Influence of Setting Error of Tool on Tooth Profile and Contact Point of Face Gear Drive

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Abstract: In order to analyze the influence of setting error of tool on both tooth profile and contact characteristic of orthogonal face gear drive, the coordinate systems with and without setting error are established. Moreover, the equations of tooth profile and contact points of face gear drive are derived by envelope principle. According to the equations, the change of tooth profile and the contact points position on face gear are analyzed. The tooth surface and contact points are obtained by numerical simulation. Results show that the tooth profile and contact characteristic of face gear drive are not sensitive to the setting error of tool.

Key words: face gear; setting error; tooth profile; contact characteristic

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1 Introduction

The face gear drive is a gear driving that a spur pinion meshes with a face gear, which the pinion shaft and face gear shaft can be intersecting or non-intersecting, and the face gear is processed by involute gear sharper cutter. Compared with the bevel gear drive, face gear drive has many advantages[1-3], such as simplified of structure, reduction of weight and noise, better split-power effect, decreased of vibration etc. Therefore, the face gear drive is suitable for application in helicopter transmissions, but a substantial step in the technology of face gear drive is based on processing of high precision face gear.

The performed research is based on application of modern theory that has been research by Litvin, Zhu Rupeng, Zhao Ning, et al[1-10]. In the references, the tooth profile and contact specialties of master face gear drive have been researched[4-11], but in machining process, setting error of tool will influence the tooth profile and contact characteristic of face gear drive. Therefore, the influence of the setting error of tool needs to be studied.

2 Tooth Profile Equation of Face Gear with Setting Error of Tool

2.1 Coordinate systems of processing

The processing error of face gear is caused by the errors of gear shaping machine, fixtures of workpiece, setting of shaper cutter, machining deformation, machine tool stiffness, motion error of machine tool, profile error of tool, etc. The errors of gear shaping machine and fixture of workpiece can be regarded as the misalignment of shaper cutter. Therefore, only the processing error caused by setting of shaper cutter is analyzed in the paper. The setting error of shaper cutter includes two forms, one is misalignment of offset of shaper cutter, and the other is misalignment of obliquity of shaper.

The coordinate systems of processing which

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are with misalignment of offset are shown in Fig. 1. The fixed coordinate system of face gear is O_F-$X_FY_FZ_F$, and the face gear rotates around the axis of O_FZ_F with the angular velocity of ω_f. The fixed coordinate system of standard shaper cutter for standard installation is O_M-$X_MY_MZ_M$, and the shaper cutter rotates around the axis of O_MZ_M with the angular velocity of ω_m. The fixed coordinate system of shaper cutter with misalignment of offset along the axis of O_MX_M is O_{T1}-$X_{T1}Y_{T1}Z_{T1}$, and the shaper rotates around the axis of $O_{T1}Z_{T1}$ with the angular velocity of ω_{t1}. The fixed coordinate system of shaper with misalignment of offset along the axis of O_MY_M is O_{T2}-$X_{T2}Y_{T2}Z_{T2}$, and the shaper rotates around the axis of $O_{T2}Z_{T2}$ with the angular velocity of ω_{t2}. The parameters a and b are the offset of coordinate systems of O_{T2}-$X_{T2}Y_{T2}Z_{T2}$ and O_{T1}-$X_{T1}Y_{T1}Z_{T1}$ from the standard system O_M-$X_MY_MZ_M$, respectively. And parameter d is the location datum of shaper from workpiece of face gear.

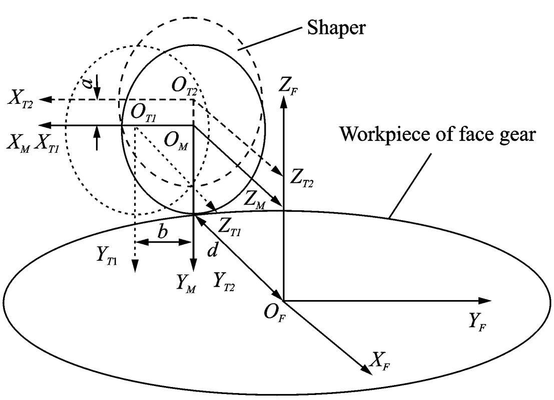


Fig. 1 Coordinate system with setting error of offset

The coordinate systems of processing which are with misalignment of obliquity are shown in Fig. 2. The fixed coordinate systems of face gear and standard shaper of standard installation are the same as coordinate systems in Fig. 1. The fixed coordinate system of shaper with misalignment of obliquity rotating around the axis of O_MY_M is O_{T3}-$X_{T3}Y_{T3}Z_{T3}$, and the shaper rotates around the axis of $O_{T3}Z_{T3}$ with the angular velocity of ω_{t3}. The fixed coordinate system of shaper with misalignment of obliquity rotating around the axis of O_MX_M is O_{T4}-$X_{T4}Y_{T4}Z_{T4}$, and the shaper rotates around the axis of $O_{T4}Z_{T4}$ with the angular velocity of ω_{t4}. The parameters ε_a and ε_b are the deflection angle of coordinates systems of O_{T3}-$X_{T3}Y_{T3}Z_{T3}$ and O_{T4}-$X_{T4}Y_{T4}Z_{T4}$ from the standard system O_M-$X_MY_MZ_M$, respectively.

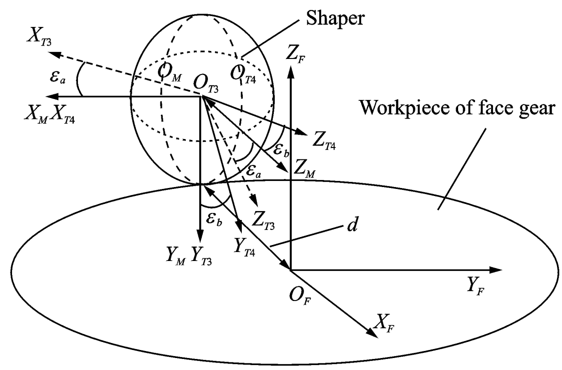


Fig. 2 Coordinate system with setting error of obliquity

2.2 Equation of standard face gear

Standard face gear is machined by the shaper cutter which is standard installation through envelope theory. In machining process, the shaper cutter is an involute cylindrical gear, and the coordinate system of shaper cutter is shown in Fig. 3. The coordinate system O_M-$X_MY_MZ_M$ is rigidly connected to the shaper.

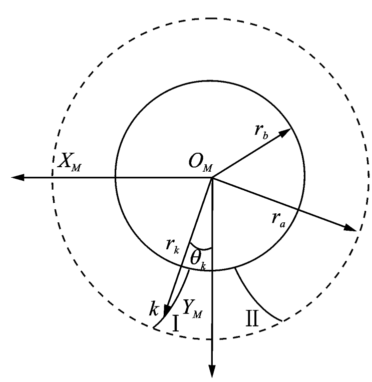


Fig. 3 Coordinate system of shaper

The equation of the tooth surface Σ_M of the shaper cutter is given by[11,12]

$$\begin{cases} X_M = \pm r_k \sin\theta_k \\ Y_M = r_k \cos\theta_k \\ Z_M = u_k \end{cases} \tag{1}$$

where the upper and lower signs correspond to profile Ⅰ and Ⅱ, respectively; r_k is radius of the point on tooth surface Σ_M of shaper which is expressed as $r_k = r_b/\cos\alpha_k$; r_b is base radius of shaper; α_k is the pressure angle of the point k; θ_k is the angle between axis of O_MY_M and vector O_Mk, and θ_k is expressed as $\theta_k = \pi/(2 \cdot N_m) - \mathrm{inv}\alpha + \mathrm{inv}\alpha_k$, N_m is the teeth number of shaper; u_k is the parameter which is along the axis of shaper.

The homogeneous matrixe of equation of tooth surface is given by

$$\boldsymbol{r}_M = [X_M \quad Y_M \quad Z_M \quad 1 \quad]^{\mathrm{T}} \tag{2}$$

According to the meshing relationship between face gear and shaper shown in Fig. 1, the homogeneous transformation matrix from the moving coordinate system of shaper O_m-$X_mY_mZ_m$ to the moving coordinate system of face gear O_f-$X_fY_fZ_f$ is the following equation

$$M_{fm} = M_{fF}M_{FM}M_{Mm} \tag{3}$$

Therefore, the tooth surface of shaper cutter is obtained from Eqs. (2, 3) by coordinate transformation.

$$\boldsymbol{R}_M = M_{fm} \cdot \boldsymbol{r}_M \tag{4}$$

The envelope condition between shaper cutter and face gear is expressed as[12-14]

$$\frac{\partial \boldsymbol{R}_M}{\partial \alpha_k} \times \frac{\partial \boldsymbol{R}_M}{\partial u_k} \cdot \frac{\partial \boldsymbol{R}_M}{\partial \varphi_m} = 0 \tag{5}$$

Therefore, the tooth surface equation of standard face gear is expressed as

$$\begin{cases} \boldsymbol{R}_M = M_{fm} \cdot \boldsymbol{r}_M \\ \dfrac{\partial \boldsymbol{R}_M}{\partial \alpha_k} \times \dfrac{\partial \boldsymbol{R}_M}{\partial u_k} \cdot \dfrac{\partial \boldsymbol{R}_M}{\partial \varphi_m} = 0 \end{cases} \tag{6}$$

where the parameter φ_m is the angle of shaper rotates around the axis O_MZ_M.

The profile of standard face gear is obtained by numerical simulation with Matlab2007, which is shown in Fig. 4. The number of teeth of the

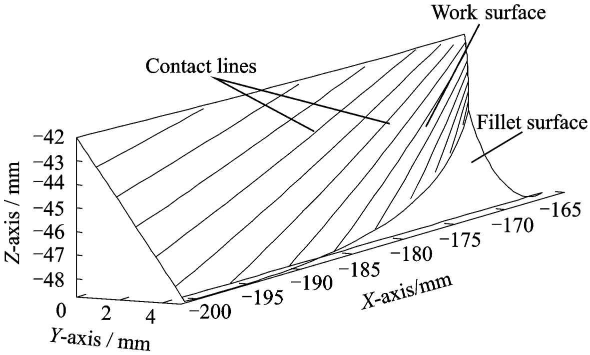


Fig. 4 Simulation of tooth surface of standard face gear

shaper is 30; the number of teeth of the face gear is 120; the module of shaper and face gear is 3 mm; the pressure angle of shaper is 25°.

2.3 Face gear with processing error of offset

Processing error of face gear with misalignment of offset contains two forms which are shown in Fig. 1. One is caused by offset along the axis O_MX_M, and the other is caused by offset along the axis O_MY_M. The equation of shaper cutter is the same as Eq. (1), and the difference is the subscript of Eq. (1) are replaced "M" with "T", which is expressed as

$$\boldsymbol{r}_T = [X_T \quad Y_T \quad Z_T \quad 1]^{\mathrm{T}} \tag{7}$$

According to the meshing relationship shown in Fig. 1, the homogeneous transformation matrix from the moving coordinate system of shaper with setting error O_t-$X_tY_tZ_t$ to the moving coordinate system of face gear O_f-$X_fY_fZ_f$ is expressed as

$$\boldsymbol{M}_{ft} = \boldsymbol{M}_{fF}\boldsymbol{M}_{FM}\boldsymbol{M}_{MT}\boldsymbol{M}_{Tt} \tag{8}$$

Matrix $\boldsymbol{M}_{MT}$ is caused by setting error of shaper which is expressed as

$$\boldsymbol{M}_{MT} = \begin{bmatrix} 1 & 0 & 0 & b \\ 0 & 1 & 0 & a \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

When parameter a is zero, matrix $\boldsymbol{M}_{MT}$ is the transformation matrix from coordinate system O_{T1}-$X_{T1}\ Y_{T1}\ Z_{T1}$ to coordinate system O_M-$X_MY_MZ_M$. When parameter b is zero, matrix $\boldsymbol{M}_{MT}$ is the transformation matrix from coordinate system O_{T2}-$X_{T2}\ Y_{T2}\ Z_{T2}$ to coordinate system O_M-$X_MY_MZ_M$.

The tooth surface Σ_T of shaper with misalignment of deflection which is expressed in coordinate system O_f-$X_fY_fZ_f$ as

$$\boldsymbol{R}_F = \boldsymbol{M}_{ft} \cdot \boldsymbol{r}_T \tag{10}$$

The envelope condition between shaper with misalignment of deflection and face gear is expressed as[12-14]

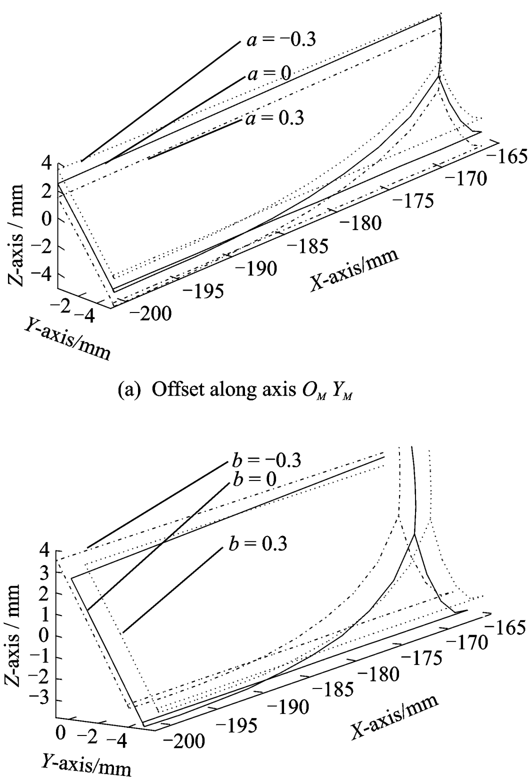
$$\frac{\partial \boldsymbol{R}_f}{\partial \alpha_k} \times \frac{\partial \boldsymbol{R}_f}{\partial u_k} \cdot \frac{\partial \boldsymbol{R}_f}{\partial \varphi_t} = 0 \tag{11}$$

where φ_t is the angle of shaper rotating around the axis O_TZ_T.

Therefore, the equation of face gear with processing error of misalignment of deflection is expressed as

$$\begin{cases} \boldsymbol{R}_F = \boldsymbol{M}_{ft} \cdot \boldsymbol{r}_T \\ \dfrac{\partial \boldsymbol{R}_F}{\partial \alpha_k} \times \dfrac{\partial \boldsymbol{R}_F}{\partial u_k} \cdot \dfrac{\partial \boldsymbol{R}_F}{\partial \varphi_t} = 0 \end{cases} \tag{12}$$

The tooth profile of face gear with processing error of misalignment of offset is obtained by numerical simulation with Matlab 2007, which is shown in Fig. 5. When the parameter a is 0.3, 0 and −0.3 mm and b is 0 mm, the surface Σ_F of face gear is shown in Fig. 5(a). When the parameter b is 0.3, 0 and −0.3 mm and a is 0 mm, the surface Σ_F of face gear is shown in Fig. 5(b).



(a) Offset along axis $O_M Y_M$

(b) Offset along axis $O_M X_M$

Fig. 5 Tooth profile of face gear with machining error of misalignment of offset

According to Fig. 5(a), the tooth profile of face gear with processing error which are misalignment of offset along the axis $O_M Y_M$ keeps invariant, but the position of tooth profile will move along the axis of face gear. If parameter a is more than zero, the tooth profile moves along the positive direction of the axis of the face gear, and if parameter a is less than zero, the moving direction is opposite.

According to Fig. 5(b), the tooth profile of face gear with processing error which is misalignment of offset along the axis $O_M X_M$ keeps invariant, too, but the position of tooth profile will rotate around the axis of face gear. If parameter b is more than zero, the tooth profile rotates in the clockwise direction of the axis of the face gear, and if parameter b is less than zero, the rotating direction is opposite.

2.4 Face gear with processing error of obliquity

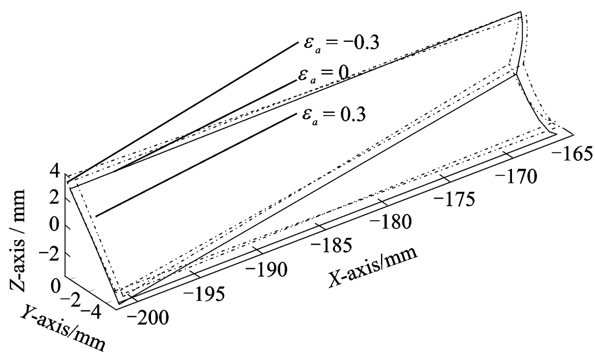
Processing error of face gear with misalignment of obliquity contains two forms which are shown in Fig. 2. One is caused by the shaper whirling around the axis $O_M Y_M$, and the other is caused by the shaper whirling around the axis $O_M X_M$. The surface equation of shaper is the same as Eq. (7).

According to the meshing relationship shown in Fig. 2, the homogeneous transformation matrix $\boldsymbol{M}_{ft}$ from the moving coordinate system with processing error of shaper O_t-$X_t Y_t Z_t$ to the moving coordinate system of face gear O_f-$X_f Y_f Z_f$ is the same as Eq. (8), but the matrix $\boldsymbol{M}_{MT}$ which is transformation matrix from the fixed coordinate O_T-$X_T Y_T Z_T$ to the fixed coordinate O_M-$X_M Y_M Z_M$ is different from Eq. (9), and matrix $\boldsymbol{M}_{MT}$ is expressed as

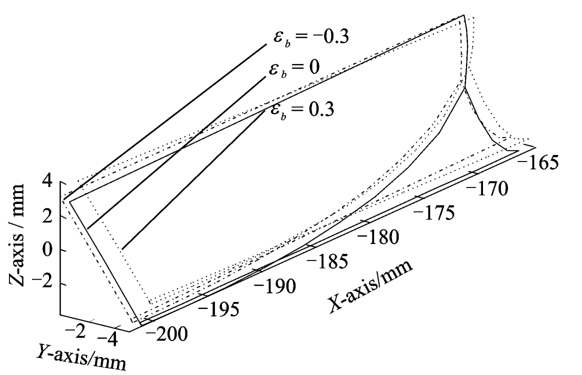
$$\boldsymbol{M}_{MT} = \begin{bmatrix} \cos\varepsilon_a & \sin\varepsilon_a \sin\varepsilon_b & \sin\varepsilon_a \cos\varepsilon_b & 0 \\ 0 & \cos\varepsilon_b & \sin\varepsilon_b & 0 \\ -\sin\varepsilon_a & \cos\varepsilon_a \sin\varepsilon_b & \cos\varepsilon_a \sin\varepsilon_b & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

where parameter ε_a is the angle of misalignment of obliquity whirling around the axis $O_M Y_M$, parameter ε_b the angle of misalignment of obliquity whirling around the axis $O_M X_M$. When parameter $\varepsilon_a = 0$, matrix $\boldsymbol{M}_{MT}$ is the transformation matrix from coordinate system O_{T4}-$X_{T4} Y_{T4} Z_{T4}$ to coordinate system O_M-$X_M Y_M Z_M$. When parameter $\varepsilon_b = 0$, matrix $\boldsymbol{M}_{MT}$ is the transformation matrix from coordinate system O_{T3}-$X_{T3} Y_{T3} Z_{T3}$ to coordinate system O_M-$X_M Y_M Z_M$.

The tooth profile of face gear with processing error which is misalignment of obliquity is obtained by numerical simulation with Matlab2007, which is shown in Fig. 6. When the parameter ε_a is 0.3°, 0° and −0.3° and ε_b is 0°, the tooth surface Σ_F of face gear is shown in Fig. 6(a). When the parameter ε_b is 0.3°, 0°, −0.3° and ε_a is 0°, the tooth surface Σ_F of face gear is shown in Fig. 6(b).



(a) Rotating around axis $O_M Y_M$



(b) Rotating around axis $O_M X_M$

Fig. 6 Profile of face gear with machining error of misalignment of obliquity

According to Fig. 6(a), the tooth profile of face gear with processing error of obliquity whirling around $O_M Y_M$ is deformation. If parameter ε_a is more than zero, the work face of right tooth profile is up, while the fillet surface of right tooth profile is down, and the left tooth profile change in opposite direction. If parameter ε_a is less than zero, the change direction is opposite.

According to Fig. 6(b), the tooth profile of face gear with processing error of obliquity whirling around $O_M X_M$ is deformation. If parameter ε_b is more than zero, the medial tooth profile is up, while the lateral tooth profile is down. If parameter ε_b is less than zero, the rotating direction is opposite.

At these parameters, the maximum error of influence of setting error of tool on tooth profile is 1.26%.

3 Contact Specialties of Face Gear Drive with Processing Error

The contact points of face gear with processing error meshing with spur pinion will be different for various types of processing error, and the chapter will analyze the influence of processing error to the position of contact points.

According to the envelope theory, the equation of contact line of face gear with processing error meshing with shaper cutter is expressed as Eq. (12), and the contact lines are shown in Fig. 4.

The transformation equation of tooth profile of spur pinion from moving coordinate of spur pinion to moving coordinate of master shaper is expressed as

$$\boldsymbol{R}_P = \boldsymbol{M}_{mp} \cdot \boldsymbol{r}_P \tag{14}$$

The number N_T of the teeth of the pinion is less than the number N_m of the teeth of the shaper: $N_m - N_T = 2$ or 3[13,14], and the pinion and the shaper are regarded as internal gear pair. $\boldsymbol{r}_P$ is the tooth surface equation of spur pinion; $\boldsymbol{M}_{mp}$ is the homogeneous transformation matrix from moving coordinate system of pinion to moving coordinate system of shaper.

The envelope condition between spur pinion and shaper is expressed as

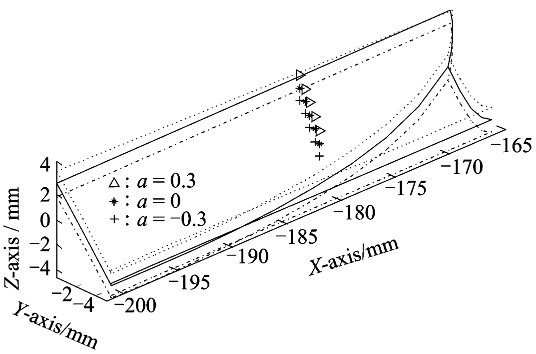
$$\frac{\partial \boldsymbol{R}_P}{\partial \partial_k} \times \frac{\partial \boldsymbol{R}_P}{\partial u_k} \cdot \frac{\partial \boldsymbol{R}_P}{\partial \varphi_t} = 0 \tag{15}$$

Therefore, the equation of contact points on the surface Σ_f is expressed as

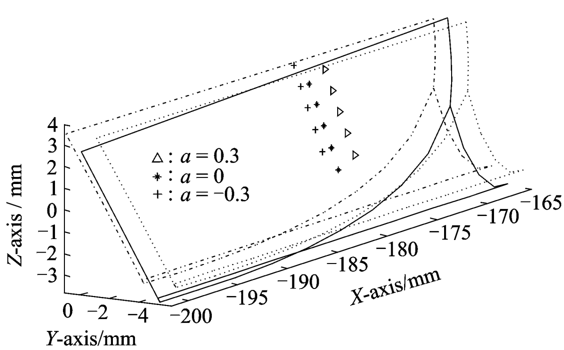
$$\begin{cases} \boldsymbol{R}_T = \boldsymbol{M}_{ft} \cdot \boldsymbol{r}_T \\ \dfrac{\partial \boldsymbol{R}_T}{\partial \alpha_k} \times \dfrac{\partial \boldsymbol{R}_T}{\partial u_k} \cdot \dfrac{\partial \boldsymbol{R}_T}{\partial \varphi_t} = 0 \\ \boldsymbol{R}_P = \boldsymbol{M}_{mp} \cdot \boldsymbol{r}_P \\ \dfrac{\partial \boldsymbol{R}_P}{\partial \alpha_k} \times \dfrac{\partial \boldsymbol{R}_P}{\partial u_k} \cdot \dfrac{\partial \boldsymbol{R}_P}{\partial \varphi_t} = 0 \end{cases} \tag{16}$$

The numerical simulations of position of contact points on tooth profile of face gear drive with processing error of misalignment of offset and obliquity are shown in Figs. 7, 8, respectively.

According to Figs. 7(a, b), the position of contact points on tooth profile of face gear will be changed with the tooth profile of face gear with processing error of misalignment of offset, but the relative position is not sensitive to the misalignment of offset. The signs "△" "*" and "+" represent the position of contact points when the parameters a and b are 0.3, 0 and −0.3 mm, respectively.



(a) Offset along axis $O_M Y_M$

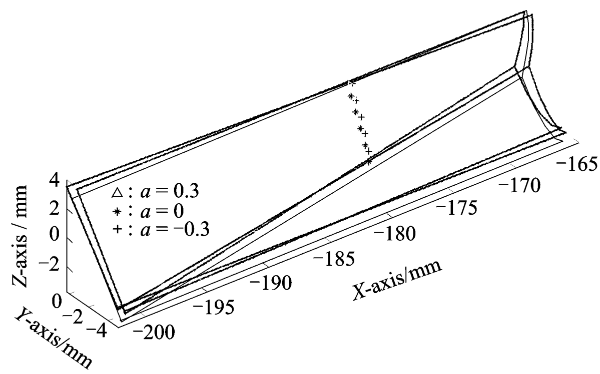


(b) Offset along axis $O_M X_M$

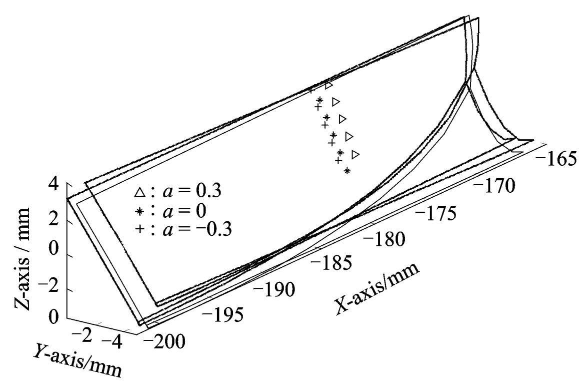
Fig. 7 Contact points of face gear with machining error of misalignment of deflection

According to Figs. 8(a, b), the position of contact points on tooth profile of face gear will be changed with the tooth profile of face gear with machining error of misalignment of obliquity, but the relative position is not sensitive to the misalignment of obliquity. The signs "△" "*" and "+" represent the position of contact points when the parameters ε_a and ε_b are 0.3°, 0° and −0.3°, respectively.

Compared with the change of various types of processing errors, the contact points of face gear drive move with the teeth profile which are processed with processing error of misalignment of offset, and the relative position keep invariant; the contact points of face gear drive will change with deformation of the tooth profile of face gear which are processed with processing error of misalignment of obliquity, and the relative position of contact points are changed a little.



(a) Rotating around axis $O_M Y_M$



(b) Rotating around axis $O_M X_M$

Fig. 8 Contact points of face gear with machining error of misalignment of obliquity

According to the above analysis and Fig. 8, the position of contact points of face gear drive will have a tiny change with the face gear which is processed by shaper cutter with the setting error. The pressure along axis "X" is zero in face gear drive. Therefore, the tiny change of contact points will not effect the transmission characteristics of face gear drive. At these parameters, the maximum error of influence of setting error of tool on position of contact points is 0.73%.

4 Conclusions

Based on the performed research, the following conclusion might be drawn:

(1) The influence of obliquity error on tooth

profile is bigger than offset error, and the offset error will make the profile of face gear translation, the obliquity error will make the profile of face gear deformation, but the influence are all tiny.

(2) The processing error will make the position of contact points moving with the tooth profile of face gear, but the relative position change a little.

In conclusion, the tooth profile and position of contact points of face gear drive are not sensitive to the processing error.

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