Conformal Multi-resolution Time-Domain Method for Scattering Curved Dielectric Objects

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(Received 15 November 2013; revised 17 December 2013; accepted 20 January 2014)

Abstract: A conformal multi-resolution time-domain (CMRTD) method is presented for modeling curved objects. The effective dielectric constant and area weighting are used to derive the update equations of CMRTD. The backward scattering bistatic radar cross sections (RCS) of the dielectric cylinder and ellipsoid are used to validate the proposed method. The results show that the proposed conformal method is more accurate to deal with the complex curved objects in electromagnetic simulations.

Key words: conformal multi-resolution time-domain (CMRTD); curved objects; radar cross sections (RCS)

CLC number: TM15 **Document code**: A **Article ID**: 1005-1120(2014)03-0269-05

1 Introduction

The multi-resolution time-domain (MRTD) method for solving electromagnetic field problems, introduced by Krumpholz and Katehi[1] and Robertson, et al[2], is based on the expansion of unknown fields in terms of scaling functions. Tentzeris, et al investigated the stability and dispersion of Battle-Lemarie MRTD method for different stencil size and for zero-resolution wavelets and concluded that MRTD had better dispersion than traditional finite difference time-domain (FDTD) method[3]. Many works on MRTD have been exerted in the past two decades. Cheong, et al firstly proposed the MRTD method based on the Duabechies′ wavelet with two vanishing wavelet moment in spatial domain and the numerical results showed the good agreement with FDTD correspondents[4]. A MRTD scheme introduced by Dogaru and Carin is based on a field expansion in terms of Cohen-Daubechies-Feauveau biorthogonal scaling and wavelet functions[5]. Wei, et al[6] described a new MRTD scheme which was developed based on Coifman compactly supporting scaling functions with a number of vanishing moments. Multiple image technique and anistropic perfectly matched layer were presented by Cao, et al[7] for boundary truncations of microwave structures. Cao, et al[8] proposed Runge-Kutta multi-resolution time-domain (RK-MRTD) with higher order both in space and time domain. The Coifman scaling function based MRTD technique was discussed in terms of applicability to model problems in microwave and wireless communication engineering[9]. The conformal scaling MRTD technique was applied for electromagnetic scattering problems containing curved perfectly conducting objects[10]. Jiang and Zhou, et al [11] constructed the MRTD cylindrical grids with perfectly matched layer and applied MRTD to calculate electromagnetic fields of lightning return stroke. Yun, et al[12] presented a robust conformal FDTD method for the accurate modeling perfectly conducting objects with curved surfaces and edges. A modified local conformal finite difference time-domain method (MLC-FDTD) which was used to analyze broad wall ra-

Foundation items: Supported by the National Natural Science Foundation of China (61172024); the Funding of Jiangsu Innovation Program for Graduate Education and the Fundamental Research Funds for the Central Universities (CXZZ12-0156).

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diating slots in a finite wall thickness waveguide was derived by Zhang, et al[13]. A modified conformal technique implemented in the high-order FDTD (2,4) was proposed by Wang, et al[14] to investigate the interaction of electromagnetic waves with three-dimensional electrically large curved dielectric objects. Gao, et al[15] discussed the conformal MRTD method on scattering perfect conducting object, but failed to discuss how to solve dielectric objects.

However, these existing methods usually lead to large staircase errors. Conformal technique is an advantage to deal with the dielectric interface. We propose a CMRTD method to deal with the curved objects.

2 Conformal MRTD Method

For simplicity, without the loss ($\sigma=0$), and the spatial step size $\Delta x=\Delta y=\Delta z$, the equation of electric field for MRTD method is updated as

$$\varepsilon_x(E^{x,n+1}_{i+0.5,j,k}-E^{x,n}_{i+0.5,j,k})=\sum_{v=0}^{L_s-1}a(v)\frac{\Delta t}{\Delta x}(H^{z,n+0.5}_{i+0.5,j+v+0.5,k}-H^{z,n+0.5}_{i+0.5,j-v-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+v+0.5}+H^{y,n+0.5}_{i+0.5,j,k-v-0.5}) \quad (1)$$

where E, H are the electric field and the magnetic field, respectively; x, y and z the x axial direction, y axial direction and z axial direction, respectively, i, j and k the indices of the computational cells, n is the index of time step, v the phase velocity, Δt the temporal step size, ε_x the dielectric constant, L_s the spatial support interval, and $a(v)$ the coefficients of Daubechies scaling function[8].

To derive the CMRTD method, Eq. (1) can be rewritten as

$$E^{x,n+1}_{i+0.5,j,k}=E^{x,n}_{i+0.5,j,k}+a(0)\frac{\Delta t}{\Delta x\varepsilon_x}(H^{z,n+0.5}_{i+0.5,j+0.5,k}-H^{z,n+0.5}_{i+0.5,j-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+0.5}+H^{y,n+0.5}_{i+0.5,j,k-0.5})+3a(1)\frac{\Delta t}{3\Delta x\varepsilon_x}(H^{z,n+0.5}_{i+0.5,j+1.5,k}-H^{z,n+0.5}_{i+0.5,j-1.5,k}-H^{y,n+0.5}_{i+0.5,j,k+1.5}+H^{y,n+0.5}_{i+0.5,j,k-1.5})+\cdots+(2L_s-1)a(L_s-1)\frac{\Delta t}{(2L_s-1)\Delta x\varepsilon_x}(H^{z,n+0.5}_{i+0.5,j+L_s-0.5,k}-H^{z,n+0.5}_{i+0.5,j-L_s+0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+L_s-0.5}+H^{y,n+0.5}_{i+0.5,j,k-L_s+0.5}) \quad (2)$$

From Ref. [16], we know that $\sum_{v=0}^{L_s-1}a(v)(2v+1)=1$, then Eq. (2) can be discomposed to L_s sub-equations as

$$a(0)\varepsilon_x(0)(E^{x,n+1}_{i+0.5,j,k}-E^{x,n}_{i+0.5,j,k})=a(0)\frac{\Delta t}{\Delta x}(H^{z,n+0.5}_{i+0.5,j+0.5,k}-H^{z,n+0.5}_{i+0.5,j-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+0.5}+H^{y,n+0.5}_{i+0.5,j,k-0.5}) \quad (3)$$

$$3a(1)\varepsilon_x(1)(E^{x,n+1}_{i+0.5,j,k}-E^{x,n}_{i+0.5,j,k})=3a(1)\frac{\Delta t}{3\Delta x}(H^{z,n+0.5}_{i+0.5,j+1.5,k}-H^{z,n+0.5}_{i+0.5,j-1.5,k}-H^{y,n+0.5}_{i+0.5,j,k+1.5}+H^{y,n+0.5}_{i+0.5,j,k-1.5}) \quad (4)$$

$$\vdots$$

$$(2L_s-1)a(L_s-1)\varepsilon_x(L_s-1)(E^{x,n+1}_{i+0.5,j,k}-E^{x,n}_{i+0.5,j,k})=(2L_s-1)a(L_s-1)\frac{\Delta t}{(2L_s-1)\Delta x}(H^{z,n+0.5}_{i+0.5,j+L_s-0.5,k}-H^{z,n+0.5}_{i+0.5,j-L_s+0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+L_s-0.5}+H^{y,n+0.5}_{i+0.5,j,k-L_s+0.5}) \quad (5)$$

where $\varepsilon_x(v)(v=0,1,2,\cdots L_s-1)$ is the dielectric constant of cell size Δx, $3\Delta x$,$\cdots$ and $(2L_s-1)\cdot\Delta x$. Summing Eqs. (3—5) can obtain E_x updating equation of CMRTD as

$$\sum_{v=0}^{L_s-1}(2v+1)a(v)\varepsilon_x(v)(E^{x,n+1}_{i+0.5,j,k}-E^{x,n}_{i+0.5,j,k})=\sum_{v=1}^{L_s-1}a(v)\frac{\Delta t}{\Delta x}(H^{z,n+0.5}_{i+0.5,j+v+0.5,k}-H^{z,n+0.5}_{i+0.5,j-v-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+v+0.5}+H^{y,n+0.5}_{i+0.5,j,k-v-0.5}) \quad (6)$$

Comparing Eq. (6) with Eq. (1), we can get the effective dielectric constant $\varepsilon_x^{\text{eff}}$ as

$$\varepsilon_x^{\text{eff}}=\sum_{v=0}^{L_s-1}(2v+1)a(v)\varepsilon_x(v) \quad (7)$$

The area weighting technique is used to deal with $\varepsilon_x(v)(v=0,1,2,\cdots L_s-1)$[15] shown in Fig. 1, Eq. (7) can be modified as

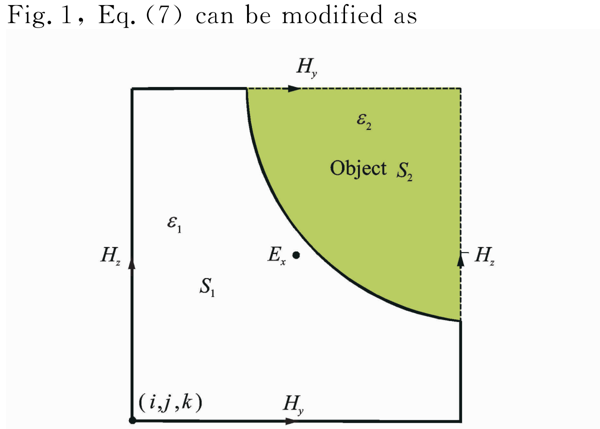

Fig. 1 Distribution of objects

$$\varepsilon_x^{\text{eff}} = \sum_{v=0}^{L_s-1} \frac{a(v)}{(2v+1)\cdot S} \cdot [\varepsilon_1 \cdot S_1 + ((2v+1)^2 \cdot S - S_1)\cdot \varepsilon_2] \quad (8)$$

where S is the unit cell area, S_1 the area out outside the object. In the same way, we can obtain $\varepsilon_y^{\text{eff}}$, $\varepsilon_z^{\text{eff}}$ and the updating equations of E_y and E_z for the CMRTD method.

3 Numerical Examples

Numerical examples of scattering cylinder and ellipsoid are used to validate the CMRTD method. The scaling function refers to Daubechies 2 scaling function. All computational simulations are conducted on a PC with Pentium dual-core 2.8 GHz CPU and 1.87 GB as memory. θ is the incident angle.

3.1 Dielectric Cylinder

The radius of the dielectric cylinder is 0.015 m and the height 0.06 m. Relative permittivity ε_r is 4, and relative permeability μ_r is 1. An incident sinusoidal wave with a wavelength 0.03 m propagates along the z-direction, and its polarization is along the x-direction. The comparisons of different methods are shown in Fig. 2. Fig. 3 shows the errors between CMRTD (MRTD) and method of moment (MoM) methods. The results indicate that the CMRTD method is more consistant with the MoM method. Table 1 shows the magnitudes of the spatial discretization, temporal discretization, total computational domain, total time steps and CPU time. From Table 1, it is found that the CPU times of these methods are similar, the accurate of CMRTD is the closest to the time of the method of moment.

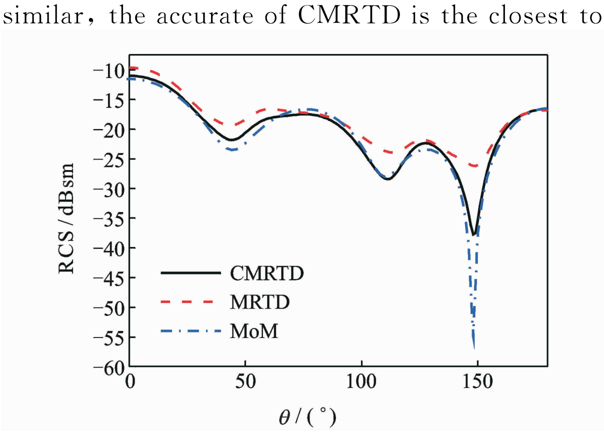

Fig. 2 RCS of E-plane different methods

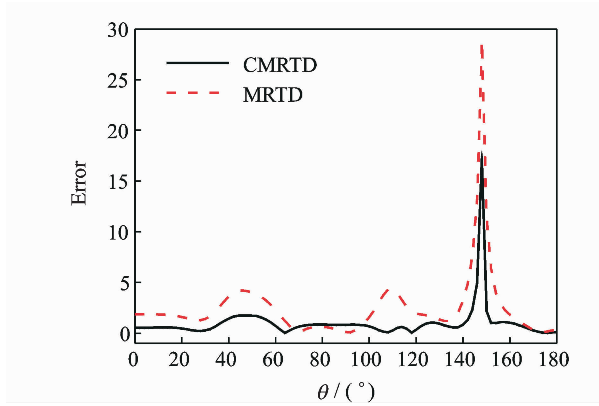

Fig. 3 Errors of different methods

Table 1 Comparison of MRTD and CMRTD

Method	Δx/m	Δt/s	Cell	Total time/s	CPU time/s
MRTD	0.003	3.2	82×82×82	2 000	1 645.43
CMRTD	0.003	3.2	82×82×82	2 000	1 756.34

3.2 Dielectric Ellipsoid

The radius of dielectric ellipsoid are 0.6, 0.6 and 0.3 m, along x-, y-, z-direction, respectively. The relative permittivity ε_r is 4, relative permeability μ_r 1, the polarization of the electric field along x-direction, and the wavelength of the incident wave is set as 0.3 m. The CFL number is chosen as 0.3.

Backward scattering bistatic RCS in different schemes are drawn in Fig. 4 and Fig. 6. Fig. 5 shows the errors between CMRTD (CFDTD) and MoM method. It is found that the CMRTD method is more accurate than other methods. The comparisons of CPU time in different methods are listed in Table 2. Fig. 7 shows the errors between CMRTD (CFDTD) and MoM methods.

Table 2 Comparison of CFDTD and CMRTD

Method	Δx/m	Δt/s	Cell	Total time/s	CPU time/s
CFDTD	0.03	30	118×118×92	2 000	4 125.37
CMRTD	0.03	30	118×118×92	2 000	4 656.38

4 Conclusions

The CMRTD method is presented for computational electromagnetic computations of some

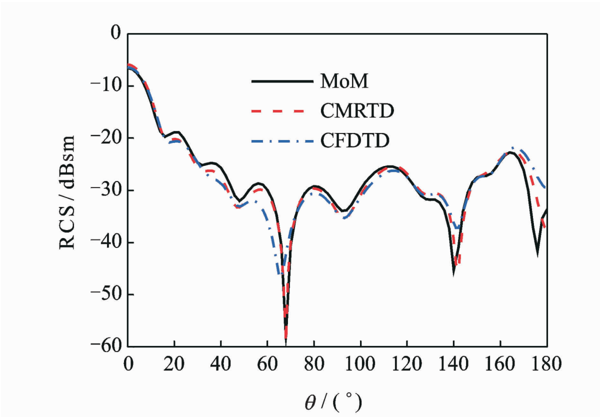

Fig. 4 RCS of *E*-plane different methods

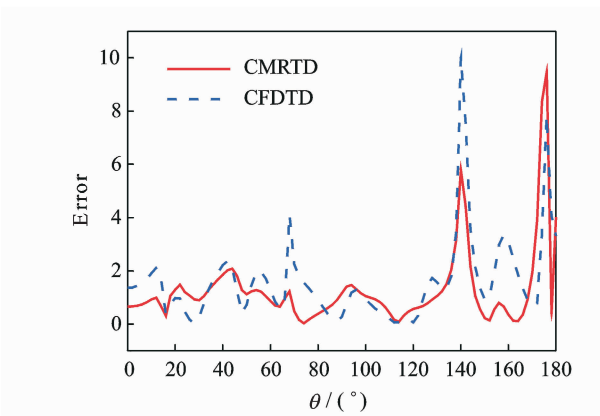

Fig. 5 Errors of different methods

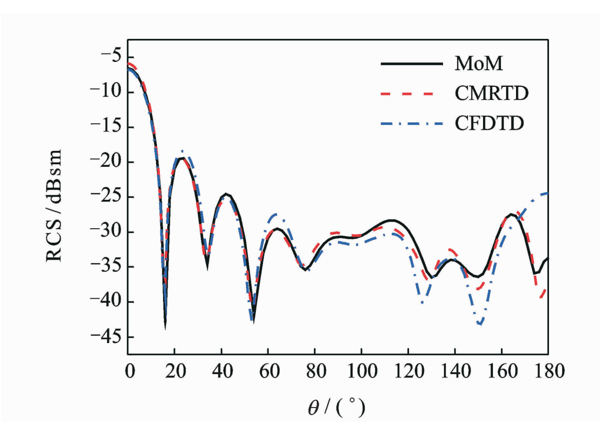

Fig. 6 RCS of *H*-plane different methods

dielectric objects. The effective dielectric constant is used to derive the updating equation of the CMRTD method. The area weighting are used to deal with the object interface. The backward scattering bistatic RCS of the dielectric cylinder and ellipsoid are given to validate the CMRTD method. And the results show that the proposed method is more close to the MoM method and are

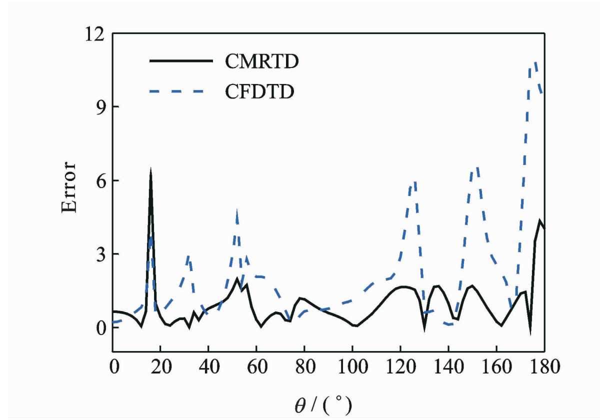

Fig. 7 Errors of different methods

more accurate when treating the curved objects.

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(Executive editor: Zhang Bei)