Improved Algorithm of Variable Bandwidth Kernel Particle Filter

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Abstract: Aiming at the large cost of calculating variable bandwidth kernel particle filter and the high complexity of its algorithm, a self-adjusting kernel function particle filter is presented. Kernel density estimation is facilitated to iterate and obtain new particle set. And the standard deviation of particle is introduced in the kernel bandwidth. According to the characteristics of particle distribution, the bandwidth is dynamically adjusted, and the particle distribution can thus be more close to the posterior probability density model of the system. Meanwhile, the kernel density is used to estimate the weight of updating particle and the system state. The simulation results show the feasibility and effectiveness of the proposed algorithm.

Key words: particle filter; kernel density estimation; kernel bandwidth; self-adjusting

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1 Introduction

Particle filter has been widely applied in nonlinear, non-Gaussian dynamic system, including target tracking, navigation, fault detection and computer vision[1-4], since it was put forward in 1999. It is necessary to further research on critical issues of particle filter, especially how to sample high quality particles, which can help to improve the performance and efficiency of particle filter.

Kernel particle filter (KPF) was proposed by Chang, et al[5]. KPF uses kernel density estimation (KDE) method[6-7] to iterate and acquire the samples of new particles, so that the particles can be distributed in the place where the weight is larger, which can improve the accuracy of posteriori probability distribution estimation. Different from the traditional practice which improves the excellent characteristic of important density function[8] to obtain high-quality particles, this method utilizes the non-parametric estimation to redistribute the particle set, and the iteration makes the particles move to the high likelihood region of the maximum posteriori probability density model. In this way, the produced particle set can reflect the density distribution of status posteriori probability.

In the kernel density estimation, the selection of kernel bandwidth is critical, and its importance is more than that of kernel function[9]. Joachim[10], et al. used the empirical value to formulate the rules for band width change. This approach relies on human experience and the change characteristics of the entire particle set cannot be correctly reflected, which seriously affects the accuracy of kernel density estimation and reduces the performance of particle filter. Avramidis[11] proposed a method of bandwidth selection based on interpolation and data, and acquired the global optimal bandwidth by solution process, which was taken as the mean bandwidth in the kernel density estimation. The performance of this kernel bandwidth is much better than that of the ker-

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nel bandwidth proposed by Chang, et al[5] and Joachim, et al[10], but the mean kernel bandwidth is not applicable to the movement of all the particles. Li, et al[12] presented the method of covariance-based variable bandwidth kernel particle filter (CVBKPF). Each particle acquires the bandwidth related to itself in the kernel density estimation to improve the accuracy of kernel density estimation. Although CVBKPF has a higher estimated accuracy, there is still a process in which the global bandwidth is solved by optimizing; meanwhile, the particle′s covariance matrix and correlated processing increase the calculating cost of the algorithm, so that the complexity of calculation is higher than that of KPF.

Based on Refs. [5,12], an algorithm of self-adjusting bandwidth kernel particle filter (SABKPF) is proposed by combining the advantages of KPF and variable bandwidth KPF. By means of the particles′ own distribution characteristics, the kernel bandwidth is regulated; meanwhile, the kernel density estimation is used to update the particle weight in the estimation so as to improve the efficiency of system estimation.

2 Particle Filter

Assume that the state transition equation of nonlinear dynamic system is as follows: $x_k = f_k(x_{k-1}, v_{k-1})$, where x_k represents the status value of the target, function $f_k(\cdot)$ the target system of state transition function, and v_k the state transition noise.

The observation equation of nonlinear dynamic system is as follows: $\boldsymbol{z}_k = h_k(x_k, u_k)$, where $\boldsymbol{z}_k$ represents the observation vector, $h_k(\cdot)$ the observation function of target system, and u_k the state observation noise.

If $\{x_{0:k}^i, w_k^i\}_{i=1}^N$ represents a series of random samples, where $\{x_{0:k}^i,\ i=1,\cdots,N\}$ is a particle collection, and $\{w_k^i, i=1,\cdots,N\}$ the corresponding weight, satisfying $\sum_{i=1}^N w_k^i = 1$; $x_{0:k} = \{x_j, j=0,\cdots,k\}$ a set of all system states at the moment k, and the system posteriori probability distribution at the moment k is

$$p(x_{0:k} \mid \boldsymbol{z}_{1:k}) \approx \sum_{i=1}^N w_k^i \delta(x_{0:k} - x_{0:k}^i) \tag{1}$$

where $\delta(\cdot)$ is a Dirac-delta function. When the number of particles $N \to \infty$ increases, the theorem of large numbers can ensure Eq. (1) to approximate the actual posterior probability distribution[13].

It is difficult to directly obtain samples from the posterior probability distribution, so the Bayesian importance sampling theorem suggests that the important density function of $q(x_{0:k} \mid \boldsymbol{z}_{1:k})$ close to the posterior probability distribution is used for sampling, the weight of the particle can be expressed as

$$w_k^i \propto \frac{p(x_{0:k}^i \mid \boldsymbol{z}_{1:k})}{q(x_{0:k}^i \mid \boldsymbol{z}_{1:k})} \tag{2}$$

In order to obtain a recursive method of posteriori probability distribution, the important density function can be decomposed into

$$q(x_{0:k} \mid \boldsymbol{z}_{1:k}) = q(x_k \mid x_{0:k-1}, \boldsymbol{z}_{1:k}) q(x_{0:k-1} \mid \boldsymbol{z}_{1:k-1}) \tag{3}$$

Then, by adding the new state of $x_k^i \sim q(x_{0:k} \mid x_{0:k-1}, \boldsymbol{z}_{1:k})$ to a known particle set of $x_{0:k-1}^i \sim q(x_{0:k-1} \mid \boldsymbol{z}_{1:k-1})$, the new particle collection of $x_{0:k}^i \sim q(x_{0:k} \mid \boldsymbol{z}_{1:k})$ is obtained.

According to Bayesian Rule and the properties of conditional probability, it is known

$$p(x_{0:k} \mid \boldsymbol{z}_{1:k}) \propto p(\boldsymbol{z}_k \mid x_k) p(x_k \mid x_{k-1}) p(x_{0:k-1} \mid \boldsymbol{z}_{1:k-1}) \tag{4}$$

Eqs. (3—4) are substituted into Eq. (2), and the updating equation of weight can be obtained

$$w_k^i \propto w_{k-1}^i \frac{p(\boldsymbol{z}_k \mid x_k^i) p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{0:k-1}^i, \boldsymbol{z}_{1:k})} \tag{5}$$

In practical engineering application, the prior transition distribution is generally used as an important density function, i.e. $q(x_k \mid x_{0:k-1}, \boldsymbol{z}_{1:k}) = p(x_k \mid x_{k-1})$, and then Eq. (5) is further simplified as

$$w_k^i \propto p(\boldsymbol{z}_k \mid x_k^i) \tag{6}$$

3 Kernel Density Estimation

Kernel density estimation is a smooth nonparametric density estimation method[6-7]. Inde-

pendent from prior knowledge of data distribution, it is a kind of method starting from sample data themselves to study the characteristics of data distribution; therefore, in the statistical theory and other related applications, it has received high attention[9].

Under one-dimensional circumstance, for a group of sample sets of independent identical distribution consisting of N data $\{x_1, \cdots, x_N\}$, at any point x, the kernel density estimation is as follows

$$\hat{f}(x) = \frac{1}{Nh}\sum_{i=1}^{N} K_h(x - x_i)$$

where $K_h(\cdot)$ is a bounded symmetric function, called the kernel function, and h the kernel bandwidth.

The accuracy of kernel density estimation is mainly relevant to Parameter h, and has little to do with the kernel function. The kernel bandwidth h is a smooth parameter. To a certain extent, it reflects the balance relation of fitting degree and smooth degree. If h is larger, the probability density function will have excessively smooth peak; if h is smaller, the probability density function will lack smoothness in its tail, and even cannot truthfully reflect the structural characteristics of sample set[9]. Taking Gaussian kernel as an example, Fig. 1 shows the influence of h on the kernel function.

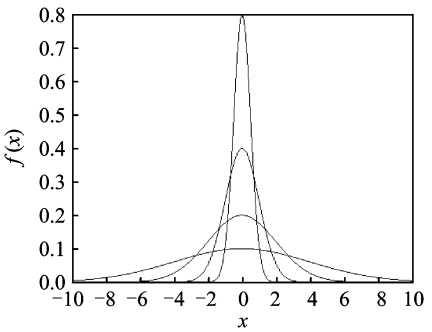

Fig. 1 Bandwidth h change in Gauss kernel function

In KPF, if the particles are relatively dense in a certain section of interval, it is expected that the small kernel bandwidth is used as far as possible to gather these particles, preventing the introduced remote particles from raising the larger smoothness error. When the particles are distributed sparsely, the larger kernel bandwidth can be used to ensure the number of particles to fall into the smooth bandwidth. Therefore, if a local kernel bandwidth is designed for each particle instead of directly using the global kernel bandwidth, the strategy of a variable bandwidth can be used to estimate the posteriori probability density.

According to the distribution characteristics of samples, it can be known that when data values are concentrated, the standard variance has smaller value; conversely, when data values are dispersed, the standard variance value is larger. It can be seen that the change of standard variance meets the demand for kernel bandwidth in the KPF; to a certain degree, it can be used as an indicator of kernel bandwidth change. Therefore, we use the standard variance of the system state to adjust the kernel bandwidth, realizing the variable bandwidth strategy that h is dynamically changed according to the characteristics of particle distribution. The corresponding density estimation function is as follows

$$\hat{f}(x, \psi(x_{0:k})) = \frac{1}{N(h + \psi(x_{0:k}))} \cdot \sum_{i=1}^{N} K_{h+\psi_{0:k}}(x - x_{0:k}^{i}) \quad (7)$$

where $\psi(x_{0:k})$ represents the standard variance of all the system states at the moment of k.

4 Self-Adjusting Bandwidth KPF

Based on the advantages of KPF and variable bandwidth KPF, an improved algorithm of variable bandwidth KPF-SABKPF is proposed. Its main idea is to substitute the kernel function for the Dirac-delta function in the posteriori probability distribution, to introduce the standard variance of particles in the bandwidth, and to dynamically adjust the bandwidth according to the particle distribution characteristics, therefore, the particle set can better reflect the posteriori probability distribution of the system model. Meanwhile, the kernel density estimation is used to update the weight of the particle and to estimate the system state.

Through the kernel density estimation, KPF seeks the method of posteriori probability distribution. Eq. (7) is substituted into Eq. (1), and

then the kernel density estimation method of posterior probability distribution is as follows

$$\hat{p}(x_{0:k} \mid \boldsymbol{z}_{1:k}) \approx \frac{1}{N(h+\psi(x_{0:k}))}\sum_{i=1}^{N} K_{h+\psi(x_{0:k})}(x_{0:k}-x_{0:k}^{i})w_k^i \quad (8)$$

where $K_{h+\psi(x_{0:k})}(\cdot)$ represents the kernel function, $h+\psi(x_{0:k})$ the bandwidth based on the particle standard variance. Considering the posteriori probability density distribution has multimodality, the kernel function is taken as Gaussian kernel in the algorithm.

Since the particle is different from the initial position after the kernel density estimation, the weight needs to be updated. The kernel density estimation is adopted to update the weight value, and Eq. (6) is changed as

$$\hat{w}_k^i \propto \frac{1}{N(h+\psi(x_{0:k}))}\sum_{l=1}^{N} K_{h+\psi(x_{0:k})}(x_{0:k}^{i}-x_{0:k}^{l}) \quad (9)$$

To increase the diversity of particles, the algorithm whitens the new particle set after re-sampling, which is described as follows:

Step 1 Initialize $k=0$

(1) For $i=1\cdots N$, extract initial particles $\{x_0^i, w_0^i\}$, $w_0^i=1/N$ from the prior probability density function $p(x_0)$;

(2) Initialize parameter h: $h=(\frac{4}{(d+2)N})^{\frac{1}{d+4}}$, d is the dimension of the system;

Step 2 While $k<m$, m is the number of iteration

(1) From initial status to the moment of k, count standard variance of the system state $\psi(x_{0:k})$;

(2) For $i=1,\cdots,N$: According to Eq. (7), move particles to obtain new particle set $\{x_k^i\}$ at the moment of k;

(3) Whiten the particles:

① Count $\hat{\boldsymbol{C}}(k)$, which is the covariance matrix of $\{x_k^i\}$;

② Obtain whitening parameter $\boldsymbol{A}(k)$ from the Cholesky decomposition on $\hat{\boldsymbol{C}}(k)$, where $\boldsymbol{A}(k)\boldsymbol{A}^{\mathrm{T}}(k)=\hat{\boldsymbol{C}}(k)$;

③ Sampl $e(\cdot)\sim N(0,I_d)$, I_d is a d-dimensional identity matrix;

④ $\hat{x}_k^i=x_k^i+(h+\psi(x_{0:k}))\cdot\boldsymbol{A}(k)\cdot e(k)$, where $h+\psi(x_{0:k})$ is the kernel bandwidth after adjusting;

(4) For $i=1,\cdots,N$: According to Eq. (9), update the weight of particles to obtain new weight value $\{\hat{w}_k^i\}$ at the moment of k;

(5) According to Eq. (8), obtain the estimation of the system state at the moment of k $\hat{x}_k\sim\hat{p}(x_{0:k}\mid\boldsymbol{z}_{1:k})=\sum_{i=1}^{N}\hat{x}_k^i\hat{w}_k^i$;

Step 3 $k=k+1$, jump to Sept 2

5 Simulation Results and Analyses

SABKPF algorithm is compared with KPF and CVBKPF filter algorithm to verify feasibility and effectiveness of the algorithm[6] by taking a typical nonlinear system model as an example. The system transition state equation and observation equation are as follows

$$x_k=1+\sin[0.04\pi(k-1)]+0.5x_{k-1}+v_{k-1}$$

$$z_k=\begin{cases}0.2x_k^2+u_k & k\leqslant 30\\ 0.5x_k-2+u_k & k>30\end{cases}$$

Among them, complying with the Gamma distribution $\zeta_a(3,2)$, v_k represents the system noise, and the measurement noise complies with Gaussian distribution $N(0,0.000\,01)$. In the experiment, the number of particles used is $N=200$, the observation time is $T=60$ s, and 100 independent experiments are conducted. The output of the algorithm is the mean particle collection, and the calculation formula is as follows

$$\hat{x}_k=\frac{1}{N}\sum_{j=1}^{N}x_k^j$$

The mean square error (MSE) of an independent experiment is as follows

$$\mathrm{MSE}=\left(\frac{1}{T}\sum_{k=1}^{T}(\hat{x}_k-x_k)^2\right)^{1/2}$$

Fig. 2 shows the state estimation results in an independent experiment through the algorithm of different-particle filter. It can be seen that at some moment, the estimation generated through KPF seriously deviates from the true data, but the states estimated by CVBKPF and SABKPF can be well in line with the true state, and the CVBKPF algorithm has the idealest effect. Table

1 shows the mean, the variance, and the running time of the MSE of different KPF algorithms after 100 independent experiments. It can be seen that the estimation accuracy of SABKPF is close to that of CVBKPF, but its running time is down more than 50% from that of CVBKPF, and the arithmetic speed has been improved effectively.

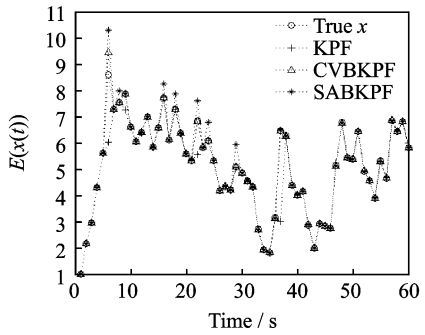

Fig. 2 State estimation of different KPF algorithms

Table 1 Comparison of estimation results of different KPF algorithms after 100 independent experiments

Algorithm	MSE		Time/s
	Mean	Var	
KPF	0.433 31	0.046 613	1.188 3
CVBKPF	0.059 557	0.004 364 3	5.157 1
SABKPF	0.059 82	0.004 503 4	2.490 8

6 Conclusions

Kernel density estimation is facilitated to seek for the posteriori probability distribution. While ensuring the algorithm accuracy, the standard variance value of particle is used to adjust the bandwidth, so that the new particle collection can be distributed in the whole space. The approach can better reflect the posteriori probability density model, therefore, effectively reduce the calculation complexity of the algorithm.

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