

# Weak GPS L1 Signal Acquisition Based on BPDC

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**Abstract:** It is difficult to achieve accurate acquisition of weak global positioning system(GPS) signals with traditional methods. A weak signal acquisition strategy based on block processing and differentially coherent (BPDC) is put forward after analyzing the advantages and disadvantages of coherent and non-coherent integration algorithms. Code phase parallel search of the pre-coherent integration is conducted by using fast Fourier transform(FFT), and the results are then differential coherent processed and block processed. BPDC method reduces computation cost compared with coherent and non-coherent(CNC) algorithm. The performance of the two algorithms is also compared based on simulated signals. The result shows that the noise suppression effect of BPDC algorithms is superior to that of traditional CNC algorithm, and the superiority of BPDC is more apparent with the reduction of carrier to noise ratio (CNR). In the case that the pre-coherent integration length is 4 ms and CNR is reduced to 28 dB-Hz, CNC algorithm cannot yet acquire signal correctly while BPDC has well acquisition performance. Therefore, for weak GPS signal acquisition, BPDC algorithm can acquire the signal with lower CNR and has better acquisition property.

**Key words:** block processing and differentially coherent (BPDC); coherent and non-coherent (CNC); weak GPS signals

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## 1 Introduction

In order to acquire weak L1 signals, a GPS receiver must increase processing gain to obtain a raised signal to noise ratio(SNR)<sup>[1]</sup>. Generally, two traditional methods are used to raise SNR. One is increasing the length of coherent integration, for example, the half-bit acquisition algorithm proposed in Ref. [2] which prolonged the length of coherent integration to 10 ms; and the whole bit algorithm suggested in Ref. [3] which reached the coherence length to 20 ms. But these methods are still limited by navigation data<sup>[4]</sup>. The other is adopting non-coherent integration. Four kinds of non-coherent integration algorithms were put forward and compared in Ref. [5]. Non-coherence can overcome the disadvantage of coherent integration, but it induces squaring loss,

which will rise with the SNR reduction<sup>[6]</sup>.

To overcome the disadvantages of the above methods, an algorithm based on block processing was put forward in Ref. [7]. On one hand, this algorithm obtained a faster operation speed by using fast Fourier transform(FFT). On the other hand, it magnified signals without raising noise by performing periodic overlap when  $I$  and  $Q$  signals are independent from each other, which can enhance the signal strength without increasing noise energy. However, if the length of coherent integration in block processing exceeds one navigation bit(20 ms), bit-hopping may occur in the data segment for block processing. To tackle this problem, a differential coherent integration algorithm was proposed in Ref. [8], which utilized the independence of sampled noise data and the high-correlation of sampled signal data in differ-

ent fore-and-aft periods. It not only offset the shortcoming of coherent integration but restrained the squaring loss induced by traditional non-coherent integration.

To avoid the disadvantages of the traditional CNC algorithm and fully avail of the advantages of block processing and differential coherent integration method, a weak GPS L1 signal acquisition strategy based on block processing and differential coherent integration is proposed.

## 2 Weak Signal Acquisition Algorithm Based on CNC

The received GPS L1 signal is down-converted and sampled, which can be expressed as follows<sup>[9]</sup>

$$y_k = Ad(t_k)c(t_k - t_s)\cos[\omega_{IF}t_k - (\omega_D t_k + \varphi_0)] + v_k = \hat{y}_k + v_k \quad (1)$$

where  $y_k$  is the output of the RF front end at sample time  $t_k$ ,  $A$  the amplitude of signal,  $d(t)$  GPS data,  $c(t)$  coarse/acquisition (C/A) code of received signals,  $t_s$  the initial point of C/A code,  $\omega_{IF}$  the normalized intermediate frequency,  $\omega_D$  the carrier Doppler frequency shift,  $\varphi_0$  the initial carrier phase,  $v_k$  the noise.

The goal of signal acquisition is to estimate  $t_s$  and  $\omega_D$  based on the intermediate frequency signal. The standard method searching for  $t_s$  and  $\omega_D$  is locating the maximum of the correlation between the signal and a reconstruction of it.

The in-phase and quadrature-phase accumulation between incoming signal and the  $i$  th local replica signal with the length of 0.001 s are as follows

$$I_i = \sum_{k=iN}^{iN+N-1} \hat{y}_k c(t_k - \hat{t}_s) \cos[(\omega_{IF} - \hat{\omega}_D)t_k] + v_{k,I} \quad (2)$$

$$Q_i = - \sum_{k=iN}^{iN+N-1} \hat{y}_k c(t_k - \hat{t}_s) \sin[(\omega_{IF} - \hat{\omega}_D)t_k] + v_{k,Q} \quad (3)$$

where  $N$  indicates the sample number in 0.001 s. The relevant correlation output function can be expressed as follows

$$U_i(\hat{t}_s, \hat{\omega}_D) = I_i + jQ_i \quad (4)$$

### 2.1 Coherent integration algorithm

Supposing the correlation integration of

length of  $M$  ms is  $Z_M^{\text{COH}}$ , then the expression of  $Z_M^{\text{COH}}$  can be shown as Eq. (5) based on Eqs. (2-4).

$$Z_M^{\text{COH}}(\hat{t}_s, \hat{\omega}_D) = \sum_{m=0}^{M-1} U_i = \sum_{m=0}^{M-1} (I_i + jQ_i) \quad (5)$$

where  $U_i$  is the correlation integration value of the  $i$ th ms.

The acquisition determination function can be written as

$$P_M^{\text{COH}}(\hat{t}_s, \hat{\omega}_D) = |Z_M^{\text{COH}}(\hat{t}_s, \hat{\omega}_D)| \quad (6)$$

If  $P(\hat{t}_s, \hat{\omega}_D)$  exceeds threshold value, the acquisition algorithm terminates successfully and the corresponding  $\hat{t}_s$  and  $\hat{\omega}_D$  are the required code phase and Doppler shift. And the coherent integration gain  $G_{\text{CHO}}$  of  $M$  ms is<sup>[10]</sup>

$$G_{\text{CHO}} = 10\lg M \quad (7)$$

However, the length of coherent integration is limited by the navigation data, and navigation data phase transition may occur in every 1 ms data. Since the frequency of navigation data is 50 Hz, one of the two sets of input data within two separate consecutive 10 ms is with a navigation data phase transition at the most, so the longest data length which can be continuously integrated is 10 ms, namely,  $M \leq 10$ .

In conclusion, it is impossible to improve SNR by limitlessly increasing the length of coherent integration.

### 2.2 Coherent and non-coherent integration

In order to solve the problem that coherent integration length is limited by navigation data, the non-coherent algorithm can be introduced after coherent integration. Supposing the non-coherent integration for  $L$  segments data is  $Z_{L,M}^{\text{NCH}}$  and each segment has a period of  $M$  ms, then the expression of  $Z_{L,M}^{\text{NCH}}$  is<sup>[11]</sup>

$$Z_{L,M}^{\text{NCH}}(\hat{t}_s, \hat{\omega}_D) = \sum_{l=0}^{L-1} (Z_{l,M}^{\text{COH}}(\hat{t}_s, \hat{\omega}_D))^2 = \sum_{l=0}^{L-1} [(I_{l,M})^2 + (Q_{l,M})^2] \quad (8)$$

where  $Z_{l,M}^{\text{COH}}(\hat{t}_s, \hat{\omega}_D)$  indicates the coherent integration of the  $l$  th segment with the length of  $M$  ms.

The relevant acquisition determination function is expressed as

$$P_{L,M}^{\text{NCH}}(\hat{t}_s, \hat{\omega}_D) = |Z_{L,M}^{\text{NCH}}(\hat{t}_s, \hat{\omega}_D)| \quad (9)$$

As can be seen from Eq. (8), the length of coherent integration is no longer limited by navigation data because of the introduction of non-coherent integration. However, there is squaring loss in non-coherent integration, which causes the SNR gain of non-coherent integration less than that of coherent integration, and the gain difference will be larger with longer integration, which is shown in Fig. 1.

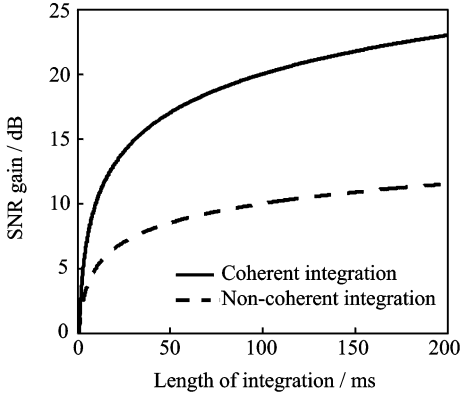


Fig. 1 Processing gain of coherent and non-coherent integrations

### 3 Weak Signal Acquisition Algorithm Based on BPDC

#### 3.1 Block processing acquisition algorithm

As can be analyzed from the above, the reason why non-coherent integration leads to SNR reduction is as follows: non-coherent integration takes the modulus of in-phase and quadrature-phase signals before accumulation for obtaining the energy strength of signal, which makes the expectation of noise mixed in the two signals no longer be zero. Therefore, the energy of noise is increased when the signal energy is improved by accumulation, resulting in the decrease of SNR. The block processing algorithm was proposed in Ref. [7] to solve the preceding problem. Block processing first takes periodic superposition for in-phase and quadrature-phase coherent result while the expectation of noise mixed in them is zero, then takes the modulus for the superposition results, in which the signal energy is enhanced without the improvement of noise energy. The block diagram of block processing acquisition al-

gorithm is shown in the Fig. 2.

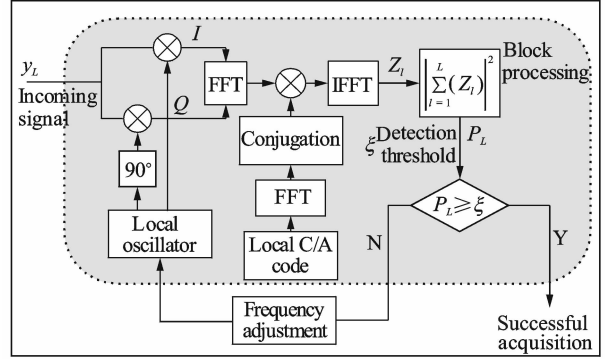


Fig. 2 Process of block processing algorithm

Block processing algorithm utilizes the following Eq. (10) to detect whether the satellite signal exists or not

$$P_L(\hat{t}_s, \hat{\omega}_D) = \left| \sum_{l=1}^N Z_l \right|^2 \quad (10)$$

where  $Z_l$  is the correlation value for the  $l$ th PRN code period.

Blocking processing algorithm can improve SNR, however, there still exists defects, namely, when the value of  $N$  exceeds a navigation data bit (20 ms), hopping may occur for the data segment of block processing.

#### 3.2 BPDC acquisition algorithm

According to the discourse in Ref. [6, 12], the differential coherent algorithm reduces the requirement for coherent integration length, therefore, differential coherence is introduced before taking modulus and square of block processing. And the ingenious differentially coherent integration algorithm proposed in Ref. [8] is adopted. Firstly, the coherent integration results of two adjacent groups of data are conjugately multiplied. Then the results of conjugate multiplication are accumulated after modulus is taken to estimate the influence of navigation data transition, which is different from the traditional way of taking modulus after accumulation. Finally the differential coherence results are taken for periodic superposition, namely, block processing, which enhances signal strength without strengthening the noise energy. In conclusion, the combination

of the two algorithms overcomes the disadvantages of traditional coherent integration algorithm and non-coherent integration algorithm and ensures higher SNR gain.

The GPS weak signal acquisition process based on BPDC is displayed in Fig. 3.

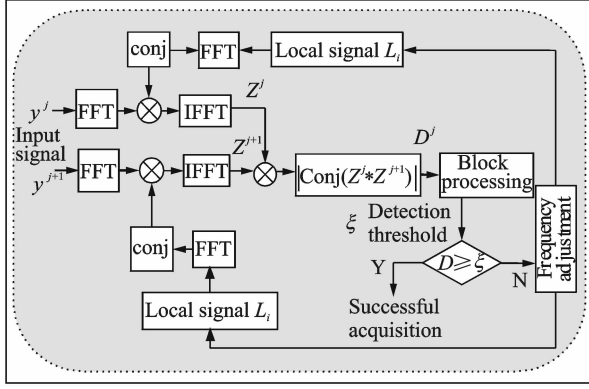


Fig. 3 Process of BPDC algorithm

The steps of weak signal acquisition algorithm based on BPDC can be detailed as follows:

The length of pre-coherent integration is taken as  $M$  ms. To make description simple, only two groups of data are performed for differential coherence integration in the following introduction. But in actual simulation, more groups of

$$L^1 = \begin{bmatrix} L^1(1) \\ L^1(2) \\ \vdots \\ L^1(M) \end{bmatrix} = \begin{bmatrix} L_0 & \cdots & L_{N-1} \\ L_N & \cdots & L_{2N-1} \\ \vdots & & \vdots \\ L_{(M-1)N} & \cdots & L_{MN-1} \end{bmatrix} =$$

$\text{conj}(\text{FFT})$

$$\begin{bmatrix} c(t_0 - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_0] + v_0 & \cdots & c(t_{N-1} - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_{N-1}] + v_{N-1} \\ c(t_N - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_N] + v_N & \cdots & c(t_{2N-1} - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_{2N-1}] + v_{2N-1} \\ \vdots & & \vdots \\ c(t_{(M-1)N} - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_{(M-1)N}] + v_{(M-1)N} & \cdots & c(t_{MN-1} - \hat{t}_s) \exp[-j(\omega_{\text{IF}} - \hat{\omega}_{\text{D}})t_{MN-1}] + v_{MN-1} \end{bmatrix} \quad (12)$$

where  $L^1$  is the locally generated signal with the length of  $M$  ms.  $L^1(i)$  the local signal of the  $i$ th ms,  $i=1,2,\dots,M$ .

(3) IFFT is exerted and the coherent integration of the first group of data is obtained

$$Z^1 = \begin{bmatrix} Z^1(1) \\ Z^1(2) \\ \vdots \\ Z^1(M) \end{bmatrix} = \begin{bmatrix} Z_0 & \cdots & Z_{N-1} \\ Z_N & \cdots & Z_{2N-1} \\ \vdots & & \vdots \\ Z_{(M-1)N} & \cdots & Z_{MN-1} \end{bmatrix} =$$

data can be selected. In addition, code-phase parallel search based on FFT is performed which can shorten acquisition time compared with serial searching algorithm.

**Step 1** Preprocess the first group of incoming signal

(1) Data of continuous  $M$  ms is input, then FFT is exerted on it, the specific process can be shown as

$$Y^1 = \begin{bmatrix} Y^1(1) \\ Y^1(2) \\ \vdots \\ Y^1(M) \end{bmatrix} = \begin{bmatrix} Y_0 & \cdots & Y_{N-1} \\ Y_N & \cdots & Y_{2N-1} \\ \vdots & & \vdots \\ Y_{(M-1)N} & \cdots & Y_{MN-1} \end{bmatrix} =$$

$$\text{FFT} \begin{bmatrix} y^1(1) \\ y^1(2) \\ \vdots \\ y^1(M) \end{bmatrix} = \text{FFT} \begin{bmatrix} y_0 & \cdots & y_{N-1} \\ y_N & \cdots & y_{2N-1} \\ \vdots & & \vdots \\ y_{(M-1)N} & \cdots & y_{MN-1} \end{bmatrix} \quad (11)$$

where  $y^1(i)$  and  $Y^1(i)$  indicate the  $i$ th ms incoming data and its FFT result in the first group,  $i=1,2,\dots,M$ ,  $M$  is the length of pre-coherent integration,  $N$  the sampling number in one C/A code period.

(2) Local replica signal of  $M$  ms is generated and FFT is exerted, then the result is complex conjugated.

$$\text{IFFT} \begin{bmatrix} L^1(1)Y^1(1) \\ L^1(2)Y^1(2) \\ \vdots \\ L^1(M)Y^1(M) \end{bmatrix} \quad (13)$$

**Step 2** Preprocess the second group of incoming signal

Data of another continuous  $M$  ms are input. Repeat (1–3) in Step 1, then obtain the coherent output of the second group of data

$$Z^2 = \begin{bmatrix} Z^2(1) \\ Z^2(2) \\ \vdots \\ Z^2(M) \end{bmatrix} = \begin{bmatrix} Z_0 & \cdots & Z_{N-1} \\ Z_N & \cdots & Z_{2N-1} \\ \vdots & & \vdots \\ Z_{(M-1)N} & \cdots & Z_{MN-1} \end{bmatrix} = \text{IFFT} \begin{bmatrix} L^2(1)Y^2(1) \\ L^2(2)Y^2(2) \\ \vdots \\ L^2(M)Y^2(M) \end{bmatrix} \quad (14)$$

**Step 3** Differential coherent process the above preprocessed result

$$D^1 = \begin{bmatrix} D^1(1) \\ D^1(2) \\ \vdots \\ D^1(M) \end{bmatrix} = [\text{conj}(Z^1) * Z^2] \quad (15)$$

**Step 4** Block-process the differential coherent processing result

$$D_1 = \begin{bmatrix} D_1(t_0, \hat{\omega}_D) \\ D_1(t_1, \hat{\omega}_D) \\ \vdots \\ D_1(t_{N-1}, \hat{\omega}_D) \end{bmatrix} = \sum_{i=1}^M |D^1(i)| \quad (16)$$

where  $D^1(i)$  is the  $i$ th differential coherence and the length of each is 1 ms,  $i=1, 2, \dots, M$ ,  $D_1$  the block processing result finally with the length of 1 ms.

**Step 5** Search in the scope of Doppler shift

$$\hat{\omega}_D = \omega_{D\min}, \omega_{D\min} + \Delta\omega_D, \omega_{D\min} + 2\Delta\omega_D \cdots \omega_{D\max},$$

and repeat = Step 1–4.

**Step 6** Acquisition determination

Compare  $D_j(\hat{t}_s, \hat{\omega}_D)$  with the acquisition threshold, if the value of a factor is above the threshold, the corresponding  $\hat{t}_s$  and  $\hat{\omega}_D$  of this factor is the acquired rough code phase and carrier Doppler shift.

## 4 Simulation Results and Analysis

### 4.1 Comparison of calculation complexity

The number of multiplication and addition operation is used to assess the calculation complexity of the two algorithms. Suppose that the length of pre-coherent integration is  $M$  ms, the integration time is  $L$  and the sampling number in 1 ms is  $N$ , then the calculation complexity of the two algorithms on a certain Doppler shift is shown in Table 1.

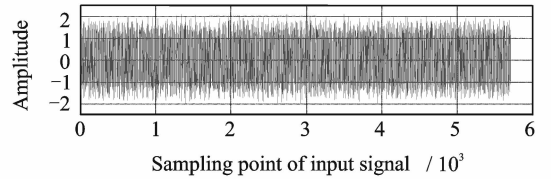
**Table 1 Comparison of calculation complexity**

Operation Method	Calculation Complexity
×	BPDC $\frac{3MNL}{2} \log_2(MN) + 2MN(L-1)$
	CNC $\frac{3MNL}{2} \log_2(MN) + 2MNL$
+	BPDC $3(MNL) \log_2(MN) + 2MN(L-1) - N$
	CNC $3(MNL) \log_2(MN) + 2MN(L-1) + MN$

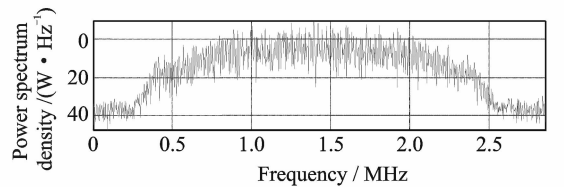
As can be seen from Table 1, the number of multiplication and addition calculation for BPDC algorithm is less than that of CNC algorithm, which indicates that BPDC is more effective for improving the acquisition speed in the same calculation environment.

### 4.2 Simulation result

BPDC and CNC are compared by using simulated GPS L1 signals to analyze the acquisition performance of BPDC algorithm. Fig. 4 shows that the band width of signal is about 2.5 MHz, and intermediate frequency is 1.405 MHz.



(a) Time domain expression of input data



(b) Power spectrum density estimation of input signal

Fig. 4 Input sampling signal and power spectrum density

Carrier to noise ratio (CNR) of signals is set as 35 dB-Hz, and code phase and Doppler shift are, set at 4 041 sampling point and 1 220 Hz, respectively. The total length of the processed data is set to 40 ms and the length of pre-coherent integration is 4 ms, the searching step of Doppler shift adopts 250 Hz, then the acquisition result after 10 times CNC integration and 10 times BPDC

integration is shown in Fig. 5, where Figs. 5(a–b) are two-dimension acquisition results while Figs. 5(c–d) three-dimension acquisition results. For convenient comparison, the coherence results are normalized.

Fig. 5 shows that both BPDC algorithm and CNC algorithm can acquire the satellite at correct code phase and carrier frequency, however, the noise suppression effect based on BPDC algorithm is better than that of CNC algorithm. Then if CNR of signals is set as 30 dB-Hz, the acquisition result is shown in

Fig. 6.

Comparing Fig. 6 with Fig. 5, it is easy to find out that when CNR of signals becomes lower, the noise suppression effect based on BPDC algorithm becomes more apparent compared to that of CNC. Then reduce CNR to 28 dB-Hz, and the acquisition result is displayed in Fig. 7.

Fig. 7 shows that when the CNR is reduced to 28 dB-Hz, CNC algorithm can not complete acquisition yet while BPDC algorithm still has good acquisition performance for weak signal in the same case.

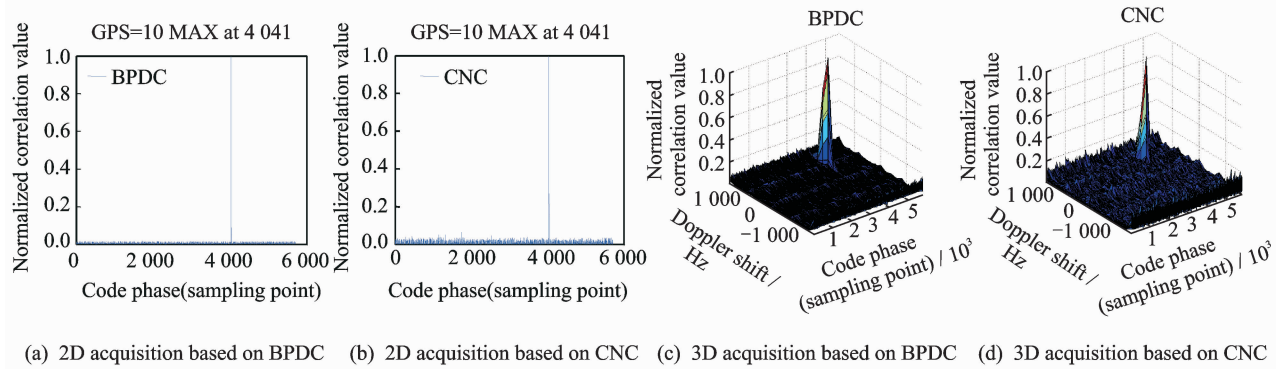


Fig. 5 Acquisition result based on BPDC and CNC algorithms (35 dB-Hz)

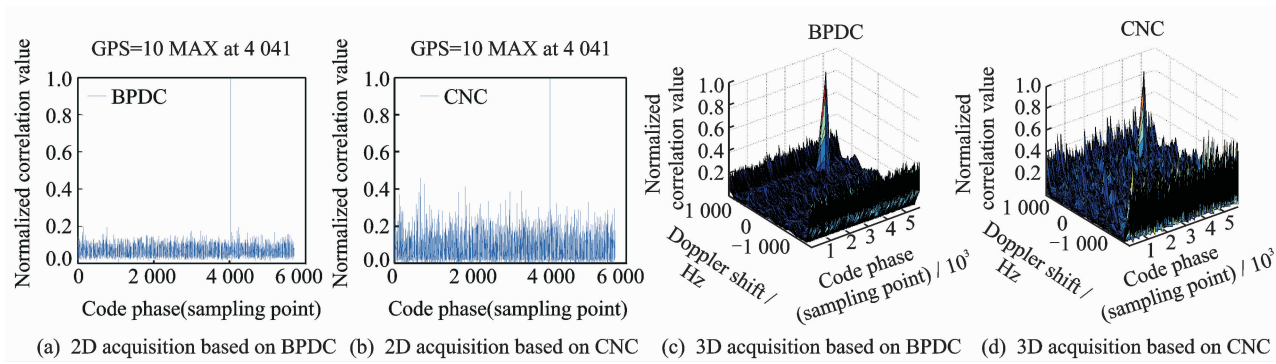


Fig. 6 Acquisition result based on BPDC and CNC algorithms (30 dB-Hz)

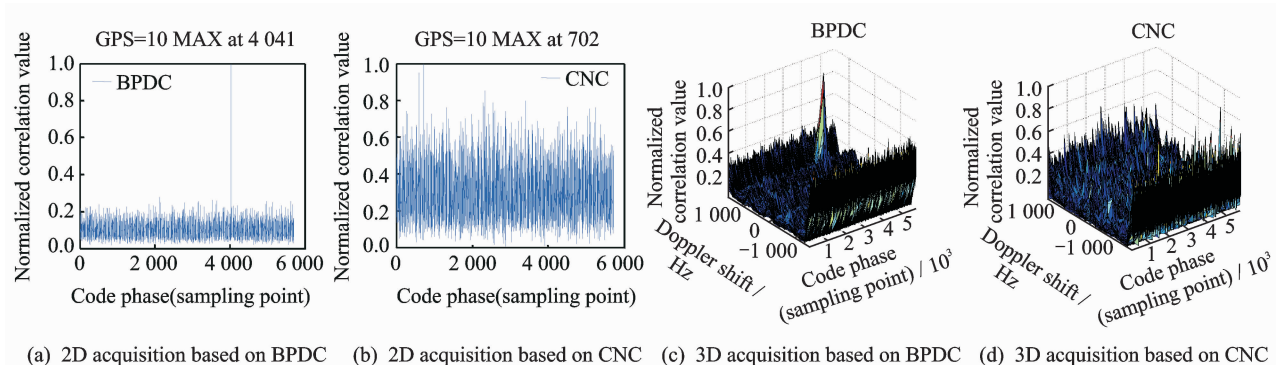


Fig. 7 Acquisition result based on BPDC and CNC algorithms (28 dB-Hz)

## 5 Conclusions

A weak GPS signal acquisition algorithm based on BPDC is proposed. It combines block processing and differential coherent integration. Not only does the suggested algorithm overcome the disadvantages of traditional coherent integration, but also the squaring loss introduced by non-coherent integration is reduced. The performance of BPDC algorithm is compared with that of CNC algorithm based on the simulated signal source. For weak GPS L1 C/A code signal acquisition, BPDC algorithm can acquire the signal with lower CNR, which is better than CNC in restraining squaring loss and improving SNR gain.

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