-i[\varepsilon_0 c^2/\lambda_{\mathrm{L}}^2(0)][1-(T/T_c)^4]/\omega \tag{10}$$

where $\lambda_{\mathrm{L}}(0)$ is the London penetration depth at absolute zero temperature, T and T_c are the temperatures of sample and phase transformation, respectively. We select a typical high-temperture superconductor $\mathrm{Bi_{1.85}Pb_{0.35}Sr_2Ca_2Cu_{3.1}O_y}$, through Eq. (10) in this model $\varepsilon_c=\varepsilon'(1-\mathrm{i}\sigma/\omega)$, a more common dielectric response of this superconductor can be investigated[17]. Fig. 3 shows the temperature-dependent performances of the real Re(ε) and imaginary Im(ε) parts of its complex dielectric constant ε_c.



(a) Re(ε_c)



(b) Im(ε_c)

Fig. 3 Variation of Re(ε) and Im(ε) of dielectric constant of superconductor with T and f

3.2 Magnetic tunability of permeability of ferrite

If the impinging wave is TE monochromatic plane wave, the magnetic field of the TE mode is perpendicular to the external magnetic field and thus it induces a precession of magnetic dipoles around this external field at this mode frequency. Thus, for this TE mode, the permeability of the ferrite μ is a function of circular frequency ω, the saturation magnetization of ferrite M_{s}, and the applied magnetic field B_{ex} are [22]

$$\mu=[(\omega_{\mathrm{ex}}+\omega_m)^2-\omega^2]/[\omega_{\mathrm{ex}}(\omega_{\mathrm{ex}}+\omega_m)-\omega^2] \tag{11}$$

where $\omega_{\mathrm{ex}}=\gamma B_{\mathrm{ex}}$, $\omega_{\mathrm{m}}=\gamma\mu_0 M_{\mathrm{s}}$. $\mathrm{Lu_{2.1}Bi_{0.9}Fe_5O_{12}}$ (LuBiIG) ferrite film is elected in this work, whose material parameters have been suggested in Ref. [23]. As shown in the Fig. 4, we can clearly see that the magnetic field has a obvious effect on

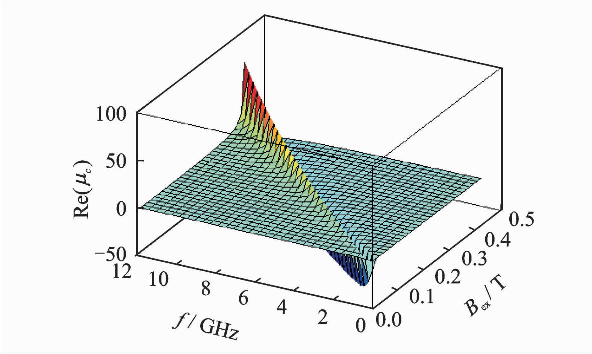


Fig. 4 Variation of ferrite magnetic permeability with changes of B_{ex} and f

permeability of this ferrite.

3.3 Electrical tunability of permittivity of ferroelectrics

The main attraction of ferroelectric materials is the intense change of their high permittivity on the applied electric field. This strong dependence can lead to tunable material application in our cloaking model by the external field. For the typical ferroelectric material $Ba_xSr_{1-x}TiO_3$ (BST), the dependence has an approximated form [24]

$$E \approx \frac{0.8 \times 10^5}{\sqrt{1-x}} \frac{\sqrt{n-1}(2+n)}{\varepsilon(0)^{3/2}} \tag{12}$$

where $n=\varepsilon(0)/\varepsilon_r$ defined as the ratio of the dielectric constant of this material at zero electric field to its permittivity at non-zero electric field. In Ref. [18], we consider that the cloaking single shell consists of ferroelectric BST with barium density $x=0.5$, and $\varepsilon(0)=200$. Fig. 5 obviously shows that the permittivity decreases from 200 to 150 when the bias electric field varies from 0 to 77 V/μm.

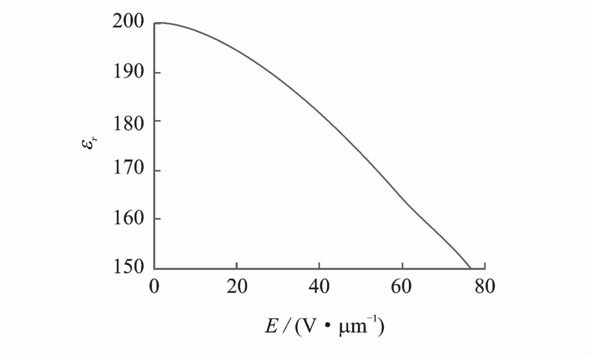


Fig. 5 Dependence of dielectric constant of BST ($x=0.5$) on applied electric field

4 Tunable Electromagnetic Cloaking by External Field

4.1 Tuning effect of temperature on CF

4.1.1 Semiconductor

As is shown in Ref. [16], we first define the scattering gain G_{sca} ($G_{\mathrm{sca}}=20\log_{10}(Q_t/Q_{\mathrm{bare}})$), which is the ratio of the SCS of the cloaked system (Q_t) to that of the bare cylinder (Q_{bare}). When the scattering gain G_{sca} is below zero, the total SCS of this cloaked system is less than than of the uncloaked object, which suggests that the cloak provides a certain degree of stealth effect. Fig. 6 reveals that this scattering gain G_{sca} of the semiconductor InSb as a function at several distinct temperatures. Clearly, CF for the minimum SCS Q_s (i. e. the minimum value corresponding to the frequency for the best CF) can be tuned by changing the temperature of the cloak.

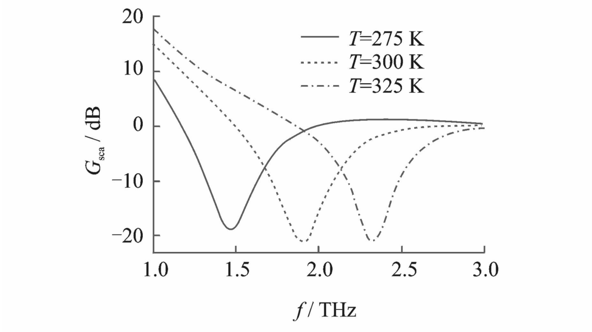


Fig. 6 Variation of G_{sca} with f at three distinct temperatures

4.1.2 Superconductor

The tuning temperature effect of the superconductor $Bi_{1.85}Pb_{0.35}Sr_2Ca_2Cu_{3.1}O_y$ on CF has been supposed in Ref. [17]. In Fig. 7, we also report scattering gain G_{sca} of this case as a function of frequency at some distinct temperatures, which obviously indicates that SCSs of theses cloaked systems are dramatically reduced at these cloaking frequencies. Compared with semiconductor invisibility cloak, this superconductor cloak has a better performance for their scattering gain G_{sca}

which is far less than zero. To further prove the performances, we simulate the propagation of the plane electromagnetic wave in the cloak systems by using the finite element method. From Fig. 8, the frequency tuning effect is very clear when the working temperature is changed in the single shell.

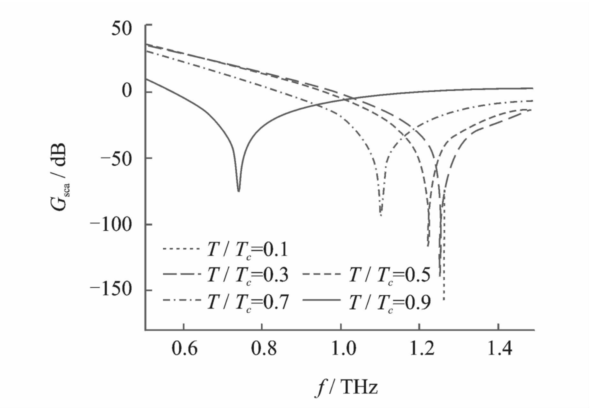
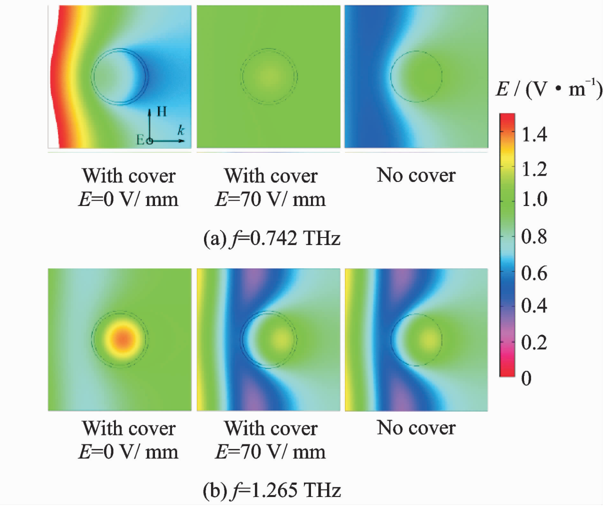


Fig. 7 Vatiation of total SCS with f at five distinct temperatures



(a) f=0.742 THz

(b) f=1.265 THz

Fig. 8 Modulus of axial electric field for three cases with two frequencies

4. 2 Tuning effect of magnetic field on CF

In order to show the tuning CF with the ferrite through the applied magnetic field, we consider a inner object ($\varepsilon=4$, $\mu=1$) with $a=5$ mm, and the external radius of the single shell consisting of LuBiIG is $a_c=8$ mm (the choice of these parameters is arbitrary in our calculation). Fig. 9 indicates that the scattering gain G_{sca} is a function of the magnetic field and frequency. Fig. 9 also displays the scattering gain G_{sca} of this case as a function of frequency at some distinct magnetic fields, which obviously reveals that SCSs of theses cloaked systems are dramatically reduced at these multi CFs. It is obvious that these multi CFs change with the external magnetic field.

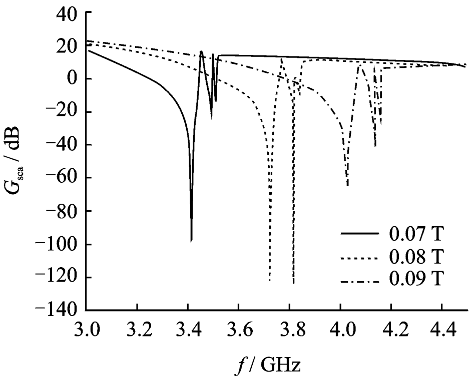


Fig. 9 Variation of G_{sca} with f for different B_{ex}

4. 3 Tuning effect of electric field on CF

In Ref. [18], we have proposed the results for tuning cloaking frequencies of the cloak with ferroelectric $Ba_{0.5}Sr_{0.5}TiO_3$ shell through the biased electric field. Fig. 10 clealy shows that there are two CFs for the minimum SCS Q_s at the two cases: (a) $E=0$ V/μm, $a_c/\lambda=0.07$ ($f=2.1$ THz) and $a_c/\lambda=0.14$ ($f=4.2$ THz), (b) $E=70$ V/μm, $a_c/\lambda=0.08$ ($f=2.4$ THz) and $a_c/$

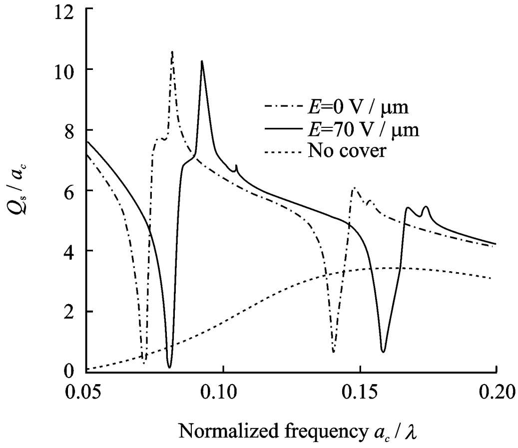
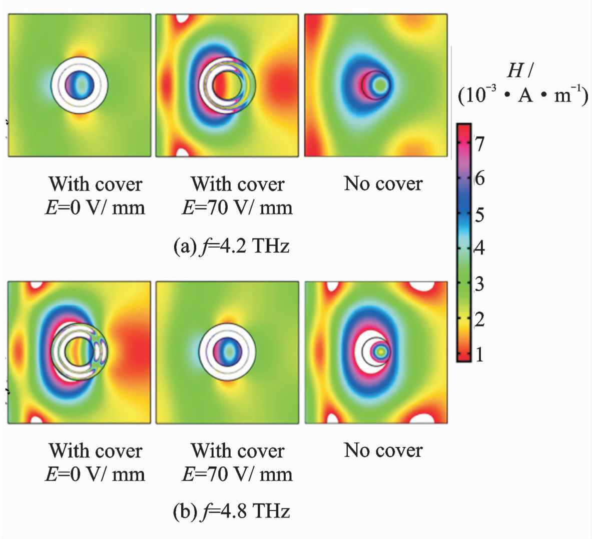


Fig. 10 Total SCS Q_s as a function of frequency for three cases

$\lambda=0.16$ ($f=4.8$ THz), which are corresponding to the conventional planar Fabry-Perot resonators $m\lambda=2(a_c-a)\sqrt{\varepsilon_c}$ at $m=1$ and 2, respectively. Fig. 11 further demonstrates simulated modulus of the total magnetic field in the orthogonal plane electromagnetic wave by the finite element method. It is obvious that these multi CFs change with the variations of the external electric field.



(a) f=4.2 THz

(b) f=4.8 THz

Fig. 11 Modulus of total magnetic field in orthogonal plane of polarization for three cases at two frequencies

5 Conclusions

The possibility of designing tunable electromagnetic cloaking in two-dimensional system is theoretically proposed with a single shell of the uniform materials, including semiconductor, superconductor, ferrite and ferroelectrics. The proposed schemes depend on the tuning of the permittivity or permeability of these materials by the applied field, such as temperature, magnetic field and electric field. The calculated and simulated results demonstrate that such cloak can drastically reduce the total SCS and these CFs can be controlled by the external field.

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