# Optimal Power Management for Antagonizing Between Radar and Jamming Based on Continuous Game Theory

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**Abstract:** To solve the problem of dynamic power resource allocation for cooperative penetration combat, the continuous game theory is introduced and a two-person general-sum continuous-game-based model is put forward with a common payoff function named collaborative detection probability of netted radar countermeasures. Comparing with traditional optimization methods, an obvious advantage of game-based model is an adequate consideration of the opposite potential strategy. This model guarantees a more effective allocation of the both sides' power resource and a higher combat efficiency during a combat. Furthermore, an analysis of the complexity of the proposed model is given and a hierarchical processing method is presented to simplify the calculating process. Simulation results show the validity of the proposed scheme.

**Key words:** game theory; radar countermeasures; power management; detection probability; penetration combat **CLC number:** N945, 25 **Document code:** A **Article ID:** 1005-1120(2014)04-0386-08

## 1 Introduction

Under the condition of information technology, the battlefield environment becomes increasingly complex. Hence, the competition between attacking and defending sides in the mastery of electromagnetic has been a new conflict intertwined point<sup>[1]</sup>. When a confrontation between radar and jamming occurs, the jamming side will take a full advantage of the synergetic superiority to suppress, disrupt and destroy the processes of early warning detection, target tracking and precision-guiding of enemy radar<sup>[2]</sup>. At the same time, the anti-jamming side will combine distributed radars to form a network for searching, tracking and identifying targets. The netted radars can gain detection range expansion and obtain better detection performance.

In order to effectively suppress the enemy radar network and improve the overall combat effectiveness, we need a reasonable allocation of various jamming resources<sup>[3]</sup>. Meanwhile, the enemy side can also take similar measures to manage the radar resources<sup>[4-8]</sup>. Therefore, in the process of jamming and anti-jamming between radars and jammers, both sides should learn to choose a better strategy to maximize one's own interests and minimize the opposite interests, thus bringing forth a game between the two sides.

Game theory is the one that the individuals or teams choose possible strategies synchronously or successively to achieve high payoff income according to the known information under a certain circumstance or constrained condition<sup>[9]</sup>. Game theory can ensure optimal decision for both sides whenever there is an interest conflict between the decision-makers in the same environment<sup>[10-12]</sup>. After fully considering the strategies that the opponent may take, the decision-making process of the game theory will be closer to the reality compared with the traditional decision-making methods that only take objective factors into account.

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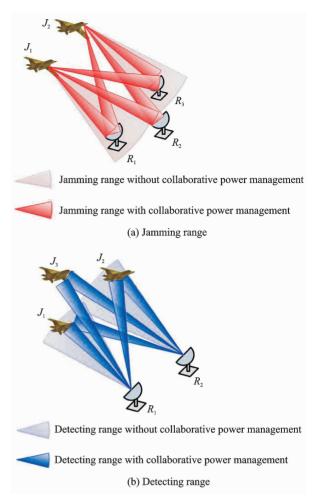
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Many scholars have been engaged in related researches on the modeling and application of game theory in decision-making field. Wei, et al [13] discussed a game theoretic target assignment in ballistic missile defense. Sang, et al<sup>[14]</sup> designed a midcourse guidance law for hypersonic cruise missile based on the differential game theory and neural network. Song, et al[15] investigated resource allocation problems using game theory. Game-based methods were discussed by Tang<sup>[16]</sup> and Song, et al<sup>[17]</sup> for multi-objective shape designing and multi-objective optimization, respectively. Yao, et al<sup>[18]</sup> presented a mission decisionmaking method for multi-aircraft cooperatively attacking multi-object based on game theory model, and Fu, et al<sup>[19]</sup> studied multi-UAV cooperative fight decision problems based on game theory. A differential game was used for the firepower-assignment in vessel formations by Li, et  $al^{[20]}$ . Wei, et  $al^{[21]}$  built a game-theoretic model for controlling military air operations with civilian players. Although game theory is more and more widely used in military, its application in power resource allocation for cooperative penetration combat has never been publicly reported. In this paper, after analyzing the nature of radar countermeasure, we propose a continuous game theory-based cooperative power management model to find an optimum allocation of power resources based on the opposite situation and the possible strategies of enemy side.

# 2 Problem Description

In a constantly changing situation, both opposing sides will effectively manage their own operating resources to pursue the more efficient operation<sup>[22-26]</sup>.

Considering an offense-defense situation as shown in Fig. 1(a), the blue side consists of three ground-based aerial defense radars, which form a radar network. The red side, in contrast, is composed of two fighters whose electronic countermeasures (ECMs) are turned on for self-screening jamming so as to guarantee the success of penetration. Suppose that there is no cooperation be-



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Fig. 1 Maps of jamming and detecting ranges

tween two red fighters and both fighters release jamming signals separately only using their own energy. In Fig. 1 (a), radars  $R_1$  and  $R_3$  are adequately included in interference sectors, while radar  $R_2$  is not. Meanwhile, the jamming regions of  $J_1$  and  $J_2$  overlap, which causes the waste of jamming energy and poor jamming effectiveness. If all the jamming power resources are scheduled and managed by an electronic warfare command center, the system can carry out a unified decision-making process. Then, the power resources will be distributed to the jammers reasonably according to the real-time situation, which can make full advantage of the multi-target jamming ability of ECM. Therefore, the overall operational effectiveness will be significantly improved. Accordingly, without coordination and collaboration of the detecting power resources, there will be a loss of overall detecting performance because of an ineffective usage of the limited power resources. Fig. 1(b) shows that without an effective coordination, only  $J_1$  and  $J_2$  are covered by the detecting sectors of radars while  $J_3$  cannot be effectively detected. Thus, it is necessary for both sides to precisely manage their own power resources in a radar-jamming counterwork.

#### 2.1 Formulation of gaming

In offense-defense gaming between the red and blue sides, the goal is to seek a power allocation strategy that can maximize one's own efficiency and minimize the opponent's efficiency. Due to the infinity and continuum features of strategy set, the collaborative power allocation model for counterwork between radar and jamming can be modeled as a two-person general sum infinite continuous game model, namely

$$\Gamma = (N, (S_i), (u_i)) \tag{1}$$

where N denotes the player set and  $N \triangleq \{R,B\}$ , here "R" represents all fighters belonging to the red side and "B" all radars of the blue side.  $(S_i) \triangleq (S_R, S_B)$ , where  $S_i$  denotes the strategy set of player i; and  $(u_i) \triangleq (u_R, u_B)$  is the payoff function of player i.

Suppose that the number of red fighters is m, the number of blue radars is n, and the total power of both sides are  $P_{\rm Jall}$  and  $P_{\rm Tall}$ , respectively. The gaming point is to deicide an optimal power transmitted from any fighter (radar) to any radar (fighter). The decision variables are defined as

$$\begin{cases} S_{R} \triangleq X = \{x_{ij} \mid 0 \leqslant x_{ij} \leqslant 1, \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = 1\} \in D_{R} \\ S_{B} \triangleq Y = \{y_{ij} \mid 0 \leqslant y_{ij} \leqslant 1, \sum_{j=1}^{m} \sum_{i=1}^{n} y_{ij} = 1\} \in D_{B} \end{cases}$$

$$(2)$$

where  $x_{ij}$  ( $i=1,2\cdots m;j=1,2\cdots n$ ) is the decision variable of the red side. Concretely speaking,  $x_{ij}$  means that the red fighter i will release a jamming with a power of  $x_{ij} \cdot P_{\text{Jall}}$  to the blue radar j. Similarly,  $y_{ij}$  ( $i=1,2\cdots n;j=1,2\cdots m$ ) indicates that the blue radar i will detect the red fighter j with a power of  $y_{ij} \cdot P_{\text{Tall}}$ .  $D_{\text{R}}$  and  $D_{\text{B}}$  are the decision spaces that can be described as

$$\begin{cases}
D_{R} = \{X \mid X \in [0,1]^{m} \times [0,1]^{n}\} \\
D_{B} = \{Y \mid Y \in [0,1]^{n} \times [0,1]^{m}\}
\end{cases}$$
(3)

In the gaming process, the jamming side will suppress enemy radar to reduce its detection probability, while the radar side seeks a detection probability increase. Therefore, we choose the detection probability of netted radar as the payoff function [3]. According to the Neyman-Pearson criterion, when the probability density of radar receiver envelope detector obeys zero-mean Gaussian distribution, detection probability  $P_d$  and false alarm probability  $P_f$  of a single radar are defined as

$$\begin{cases} P_{\rm f} = \exp\left(-\frac{U_{\rm T}^2}{2}\right) \\ P_{\rm d} = P(U_{\rm T} \leqslant r \leqslant \infty) = \int_{U_{\rm T}}^{\infty} r \exp\left(-\frac{r^2 + A^2}{2}\right) \\ I_{\rm 0}(-rA) \, \mathrm{d}r \end{cases}$$

$$(4)$$

where  $U_T$  is the detection threshold and  $I_0(z)$  the zero-order modified Bessel Function. Variable A satisfies the following formula

$$SNR = \frac{A^2}{2} \tag{5}$$

where SNR is the ratio of signal to jamming and it can be calculated by  $^{\text{[4]}}$ 

$$SNR = \frac{P_{t}G_{t}^{2}\sigma R_{j}^{2}}{4\pi V_{i}P_{i}G_{i}G_{t}^{\prime}R_{t}^{4}}$$
 (6)

where  $P_{\rm t}$  is the radar transmit power;  $G_{\rm t}$  the radar antenna gain;  $\sigma$  the cross-sectional area of the target;  $R_{\rm t}$  the range from target to radar;  $R_{\rm j}$  the range from jammer to radar;  $P_{\rm j}$  the jammer transmit power;  $G_{\rm j}$  the jammer antenna gain;  $V_{\rm j}$  the polarization loss from jammer to radar antenna; and  $G_{\rm t}'$  the antenna gain of radar antenna in the direction of the jammer. When the fighters ultilize a self-screening jamming, we get  $R_{\rm t}=R_{\rm j}$  and  $G_{\rm t}'=G_{\rm t}$ , then

$$SNR = \frac{P_t G_t \sigma}{4\pi V_i P_i G_i R_t^2} \tag{7}$$

When the detection side builds a radar network composed of multiple dispersive radars, it uses rank-K fusion rule for decision-making. Then, the overall detection probability  $P_{\rm DKn}$  and the overall false alarm probability  $P_{\rm FKn}$  of the network can be calculated as follows<sup>[27]</sup>

$$\begin{cases}
P_{\text{DK}n} = \sum_{i=K}^{n} \left\{ \left[ \sum_{p=0}^{i-k} (-1)^{p} \cdot C(i,p) \right] \cdot \left[ \sum_{C_{in}} \left( \prod_{j} P_{dj} \right) \right] \right\} \\
P_{\text{FK}n} = \sum_{i=K}^{n} \left\{ \left[ \sum_{p=0}^{i-k} (-1)^{p} \cdot C(i,p) \right] \cdot \left[ \sum_{C_{in}} \left( \prod_{j} P_{fj} \right) \right] \right\}
\end{cases}$$
(8)

where  $\sum_{C_{in}} (\prod_j P_{dj})$  is the sum of all possible products of detection probability of the i local detectors among the n local detectors. This definition applies equally to  $\sum_{C_{in}} (\prod_j P_{fj})$ . C(i,p) =

 $\frac{i!}{p! (i-p)!}$  is the combination of i different objects taken p.

According Eqs. (2–8),  $P_{\rm DKn}$  can be abstracted as a function of the decision variables of both sides. Thus, in the game model, we select  $P_{\rm DKn}$  as the benefit of jamming side and  $-P_{\rm DKn}$  as the profit of detecting side, i. e.,

$$\begin{cases} u_{R} = P_{DK_{n}}(S_{R}, S_{B}) = P_{DK_{n}}(X, Y) \\ u_{B} = -P_{DK_{n}}(S_{R}, S_{B}) = -P_{DK_{n}}(X, Y) \end{cases}$$
(9)

During the decision process, the jamming side expects a detection probability as low as possible with a choice of (X,Y) and contrarily the detecting side wants the highest probability. Then, the game-based decision model can be formulated as

$$\begin{cases} V_{R}^{*} = \min_{X \in D_{R}Y \in D_{B}} \{P_{DKn}(X,Y)\} \\ V_{B}^{*} = \max_{X \in D_{R}Y \in D_{B}} \{-P_{DKn}(X,Y)\} \end{cases}$$

$$(10)$$

$$(P_{DKn}(X^{*},Y^{*}) \leq P_{DKn}(X,Y^{*}) \forall X \in D_{R}$$

s.t. 
$$\begin{cases} P_{\mathrm{D}\mathit{K}n}(X^*, Y^*) \leqslant P_{\mathrm{D}\mathit{K}n}(X, Y^*) \; \forall \; X \in D_{\mathrm{R}} \\ P_{\mathrm{D}\mathit{K}n}(X^*, Y^*) \geqslant P_{\mathrm{D}\mathit{K}n}(X^*, Y) \; \forall \; Y \in D_{\mathrm{B}} \end{cases}$$

where  $V_{\rm R}^*$  and  $V_{\rm B}^*$  are the optimal game values of red and blue sides, respectively and  $(X^*,Y^*)$  is the final Nash Equilibrium strategy of the game model.

### 2.2 Problem solving

The game-based collaborative power allocation model seeks the most appropriate powers for both sides according to the situations and strategy of the opponent. Since there exists huge decision space when the combat unit increases, it will be much more difficult to reach the game equilibri-

um. To improve the efficiency, we present a stage-hierarchical method to turn a complex multidimensional game<sup>[28]</sup> into numerous simple one-dimensional games, thus simplifying the optimization process.

In Fig. 2, after decomposing, the first round gaming centralizes on the red side. It works out the jamming power allotted to each fighter for jamming the n radars, as well as the total detection powers of n radars for detecting each fighter. Then, the decision variables of the first round gaming are denoted as

$$\begin{cases}
S_{R}^{1} \triangleq \alpha = \{\alpha_{i} \mid 0 \leqslant \alpha_{i} \leqslant 1, \sum_{i=1}^{m} \alpha_{i} = 1\} \\
S_{B}^{1} \triangleq \beta = \{\beta_{i} \mid 0 \leqslant \beta_{i} \leqslant 1, \sum_{i=1}^{m} \beta_{i} = 1\}
\end{cases} (11)$$

where  $\alpha_i$  and  $\beta_i$  are the decision variables of both sides of the first round gaming. They indicate that the *i*th fighter of red side gets a jamming power of  $\alpha_i \cdot P_{\text{Jall}}$  while the blue side allocates a detection power of  $\beta_i \cdot P_{\text{Tall}}$  for detecting the *i*th fighter.

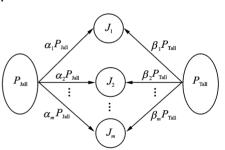


Fig. 2 Situation of the first round gaming

Following the first round gaming, the original complicated game model is broken into m one-dimensional models, whose game situations are 1 versus n. That is to say, the ith  $(i=1,\cdots,m)$  fighter of the red side with a total jamming power of  $\alpha_i \cdot P_{\text{Jall}}$  will counter against all the n radars of the blue side with a total detection power of  $\beta_i \cdot P_{\text{Tall}}$ .

The second round gaming centralizes on the blue side which figures out the jamming power that the ith  $(i=1,\dots,m)$  fighter emits to the jth  $(j=1,\dots,n)$  radar and the detection power that the jth  $(j=1,\dots,n)$  radar emits to the ith  $(i=1,\dots,m)$  fighter. As shown in Fig. 3, the decision

variables of the second round gaming are denoted

$$\begin{cases} S_{\mathrm{R}}^{2} \triangleq \lambda^{i} = \{\lambda_{j}^{i} \mid 0 \leqslant \lambda_{j}^{i} \leqslant 1, \sum_{j=1}^{n} \lambda_{j}^{i} = 1\} \\ S_{\mathrm{B}}^{2} \triangleq \eta^{i} = \{\eta_{j}^{i} \mid 0 \leqslant \eta_{j}^{i} \leqslant 1, \sum_{j=1}^{n} \eta_{j}^{i} = 1\} \end{cases}$$
(12)

where  $\lambda_j^i$  and  $\eta_j^i$  are the decision variables of both sides in the second round gaming, which means that the ith  $(i=1,\cdots,m)$  fighter will emit a jamming power of  $\lambda_j^i \cdot \alpha_i \cdot P_{\text{Jall}}$  to the jth  $(j=1,\cdots,n)$  blue radar and in return, the jth  $(j=1,\cdots,n)$  blue radar will emit a detection power of  $\eta_j^i \cdot \beta_i \cdot P_{\text{Tall}}$  to the ith  $(i=1,\cdots,m)$  red fighter.

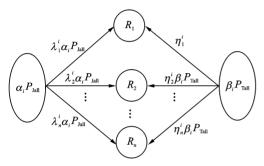


Fig. 3 Situation of the second round gaming

The stage-hierarchical method described above can simplify the problem solving process and improve the problem solving efficiency deeply. The one-dimensional continuous game models can be solved following the method introduced in Ref. [29], where the continuous model is discretized to obtain an infinite approximation of the two-person zero-sum game. Then, the optimal mixed strategy will be achieved as

$$\begin{cases}
X \triangleq \{x_{ij} \mid x_{ij} = \lambda_j^i \cdot \alpha_i, i = 1, 2, \dots, m; j = 1, 2, \dots, n\} \\
Y \triangleq \{y_{ij} \mid y_{ij} = \eta_j^i \cdot \beta_i, i = 1, 2, \dots, m; j = 1, 2, \dots, n\}
\end{cases}$$
(13)

# 3 Results and Discussion

A scenario of two fighters versus two radars is analyzed here. As an offence side, the red formation consists of two fighters whose ECMs work at a self-screening jamming mode. In addition, the blue side is composed of two ground defense radars which have already been combined into a detecting network. To decrease the detection probability, the red fighters collaboratively

manage the jamming power according to the relative situation and possible strategies that the blue side may choose. Meanwhile, the blue radars also manage their power to obtain a higher detection probability. Then, a gaming begins between the red and blue sides.

Set the simulation parameters as follows:  $P_{\text{Jall}} = 800 \text{ W}$ ,  $G_{\text{jl}} = G_{\text{j2}} = 10 \text{ db}$ ,  $\sigma_1 = \sigma_2 = 10 \text{ m}^2$ ,  $V_{\text{jl}} = V_{\text{j2}} = 0.5$ ;  $P_{\text{Tall}} = 2 000 \text{ kW}$ ,  $G_{\text{tl}} = G_{\text{t2}} = 80 \text{ db}$ . The rank fusion rule of the detecting probability of the radar network is set as K = 2. All the radars have the same constant false alarm probability (CFAR) of  $P_{\text{f}} = 1.0 \text{ e}^{-6}$ . With the above parameters, a problem in the typical situation is solved and discussed in the following.

Assuming a typical situation as shown in Fig. 4, we obtain the numerical expression in Table 1 by quantifying the positions in the coordination of North-Sky-East.

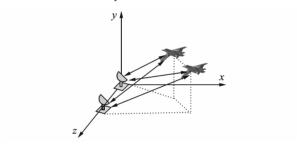


Fig. 4 Situation of offense and defense

Table 1 Quantified situation

Player -			Position	
		$R_x$	$R_{y}$	$R_z$
ъ 1	$J_1$	50 000	8 000	4 000
Red	$J_{\scriptscriptstyle  2}$	65 000	8 000	5 000
DI	$R_1$	0	0	0
Blue	$R_2$	0	0	4 000

#### 3.1 The first round gaming

According to the hierarchical method described in Section 3, the first round gaming decides how to allocate the 800 W jamming power to  $J_1$ ,  $J_2$  for jamming the blue radars, and how to divide the 2 000 kW detection power into two pieces which are used to detect  $J_1$ ,  $J_2$ , separately. To solve this high level gaming model, we carry out a discretization of N=10 and figure out the mixed Nash equilibrium (MNE) with the help

of lingo programming toolkit (LPT). The results of the first round gaming are shown in Table 2.

Table 2 Results of the first round gaming

Ctt.	Game solution		Power allocation	
Strategy	$\alpha_1/\beta_1$	$\alpha_2/eta_2$	$P^r_{J1}/P^b_{J1}$	$P^r_{J2}/P^b_{J2}$
Red	0.64 673	0.35 327	517.386 W	282.614 W
Blue	0.57 545	0.42 455	1150.9 kW	849.1 kW

#### The second round gaming

The second round gaming consists of two one-dimensional games. As one of them, the game  $J_1$  versus  $R_1$ ,  $R_2$  works out the optimum strategy of how to divide the jamming power  $P_{J1}^r$ into two pieces for jamming  $R_1$ ,  $R_2$  and how to allocate the detection power  $P_{J1}^b$  to  $R_1$ ,  $R_2$  for detecting  $J_1$ . Similarly, the game  $J_2$  versus  $R_1$ ,  $R_2$  is used to decide how to divide the jamming power  $P_{J2}^r$  into two pieces for jamming  $R_1$ ,  $R_2$  and how to allocate the detection power  $P^b_{\it J2}$  to  $R_1$  ,  $R_2$ for detecting  $J_2$ . The results of the second round gaming are shown in Tables 3,4.

Table 3 Results of the gaming  $J_1$  versus  $R_1$ ,  $R_2$ 

Ctuata	Game solution		Power allocation	
Strategy	$\overline{\lambda_1^1/\eta_1^1}$	$\lambda_2^1/\eta_2^1$	$P_{R1}^{J1}/P_{J1}^{R1}$	$P_{R2}^{J1}/P_{J1}^{R2}$
Red	0.41 835	0.58 165	216.449 W	300.937 W
Blue	0.49 900	0.50 100	574. 293 kW	576.607 kW

Table 4 Results of the gaming  $J_2$  versus  $R_1$ ,  $R_2$ 

Ctuataggy	Game solution		Power allocation	
Strategy	$-\lambda_1^2/\eta_1^2$	$\lambda_2^2/\eta_2^2$	$P_{R1}^{J2}/P_{J2}^{R1}$	$P_{ m R2}^{ m J2}/P_{ m J2}^{ m R2}$
Red	0.44 859	0.55 141	126.778 W	155.837 W
Blue	0.42 642	0.57 358	362.077 kW	487.023 kW

#### Discussion of the results

Synthesizing the two round gaming results, the final results are shown in Table 5.

Table 5 Results of final strategy

	Decision n	natrix	Power allocation	
Red	$S_{\rm R} = \begin{bmatrix} 0.271 \\ 0.148 \end{bmatrix}$	0.376 0.205	[216.8 W 118.4 W	300.8 W 164 W
Blue	$S_{\rm B} = \begin{bmatrix} 0.287 \\ 0.181 \end{bmatrix}$	0.288 0.244	[574 kW 362 kW	576 kW 488 kW

Corresponding with the equilibrium strategy in Table 5, we define marginal strategy as  $S_{\mathrm{M}} =$ 

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$
 and equipotent strategy as  $S_{\mathrm{E}} =$ 

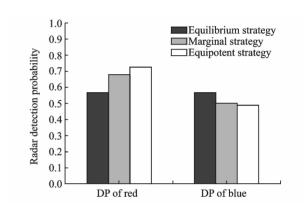
 $\begin{bmatrix} 0.25 & 0.25 \\ 0.25 & 0.25 \end{bmatrix}$ . To verify the optimality of the equilibrium strategy, a series of comparative trials have been conducted.

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Table 6 shows the comparative trials of typical strategies and Fig. 5 compares the detecting probabilities of all the trials.

Table 6 Comparative trials of typical strategies

Ī	Group	Red	Blue	Detecting probability
	1	$S_{ m R}$	$S_{ m B}$	0.567
	2	$S_{\mathrm{M}}$	$S_{\scriptscriptstyle  m B}$	0.679
	3	$S_{\rm E}$	$S_{ m B}$	0.725
	4	$S_{\mathrm{B}}$	$S_{\mathrm{M}}$	0.501
	5	$S_{ m B}$	$S_{\mathrm{E}}$	0.489



Bar chart of comparative trial results

In Fig. 5, compared with equilibrium strategy, both the marginal strategy and equipotent strategy will produce a higher detection probability for the red side, which means that it will be more easily detected by the blue radars when the red side adopts the marginal strategy or equipotent strategy. Accordingly, the blue side will get a worse detection performance when the marginal strategy or equipotent strategy is adopted. In fact, any strategy that differs from the equilibrium strategy can reach the same conclusion. Only the equilibrium strategy

$$(S_{\mathbf{R}}^*, S_{\mathbf{B}}^*) = \begin{pmatrix} \begin{bmatrix} 0.271 & 0.376 \\ 0.148 & 0.205 \end{bmatrix}, \begin{bmatrix} 0.287 & 0.288 \\ 0.181 & 0.244 \end{bmatrix} \end{pmatrix}$$

can meet the limits of

$$\begin{cases} P_{\mathrm{DK}_{n}}(S_{\mathrm{R}}^{*}, S_{\mathrm{B}}^{*}) \leqslant P_{\mathrm{DK}_{n}}(S_{\mathrm{R}}, S_{\mathrm{B}}^{*}) \ \forall \ S_{\mathrm{R}} \\ P_{\mathrm{DK}_{n}}(S_{\mathrm{R}}^{*}, S_{\mathrm{B}}^{*}) \geqslant P_{\mathrm{DK}_{n}}(S_{\mathrm{R}}^{*}, S_{\mathrm{B}}) \ \forall \ S_{\mathrm{B}} \end{cases}$$

Therefore, the equilibrium strategy shown in Table 5 is optimal in the strategy set of the constructed game model.

The comparison between allocating results of the red side's jamming power is shown in Fig. 6. It shows that both  $J_1$  and  $J_2$  allocate more jamming power to  $R_2$  rather than  $R_1$ . The reason is that  $R_2$  is closer to  $J_1$ ,  $J_2$  than  $R_1$  in the situation (as shown in Table 1) and  $R_2$  radiates more detection power to  $J_1$ ,  $J_2$  than  $R_1$  (as shown in Fig. 7), which indicates that the red side is apt to emit more jamming power to the radar with a bigger situational threat or energy threat. Similarly, the optimal strategy of the blue side makes  $R_1$ ,  $R_2$  emit more detection power to the fighter with a higher degree of situation threat or energy threat, as shown in Figs. 6,7. According to the above discussion, the game model can provide more practical approximation of the actual military conflict, and certainly it will provide a more reasonable decision.

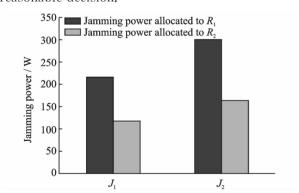


Fig. 6 Power allocation contrast of red side

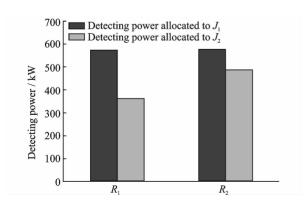


Fig. 7 Power allocation contrast of blue side

# 4 Conclusions

The collaborative management of power resources for offence-defense counterwork between

radar and jamming is addressed. A two-person general-sum continuous-game-based model is built for dynamic allocation of the power resources. With consideration of the situation and strategies the opposing side may take, the optimum solution can be more practical. To simplify the solving process, on the basis of a discussion about the model complexity, we work out a hierarchical processing method to promote computational efficiency by breaking the complicated model into numerous small one-dimensional games. With a contrastive analysis of the simulation results, we draw a conclusion that the equilibrium strategy of the game model can realize the optimal allocation of the power resources. When using the equilibrium strategy, more power will be allocated to the target that produces a higher degree of situational threat or energy threat, which means that the gamed-based model and the equilibrium strategy can provide a reasonable decision.

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