Joint Iterative Decoding for Network-Coding-Based Multisource LDPC-Coded Cooperative MIMO

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Abstract: A network-coding-based multisource LDPC-coded cooperative MIMO scheme is proposed, where multiple sources transmit their messages to the destination with the assistance from a single relay. The relay cooperates with multiple sources simultaneously via network-coding. It avoids the issues of imperfect frequency/timing synchronization and large transmission delay which may be introduced by frequency-division multiple access (FDMA)/code-division multiple access (CDMA) and time-division multiple access (TDMA) manners. The proposed joint "Min-Sum" iterative decoding is effectively carried out in the destination. Such a decoding algorithm agrees with the introduced equivalent joint Tanner graph which can be used to fully characterize LDPC codes employed by the sources and relay. Theoretical analysis and numerical simulation show that the proposed scheme with joint iterative decoding can achieve significant cooperation diversity gain. Furthermore, for the relay, compared with the cascade scheme, the proposed scheme has much lower complexity of LDPC-encoding and is easier to be implemented in the hardware with similar bit error rate (BER) performance.

Key words: cooperative MIMO; network coding; LDPC codes; equivalent joint Tanner graph; joint iterative decoding

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1 Introduction

Multiple-input multiple-output (MIMO) technology, in which both the transmitter and receiver of a communication link are equipped with multiple antennas, can significantly increase communication reliability. However, many mobile devices may not be able to support multiple antennas due to size or weight limit, etc. To overcome the issues in MIMO technology and maintain its benefits, cooperation techniques[1-3] have attracted much attention, via which the single-antenna mobiles can share the use of their antennas to form a virtual antenna array. For single-source single-relay cooperation system, three main categories of protocols that support cooperative communications have been introduced recently, i. e., the amplify-and-forward (AF)[1], detect-and-forward (DF)[2] and coded cooperation[3] protocols. In the AF protocol, the relay node simply amplifies signals received from the source to destination. In the DF protocol, the relay node first detects the noise-corrupted received signals by hard decision and then forwards the estimated signals to the destination. Generally, both AF and DF modes may not be suitable for the systems where high error-performance is strictly required in the destination. To further pursue higher transmission reliability, many previous studies[4-9] concentrated on these attractive topics of high-performance coded cooperation, in

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particular, using various low-density-parity-check codes (LDPC)[10]. They are widely employed by digital video broadcasting (DVB), worldwide interoperability for microwave access (WIMAX), and Wi-Fi standards. We refer to that coded cooperation protocol employing LDPC codes as LDPC-coded cooperation[11].

Here we focus on the so-called LDPC-coded cooperation with multiple sources rather than a single one. Compared with a single-source single-relay coded cooperation system, the multisource cooperation system faces new challenges. If the relay cooperates with multiple sources simultaneously by frequency-division multiple access (FDMA)/code-division multiple access (CDMA), it introduces the issue of imperfect frequency/timing synchronization. Similarly large transmission delay is inevitable in time-division multiple access (TDMA) manner. To overcome these challenges, network coding[12,13], a technique originally developed for routing in networks, has been recently used in multisource cooperation system[14-22]. In Ref. [14] a network coding approach to achieve cooperation diversity is proposed, where the partner transmits the algebraic superposition of the locally generated codeword and the relayed codeword. A kind of space-time network coding is adopted in Refs. [15,16] to achieve full spatial diversity with low transmission delay and eliminate the issue of imperfect synchronization. In Ref. [18] the authors introduce a complex field network coding for multiuser cooperative communications, via which full diversity gain can be achieved regardless of the underlying signal-to-noise ratio (SNR) and the constellation used. However, the complexity of encoding and decoding is too high. Relative to complex field network coding, the complexity of network coding in Galois field (GF) is much lower. In Refs. [20, 21] the joint designs of network coding and channel coding in GF are considered. Another advantage of network coding in GF is that it is practical to implement the joint decoding for the network coding and channel coding in the destination. To efficiently perform the joint decoding for network-coding-based LDPC-coded cooperation is the main contribution of this paper, which is rarely discussed in the previous literatures.

In this paper, the network-coding-based multisource LDPC-coded cooperation is formulated in the following scenario. As mobile users, the sources and relay have a single antenna; while as a base station, the destination is equipped with multiple receiving antennas. This kind of cooperation model is known as the cooperative MIMO[23,24], which can achieve the receiving diversity as well as cooperation diversity gains. We consider the network-coding-based multisource LDPC-coded cooperative MIMO over Rayleigh fading channels, where multiple sources transmit their messages to the destination with the assistance from a single relay. The equivalent overall parity-check matrix in view of the destination is obtained based on the network-coding scheme, and the equivalent joint Tanner graph corresponding to the overall parity-check matrix is described in detail. A novel joint iterative decoding algorithm is introduced to decode multiple detected signals coming from the sources and relay in the destination based on the proposed equivalent joint Tanner graph that can fully characterize all the individual LDPC codes adopted by the sources and relay.

2 System Description

A network-coding-based multisource LDPC-coded cooperative MIMO with K sources, one relay and one destination is shown in Fig. 1. As mobile users, the sources (S) and relay (R) have single antenna; while as a base station, the destination (D) is equipped with L receiving antennas. Sources S_1, …, S_K broadcast their messages encoded by LDPC-1, …, LDPC-K to the relay and destination by TDMA manner. After receiving and decoding all the messages from sources, firstly the relay R combines the decoded messages by network coding scheme which is denoted as bitwise exclusive OR (XOR) operation in the Galois field GF(2), then R encodes the combined messages by LDPC-R to generate parity-check bits, which are finally transmitted to D. It is known that $K+1$ time slots are needed to accomplish the transmitting period, however, for the separate processing by TDMA manner without network coding in the relay, $2\times K$ time slots are required.

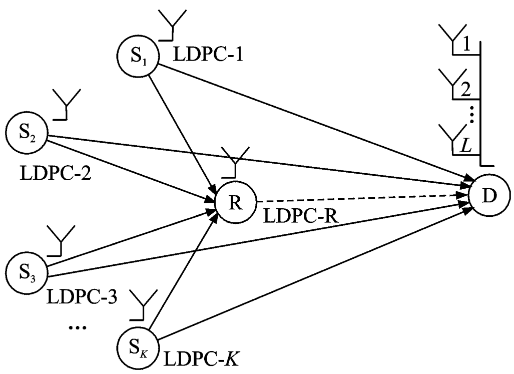

Fig. 1 Network-coding-based multisource LDPC-coded cooperative MIMO

Assume that S-R channels linking sources and relay R are ideal, which implies that R can inerrably recover all the messages from sources. Suppose all messages over S-D and R-D channels suffer from independent Rayleigh block fading and complex additive white Gaussian noise (AWGN), where channel gains are independent and identically distributed (IID) zero-mean unit variance circularly symmetric complex Gaussian random variables.

3 Encoding Scheme for Network-Coding-Based Multisource LDPC-Coded Cooperative MIMO

In this section, encoding scheme for network-coding-based multisource LDPC-coded cooperative MIMO that employs regular LDPC codes is designed, and the overall parity-check matrix in view of the decoder in the destination is obtained. Here, the superscript "T" denotes the transpose of a matrix/vector, and $\mathbf{0}_{M\times N}$ is an $M\times N$ zero matrix.

3.1 Encoding scheme for sources

Assume that $S_1,\cdots,S_K$ transmit independent messages. For $S_k(k=1,\cdots,K)$, a block of information bits is encoded into a codeword

$$\boldsymbol{c}_k=(c_1^{(k)},\cdots,c_N^{(k)})^{\mathrm{T}} \tag{1}$$

by encoder LDPC-k, which is defined by its sparse parity-check matrix $(\boldsymbol{H}_k)_{M_1\times N}$. The degree distributions for the variable and check nodes are

$$\boldsymbol{\lambda}(x)=x^{d_v-1},\ \boldsymbol{\rho}(x)=x^{d_c-1} \tag{2}$$

where each bit of the codeword participates in exactly d_v parity-check equations and each such equation exactly involves d_c codeword bits. Sources $S_1,\cdots,S_K$ broadcast $\boldsymbol{c}_1,\cdots,\boldsymbol{c}_K$ to R and D by TDMA manner, respectively.

3.2 Encoding scheme for relay

In the proposed network-coding-based multisource LDPC-coded cooperative MIMO, the encoding scheme for the relay is depicted in Fig. 2, which includes three steps as follows.

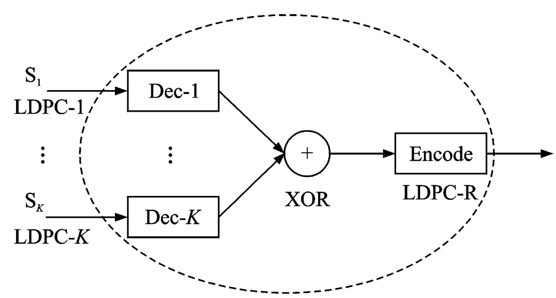

Fig. 2 Encoding scheme for relay in the proposed network-coding-based multisource LDPC-coded cooperative MIMO

Step 1 R decodes the contaminated messages from the sources, and the codewords $\boldsymbol{c}_1,\cdots,\boldsymbol{c}_K$ are recovered.

Step 2 R operates the recovered codewords $\boldsymbol{c}_1,\cdots,\boldsymbol{c}_K$ with bitwise exclusive XOR operator denoted as $\oplus$, and the new corresponding codeword $\boldsymbol{c}'$ is achieved.

$$\boldsymbol{c}'=\boldsymbol{c}_1\oplus\cdots\oplus\boldsymbol{c}_K=(c'_1,\cdots,c'_N)^{\mathrm{T}} \tag{3}$$

Step 3 As information bits, the whole block of achieved bits $\boldsymbol{c}'$ is encoded by LPDC-R to generate a new codeword

$$\boldsymbol{c}_{\mathrm{R}}=(c'_1,\cdots,c'_N,p_1,\cdots,p_{M_2})^{\mathrm{T}} \tag{4}$$

by appending M_2 additional parity-check bits $\boldsymbol{p}=(p_1,\cdots,p_{M_2})^{\mathrm{T}}$. The parity-check matrix of LDPC-R is given as

$$\boldsymbol{H}_{\mathrm{R}}=[\boldsymbol{A}_{M_2\times N}\quad \boldsymbol{B}_{M_2\times M_2}] \tag{5}$$

whose degree distributions for the variable and check nodes are $\bar{\lambda}(x)=x^{\bar{d}_v-1}$ and $\bar{\rho}(x)=x^{\bar{d}_c-1}$, respectively, with $\bar{d}_v$ and $\bar{d}_c$ having the similar meaning as d_v and d_c in Eq. (2). To ensure that $\boldsymbol{c}_{\mathrm{R}}$ is systematic as Eq. (4), the rank of sub-matrix $\boldsymbol{B}_{M_2 \times M_2}$ should be M_2, i. e., $\mathrm{rank}(\boldsymbol{B}_{M_2 \times M_2}) = M_2$. The parity-check bits $\boldsymbol{p}$ of the codeword $\boldsymbol{c}_{\mathrm{R}}$ are sent to the destination.

3.3 Equivalent overall parity-check matrix in view of destination

Subjected to the constraint between the parity-check matrix and the codeword, the following equations are given as

$$\boldsymbol{H}_k \boldsymbol{c}_k = \boldsymbol{0} \quad (k = 1, \cdots, K) \tag{6}$$

$$\boldsymbol{H}_{\mathrm{R}} \boldsymbol{c}_{\mathrm{R}} = [\boldsymbol{A} \quad \boldsymbol{B}] \begin{bmatrix} \boldsymbol{c}' \\ \boldsymbol{p} \end{bmatrix} = \boldsymbol{A}\boldsymbol{c}' + \boldsymbol{B}\boldsymbol{p} = \boldsymbol{0} \tag{7}$$

where the subscript of the matrix indicating its size is ignored.

It is noticed that "+" in the GF(2) is equivalent to "⊕".

$$\boldsymbol{A}\boldsymbol{c}' = \boldsymbol{A}(\boldsymbol{c}_1 \oplus \cdots \oplus \boldsymbol{c}_K) = \boldsymbol{A}\boldsymbol{c}_1 \oplus \cdots \oplus \boldsymbol{A}\boldsymbol{c}_K \tag{8}$$

Hence, Eq. (7) is equal to

$$[\boldsymbol{A} \quad \cdots \quad \boldsymbol{A} \quad \boldsymbol{B}]\boldsymbol{c} = \boldsymbol{0} \tag{9}$$

where $\boldsymbol{c} = [\boldsymbol{c}_1^{\mathrm{T}} \cdots \boldsymbol{c}_K^{\mathrm{T}}\ \boldsymbol{p}^{\mathrm{T}}]^{\mathrm{T}}$ is the overall codeword in view of the destination.

According to Eqs. (6, 7, 9), the equivalent constraint relationship between the parity-check matrix and the codeword can be rewritten as

$$\begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_2 & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{H}_K & \boldsymbol{0} \\ \boldsymbol{A} & \boldsymbol{A} & \cdots & \boldsymbol{A} & \boldsymbol{B} \end{bmatrix} \boldsymbol{c} = \boldsymbol{0} \tag{10}$$

Finally, the equivalent overall parity-check matrix corresponding to the overall codeword in view of the destination is obtained as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_2 & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{H}_K & \boldsymbol{0} \\ \boldsymbol{A} & \boldsymbol{A} & \cdots & \boldsymbol{A} & \boldsymbol{B} \end{bmatrix} \tag{11}$$

Assuming that the code rate of LDPC-k is r_k, the overall code rate in view of the destination can be calculated as

$$r = \frac{\sum_{k=1}^{K} r_k N}{KN + M_2} \tag{12}$$

The efficient joint iterative decoding algorithm is performed in the destination based on the equivalent joint Tanner graph, which is related to the obtained overall parity-check matrix.

4 Joint Min-Sum Iterative Decoding Based on Equivalent Joint Tanner Graph

4.1 Equivalent joint Tanner graph

To understand in-depth the proposed equivalent joint Tanner graph structure that is directly related to the subsequent analysis on the joint iterative decoding for network-coding-based multi-source LDPC-coded cooperative MIMO, we present a simple example of block codes with short lengths, which have the similar Tanner graph structure as the LDPC codes.

Example 1 For simplicity, assume $K = 2$, which means there are only two sources S_1, S_2 in Fig. 1. Three block codes $\boldsymbol{C}_1$, $\boldsymbol{C}_2$ and $\boldsymbol{C}_{\mathrm{R}}$, defined by the following parity-check matrices $\boldsymbol{H}_1$, $\boldsymbol{H}_2$ and $\boldsymbol{H}_{\mathrm{R}}$ are employed by S_1, S_2 and R, respectively.

$$\boldsymbol{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$
$$\boldsymbol{H}_2 = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{13}$$

$$\boldsymbol{H}_{\mathrm{R}} = [\boldsymbol{A} \quad \boldsymbol{B}] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The equivalent overall parity-check matrix corresponding to the overall codeword in view of the destination is calculated as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_2 & \boldsymbol{0} \\ \boldsymbol{A} & \boldsymbol{A} & \boldsymbol{B} \end{bmatrix} \tag{15}$$

From the viewpoint of the decoder in the destination, the equivalent joint Tanner graph related to $\boldsymbol{H}_1$, $\boldsymbol{H}_2$, $\boldsymbol{H}_{\mathrm{R}}$ is shown in Fig. 3.

As shown in Fig. 3, there are three layers in the equivalent Tanner graph, i. e. the first, the second and the third layers (for R) corresponding to the block codes $\boldsymbol{C}_1$, $\boldsymbol{C}_2$ and $\boldsymbol{C}_R$, respectively. Clearly, the check nodes $c_m^{(1)}$ ($m=1, 2, 3$) and $c_m^{(2)}$ ($m=1, 2, 3$) in the first and the second layers are only related to the variable nodes $v_n^{(1)}$ ($n=1, \cdots, 6$) and $v_n^{(2)}$ ($n=1, \cdots, 6$) in their own layers, respectively. However, in the third layer, the check nodes $c_m^{(3)}$ ($m=1, 2, 3$) are not only related to the variable nodes $v_n^{(3)}$ ($n=1, 2, 3$) in its own layer but also related to $v_n^{(1)}$ ($n=1, \cdots, 6$) and $v_n^{(2)}$ ($n=1, \cdots, 6$) in the layers corresponding to two sources. It should be noted that for a certain $c_m^{(3)}$, which is related to one variable node $v_n^{(1)}$, it must also be related to $v_n^{(2)}$. In other word, each $c_m^{(3)}$ has the same relationship with all the layers corresponding to the sources. For example, $c_1^{(3)}$ is related to $v_1^{(1)}$, $v_4^{(1)}$ and $v_1^{(2)}$, $v_4^{(2)}$ simultaneously.

The aforementioned equivalent joint Tanner graph for the block codes adopted in the proposed network-coding-based two-source coded cooperative MIMO can be easily extended to more complicated scenarios of network-coding-based multisource LDPC-coded cooperative MIMO as described in Section 3.

Fig. 4 illustrates the equivalent joint Tanner graph corresponding to the overall parity-check matrix in Eq. (11) for network-coding-based K-source LDPC-coded cooperative MIMO in view of the decoder in the destination. The kth layer ($k=1, \cdots, K$) is corresponding to $\boldsymbol{H}_k$ ($k=1, \cdots, K$)

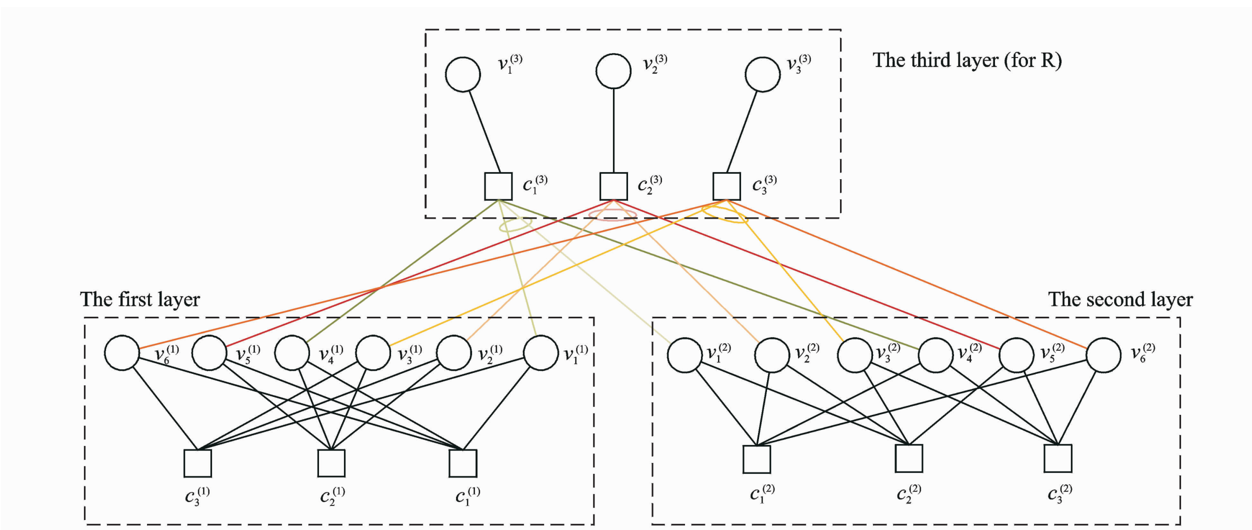

Fig. 3 Equivalent joint Tanner graph used to characterize the overall parity-check relationship of network coding-based two-source coded cooperative MIMO using three block codes $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, $\boldsymbol{C}_R$

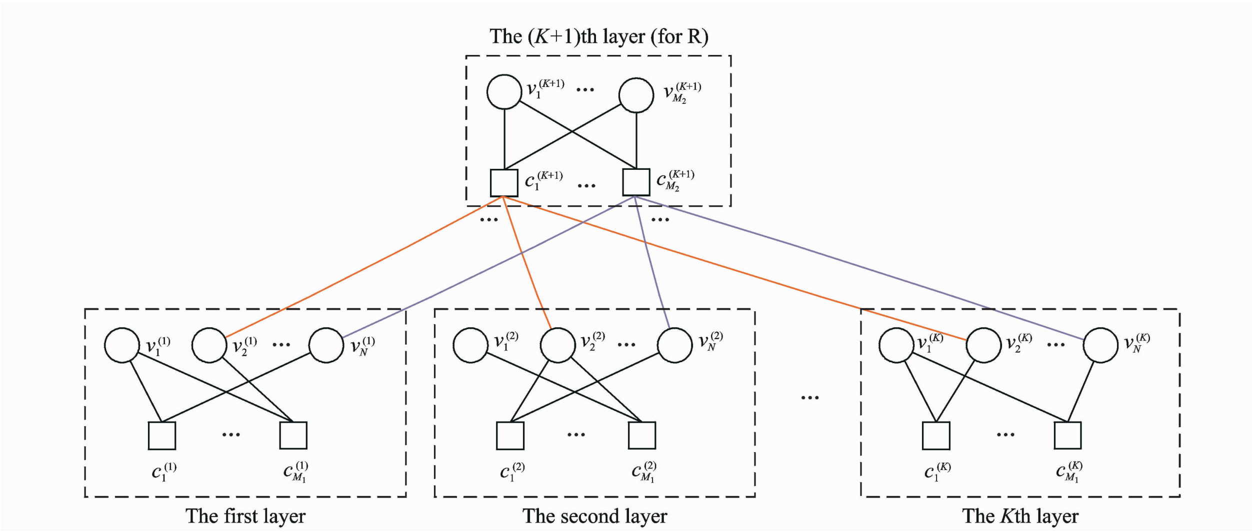

Fig. 4 Equivalent joint Tanner graph used to characterize the overall parity-check relationship of network-coding-based multisource LDPC-coded cooperative MIMO

which characterizes LDPC-k ($k=1, \cdots, K$) employed by source S_k ($k=1, \cdots, K$), and the ($K+1$)th layer is corresponding to $\boldsymbol{H}_R$ which characterizes LDPC-R used by relay R.

A new decoding approach based on the described equivalent joint Tanner graph is implemented in the destination to perform the joint iterative decoding for all signals from the sources and relay. The extrinsic information obtained from the variable and check nodes in the equivalent joint Tanner graph is exchanged sufficiently during each iteration step, which differs greatly from the traditional approach that decodes the received signals successively[7].

4.2 Joint Min-Sum iterative decoding based on equivalent joint Tanner graph

Standard belief propagation (BP) is the best performing, yet the most complex algorithm for decoding LDPC codes. To balance the performance and complexity, another iterative decoding algorithm named "Min-Sum"[25] is widely used in practice.

In this paper, we propose a joint "Min-Sum" iterative decoding in the destination based on the equivalent joint Tanner graph, which is virtually an incorporated Tanner graph associated with the multiple LDPC codes used by the sources and relay.

In the equivalent joint Tanner graph shown in Fig. 4, all the variable nodes in $K+1$ layers form the set $\{v_n: n=1, \cdots, KN+M_2\}$, where $v_n^{(i)}$ is noted as $v_{(i-1)N+n}$ to describe the algorithm concisely; and the sets of the check nodes corresponding to sources and relay are $\{c_m^{(i)}: m=1, \cdots, M_1\}$ ($i=1, \cdots, K$) and $\{c_m^{(K+1)}: m=1, \cdots, M_2\}$, respectively. The set $C(v_n)$ contains all the check nodes in the joint Tanner graph related to the variable node v_n; $V(c_m^{(i)})$ ($i=1, \cdots, K+1$) is the set of all variable nodes associated with $c_m^{(i)}$. Assume maximal-ratio combining (MRC) as a special case of matrix matched filter (MMF) is adopted to detect the received sequences of multiple antennas in the destination, and the output sequences of that MRC associated with the codewords $\boldsymbol{c}_1, \cdots, \boldsymbol{c}_K$ transmitted by sources and the codeword $\boldsymbol{p}$ transmitted by relay are $\boldsymbol{y}^{(1)}, \cdots, \boldsymbol{y}^{(K)}$ and $\boldsymbol{y}^{(K+1)}$, respectively. Let $\boldsymbol{y}=(\mathrm{Re}(\boldsymbol{y}^{(1)}), \cdots, \mathrm{Re}(\boldsymbol{y}^{(K)}), \mathrm{Re}(\boldsymbol{y}^{(K+1)}))$, which is the direct input of the joint iterative decoding.

Define $Lq_{m,n}^{(i)}$ as the extrinsic information from a variable node v_n to an incident check node $c_m^{(i)}$ and $Lr_{m,n}^{(i)}$ the extrinsic information from a check node $c_m^{(i)}$ to an incident variable node v_n. Following the equivalent joint Tanner graph, the joint "Min-Sum" iterative decoding algorithm is summarized in the following.

Preparations Initially, the decoder in the destination only acquires the received signals, and does not have any a prior information from the check nodes. For binary phase shift keying (BPSK), each bit n is assigned a log-likelihood ratio (LLR) as

$$Lp_n = \log \frac{P(c_n=0 \mid y_n)}{P(c_n=1 \mid y_n)} = \frac{4g}{N_0} y_n$$
$$n=1, \cdots, KN+M_2 \tag{16}$$

where g is the gain of Rayleigh fading channel after MRC and N_0 is the variance of the complex additive noise. The factor $4g/N_0$ is a fixed positive number which can be ignored during the joint iteration for the joint "Min-Sum" algorithm. Hence, Lp_n in Eq. (16) can be further evaluated as

$$Lp_n = y_n \quad (n=1, \cdots, KN+M_2) \tag{17}$$

Step 1 (Initialization) Before commencing the iterative decoding, $Lq_{m,n}^{(i)}$ can be initialized as Lp_n in Eq. (17).

Step 2 (Horizontal process) The extrinsic information $Lr_{m,n}^{(i)}$ sent from a check node $c_m^{(i)}$ in the ith layer to an incident variable node v_n is evaluated as

$$Lr_{m,n}^{(i)} = \Big(\prod_{v_f \in V(c_m^{(i)}) \setminus v_n} \mathrm{sign}(Lq_{m,f}^{(i)})\Big) \times \Big(\min_{v_f \in V(c_m^{(i)}) \setminus v_n} (\mid Lq_{m,f}^{(i)} \mid)\Big) \tag{18}$$

where sign(•) and min(•) are the sign function and minimal function, respectively. The updated extrinsic information $Lr_{m,n}^{(i)}$ from the check nodes $c_m^{(i)}$ in the joint Tanner graph is obtained.

Example 2 The extrinsic information $Lr_{1,4}^{(3)}$

sent from the check node $c_1^{(3)}$ to the variable node $v_4^{(1)}(v_4)$ in Example 1 is shown in Fig. 5.

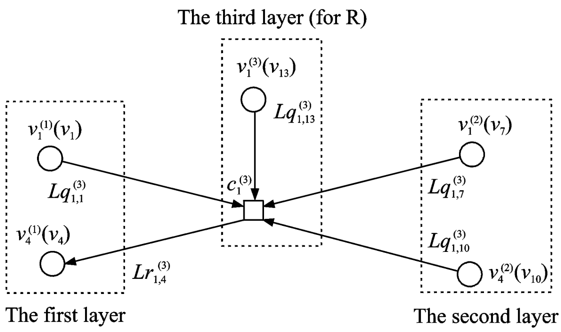

Fig. 5 Updated extrinsic information $Lr_{1,4}^{(3)}$ from the check node $c_1^{(3)}$ to the variable node $v_4^{(1)}(v_4)$ in Example 1 as shown in Fig. 3

The updated $Lr_{1,4}^{(3)}$ is relevant to the input $Lq_{1,1}^{(3)}$ from the first layer, $Lq_{1,7}^{(3)}$, $Lq_{1,10}^{(3)}$ from the second layer, and $Lq_{1,13}^{(3)}$ from the third layer. Since different layers are corresponding to signals over different fading channels, this can achieve better diversity than the non-joint decoding scheme.

Step 3 (Vertical process) Update the extrinsic information $Lq_{m,n}^{(i)}$ sent from a variable node v_n to an incident check node $c_m^{(i)}$ in the ith layer.

$$Lq_{m,n}^{(i)} = Lp_n + \sum_{c_l^{(i)} \in C(v_n)\backslash C_m^{(i)}} Lr_{l,n}^{(i)} + \sum_{\substack{j=1 \\ j\neq i}}^{K+1} \sum_{c_s^{(j)} \in C(v_n)} Lr_{s,n}^{(j)} \tag{19}$$

Step 4 (Final decision) Repeat Steps 2, 3 until reach the maximum number of decoding iterations. The a posterior LLR concerning each codeword bit is calculated as

$$R_n = Lp_n + \sum_{i=1}^{K+1} \sum_{C_m^{(i)} \in C(v_n)} Lr_{m,n}^{(i)}$$
$$n = 1, \cdots, KN + M_2 \tag{20}$$

Therefore, the final decoded block of $KN + M_2$ bits is obtained as

$$\hat{c}_n = \begin{cases} 0 & R_n \geqslant 0 \\ 1 & R_n < 0 \end{cases} \quad (n = 1, \cdots, KN + M_2) \tag{21}$$

4.3 Comparison of joint and traditional Min-Sum iterative decoding methods for network-coding-based multisource LDPC-coded cooperative MIMO

In network-coding-based multisource LDPC-coded cooperative MIMO, the traditional decoding method[7] in the destination has to employ multiple decoders for the received signals from sources and relay. Hence, the traditional decoding method increases the decoding delay, and the decoding processing in the destination is more complicated due to multiple decoders. In the proposed joint iterative decoding, only one decoder is needed in the destination, and the extrinsic information obtained from the variable and check nodes in the equivalent joint Tanner graph is exchanged sufficiently in each iteration step, which can accelerate the decoding convergence and reduce the iteration time compared with the traditional one.

5 Simulation Results

Numerical simulations are performed to investigate the performance of the network-coding-based multisource LDPC-coded cooperative MIMO employing the proposed joint "Min-Sum" iterative decoding algorithm in the destination. S-D and R-D are Rayleigh block fading channels with perfect channel state information (CSI) in the destination. The fading coefficient for each channel remains constant over each codeword. The average received per bit per antenna SNRs of the signals from sources and relay are equal, and BPSK modulation is assumed in the simulations.

Random rate $-1/2$ regular LDPC codes with the length $N = 1\ 000$, whose degree distributions for variable and check nodes are $\lambda(x) = x^2$ and $\rho(x) = x^5$, are employed by the sources. Rate $-2/3$ regular LDPC codes with the length $N_R = 1\ 500$, whose degree distributions for variable and check nodes are $\lambda(x) = x^2$ and $\rho(x) = x^8$, are adopted by the relay. Particularly, the LDPC codes for the relay are systematic as described in Section 3.

5.1 Network- coding-based multisource LDPC-coded cooperative MIMO v. s. noncooperation

For comparing under the same condition, in view of the destination, equal LDPC codes as the proposed scheme are employed by the noncooperation scheme. Fig. 6 compares the BER performances of the network-coding-based LDPC-coded cooperative MIMO with two sources ($K=2$) and coded noncooperation under the identical condition of three receiving antennas ($L=3$). The results show that the performance of the proposed cooperative MIMO scheme clearly outperforms the coded noncooperation in all ranges of SNR. For instance, at the BER of 2×10^{-3} with five decoding iterations, the proposed scheme achieves about 1.4 dB gain over its respective coded noncooperation one. The significant gain is due to the fact that three detected signals from two sources and the relay, which are through independent fading channels, are jointly decoded by the proposed high-efficient joint "Min-Sum" iterative decoding algorithm. Hence, it can dramatically overcome the signals fading to achieve the diversity gain.

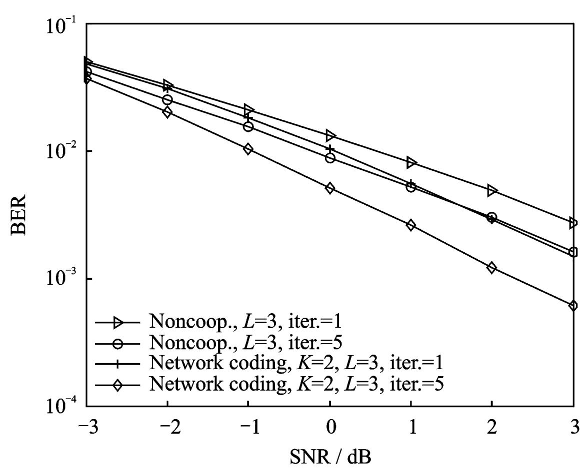

Fig. 6 BER comparison of network-coding-based LDPC-coded cooperative MIMO and noncooperation schemes with $L=3$ receiving antennas and various decoding iterations

5.2 Network- coding-based multisource LDPC-coded cooperative MIMO with various numbers of receiving antennas

To investigate the receive diversity of multiple antennas in network-coding multisource LDPC-coded cooperative MIMO, various numbers of receiving antennas are applied in the destination. BER performances of the proposed cooperative MIMO with two sources are shown in Fig. 7,

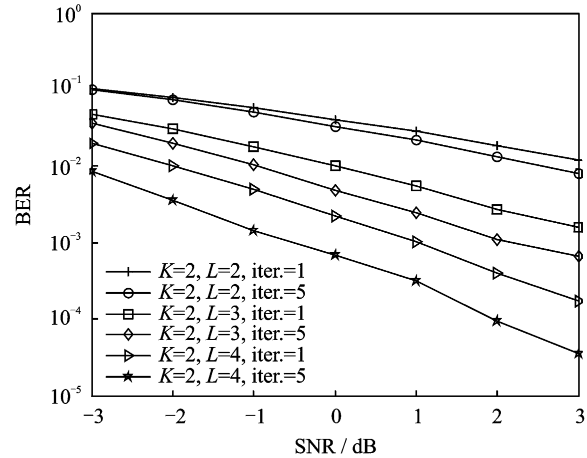

Fig. 7 BER performance of network-coding-based LDPC-coded cooperative MIMO schemes with two sources

where numbers of receiving antennas are 2, 3 and 4, respectively. Significant gains can be obtained with the increasing number of receiving antennas. For instance, at the BER of 10^{-2} and five joint decoding iterations, about 3, −1, and −3 dB of SNR are required for 2, 3 and 4 receiving antennas, respectively. This merit can be credited to the fact that the more the receiving antennas, the more the receive spatial diversity gain can be achieved in the proposed scheme.

5.3 Network- coding-based multisource LDPC-coded cooperative MIMO with various numbers of sources

In this part, we investigate the BER performance of the network-coding-based LDPC-coded cooperative MIMO with various sources. Fig. 8 depicts the BER curves for the proposed scheme with the number of sources $K=2, 3, 4$ under the conditions of $L=2$ receiving antennas and five joint decoding iterations in the destination. In view of the destination, the overall code rates corresponding to $K=2, 3, 4$ are 2/5, 3/7, 4/9, respectively. It is shown in Fig. 8 that the BER performance slightly decreases as the number of sources increases. For instance, the BER performance in the three-source case is inferior to that in the two-source case and slightly superior to that in the four-source case. The reason relates to the fact that the overall code rate becomes lar-

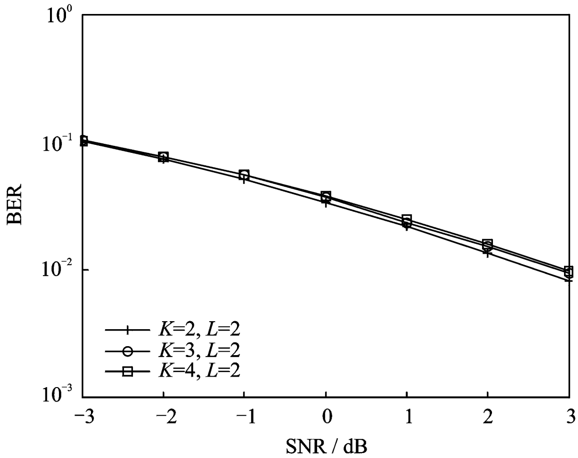

Fig. 8 BER performances of network-coding-based LDPC-coded cooperative MIMO schemes with $K=2, 3, 4$ sources

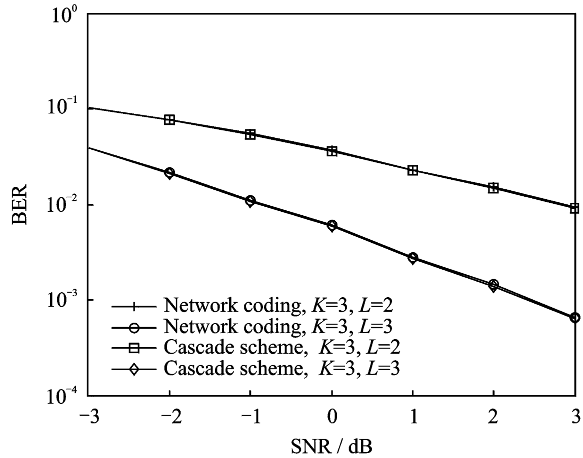

Fig. 9 BER comparison of network-coding-based LDPC-coded cooperative MIMO and the cascade schemes with the number of sources $K=3$

ger with the number of sources increasing, which results in the BER performance losses. It indicates that for one relay there is a tradeoff between the number of sources that it cooperates with and the transmission reliability that can be achieved. This tradeoff should be considered according to the actual situation.

5.4 The proposed scheme v. s. the cascade scheme

In Ref. [26] another encoding scheme is proposed, where the relay firstly cascades the recovered codewords from sources, and then encodes the cascaded message to generate parity-check bits. It is called cascade scheme here, which is much different from the proposed scheme in this paper, where the relay first operates the recovered codewords from the sources with XOR, and then encodes the XORed message. In this part, we compare these two schemes in the identical scenario as the proposed scheme in this paper. For the cascade scheme, LDPC codes for the sources are the same as the proposed scheme, and LDPC codes for the relay depends on the number of sources K (For example, $K=2$, rate $-4/5$ regular LDPC codes with length $N=2\ 500$ and degree distributions $\lambda(x)=x^2$, $\rho(x)=x^{14}$; $K=3$, rate $-6/7$ regular LDPC codes with length $N=3\ 500$ and degree distributions $\lambda(x)=x^2$, $\rho(x)=x^{20}$). Fig. 9 compares the BER performance of the proposed network-coding-based LDPC-coded cooperative MIMO and the cascade schemes with the number of sources $K=3$, under the same conditions of $L=2$, 3 receiving antennas and five joint decoding iterations in the destination. It is shown that the proposed scheme achieves the similar BER performance of the cascade scheme with the same receiving antennas. It can be explained by the two facts as follows: (1) In these two schemes, the overall code rates in view of the destination are equal; (2) In these two schemes, the four received signals corresponding to the overall codeword are from four independent fading channels (three from the sources and one from the relay, respectively). Hence, these two schemes can achieve the same diversity gain due to the joint iterative decoding.

It should be noticed that for the relay, the complexity of LDPC encoding in these two schemes is much different. Considering encoding by Gaussian elimination method, it can be easily obtained that bringing the sparse parity-check matrix into the desired lower triangular form requires $O(N_R^3)$ operations of preprocessing, and the actual encoding requires other $O(N_R^2)$ operations on the non-sparse triangular matrix, where $O(N_R^3)$ and $O(N_R^2)$ mean that the numbers of operations are cubic and quadratic with N_R, respectively. Here N_R is the length of LDPC code employed by the relay and the operations "+" or "×" in the GF(2). For the cascade scheme, N_R increases with the number of sources K, where $N_R = NK + M_2 = 1\,000K + 500$ ($K=3$, $N_R = 3\,500$), however, for the proposed scheme, $N_R = N + M_2$ is fixed at 1 500. The proposed scheme sharply decreases the encoding complexity compared with the cascade scheme. Furthermore, the proposed scheme is much easier to be implemented in the hardware than the cascade scheme, especially in the case of large K.

6 Conclusions

A network-coding-based multisource LDPC-coded cooperative MIMO scheme is investigated over Rayleigh block fading channels. The proposed scheme well combines cooperation diversity, multi-receive diversity and channel-coding gains. By the features of the operations in the GF(2), the equivalent overall parity-check matrix corresponding to the overall codeword in view of the destination is obtained based on the network-coding and LDPC-encoding. A novel joint iterative decoding algorithm is adopted to decode multiple detected signals coming from the sources and relay in the destination based on the equivalent joint Tanner graph related to the obtained equivalent overall parity-check matrix. Theoretical analysis and numerical simulation have demonstrated the superiorities of the proposed scheme.

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