

# Performance of Cross-Layer Design with Power Allocation in Space-Time Coded MIMO Systems

Yang Dingli(杨定礼)<sup>1\*</sup>, Liu Yan(刘岩)<sup>2</sup>, Yu Xiangbin(虞湘宾)<sup>2</sup>

1. Faculty of Electronic and Electrical Engineering, Huaiyin Institute of Technology, Huaian, 223003, P. R. China;

2. College of Electronic Information Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, 210016, P. R. China

(Received 14 November 2013; revised 9 May 2014; accepted 14 May 2014)

**Abstract:** A cross-layer design (CLD) scheme with combination of power allocation, adaptive modulation (AM) and automatic repeat request (ARQ) is presented for space-time coded MIMO system under imperfect feedback, and the corresponding system performance is investigated in a Rayleigh fading channel. Based on imperfect feedback information, a suboptimal power allocation (PA) scheme is derived to maximize the average spectral efficiency (SE) of the system. The scheme is based on a so-called compressed SNR criterion, and has a closed-form expression for positive power allocation, thus being computationally efficient. Moreover, it can improve SE of the presented CLD. Besides, due to better approximation, it obtains the performance close to the existing optimal approach which requires numerical search. Simulation results show that the proposed CLD with PA can achieve higher SE than the conventional CLD with equal power allocation scheme, and has almost the same performance as CLD with optimal PA. However, it has lower calculation complexity.

**Key words:** cross-layer design (CLD); power allocation (PA); space-time coding; spectral efficiency (SE); packet error rate (PER)

**CLC number:** TN913.3

**Document code:** A

**Article ID:** 1005-1120(2014)04-0431-09

## 1 Introduction

With the fast development of modern communication techniques, the demand for high data rate service is grown increasingly in the limited radio spectrum. For this reason, the future wireless communication system will require spectrally efficient techniques to increase the system capacity. Cross-layer design (CLD), as a good work to improve the spectral efficiency (SE) and system throughput while meeting the prescribed quality of service requirements, has received much attention recently<sup>[1]</sup>. Especially, CLD combining adaptive modulation (AM) and automatic repeat request (ARQ) is widely accepted as an efficient means to improve the overall performance of transmission in fading channels<sup>[2-8]</sup>. Multiple an-

tennas approach is another well known SE technique with diversity and/or coding gain<sup>[9-13]</sup>. Therefore, effective combination of cross-layer design and multiple antenna techniques has received much attention<sup>[2-6]</sup>.

A CLD combined adaptive modulation and coding at the physical layer and ARQ protocol at the data link layer over single antenna Nakagami-m fading channels is developed in Ref. [2]. Based on the principle of cross-layer design in Ref. [2], the CLD schemes with space-time block coding (STBC) are presented for multiple input and multiple output (MIMO) systems in Refs. [3-6] over different fading channels. The performance of CLD scheme with antenna selection (CLD-AS) is analyzed over MIMO Nakagami-m fading channel in Ref. [7]. CLD-AS can obtain higher SE

**Foundation items:** Supported by the Foundation of Huaian Industrial Projects (HAG2013064); the Foundation of Huaiyin Institute of Technology (HGB1202); the Doctoral Fund of Ministry of Education of China (20093218120021).

\* **Corresponding author:** Yang Dingli, Lecturer, E-mail: yangdingli@163.com.

than the space-time coded CLD scheme, but it is more sensitive to the imperfect channel state information (CSI) than the latter. Under the feedback constraint, a CLD scheme is presented for multiuser MIMO systems, and the corresponding performance is investigated in Rayleigh fading channel<sup>[8]</sup>.

In all these studies, CLD schemes are basically based on the perfect CSI, whereas in practice, CSI will be far from perfect due to imperfect feedback. Moreover, these schemes do not exploit the channel feedback information effectively, and often employ equal power allocation (PA) for data transmission, thus the system performance is limited. For this reason, the system should take imperfect CSI into account, and exploit the imperfect CSI to design CLD and adaptive PA scheme. Thus, some superior performance can be obtained. In Refs. [14–15], the optimal PA schemes for minimizing the upper bound of the pair error probability (PEP) are developed for MIMO systems with STBC and beamforming. A suboptimal PA scheme based on the criterion of minimizing an upper bound on the symbol error rate is proposed in Ref. [16]. An adaptive PA scheme is presented in Ref. [17] to maximize SNR at the receiver under perfect feedback. Based on mean or covariance feedback, the power allocation for approaching the ergodic capacity of a MIMO channel is proposed to achieve near-optimal capacity performance<sup>[18]</sup>. A linear precoder which minimizes an upper bound of the average PEP is designed for the orthogonal STBC system based on non-zero mean feedback in Ref. [19]. The above-mentioned power control algorithms basically need numerical search for the value of the Lagrange multiplier of the constrained optimization and iterative calculations to determine the number of eigenbeams of positive power. Therefore, the calculation complexity is much higher. Moreover, the above PA schemes are suitable for the MIMO system with fixed modulation mode only. Namely, these schemes do not consider the superiority of adaptive modulation and cross-layer design, thus the SE improvement is limited.

Based on the reasons above, we will develop a CLD scheme with power allocation in space-time coded MIMO system under imperfect feedback, and investigate the spectrum efficiency and packet error rate (PER) performance in Rayleigh fading channel. By exploiting the delayed CSI at the transmitter, a suboptimal power allocation scheme for maximizing the average spectral efficiency (ASE) of system is derived. The scheme is based on a compressed SNR (CSNR) criterion, where a single compression factor is used to minimize PER and can be determined analytically. The derivation yields a closed-form power allocation which is shown to be applicable in the whole range of SNR of interest. Moreover, the derived power allocations are always positive, and thus they avoid the numerical search and iteration of the existing optimal scheme to find the positive power. Namely, the developed PA can make the calculation of power coefficients become straightforward due to its closed-form expression. With the PA scheme, the presented CLD will obtain higher ASE than the conventional CLD with equal power, and it has ASE similar to CLD with optimal PA scheme due to better approximation.

Throughout the paper, the superscripts  $(\cdot)^H$ ,  $(\cdot)^T$  and  $(\cdot)^*$  denote the Hermitian transposition, transposition and complex conjugation, respectively.  $\mathbf{A}_{ij}$  denotes the element in the  $i$ th row and the  $j$ th column of matrix  $\mathbf{A}$ . "CN" denotes complex Gaussian distribution.  $\mathbf{I}_n$  represents the  $n \times n$  identity matrix.

## 2 System Model

In this section, we will give a cross-layer design combined adaptive modulation at the physical layer and ARQ at the data link layer for space-time coded MIMO system with power allocation. The MIMO system is equipped with  $N_t$  transmit antennas and  $N_r$  receive antennas, and it operates over a flat and quasi-static Rayleigh fading channel represented by a  $N_r \times N_t$  fading channel matrix  $\mathbf{H} = \{h_{ji}\}$ . The complex element  $h_{ji}$  denotes the channel gain from the  $i$ th transmit antenna to the  $j$ th receive antenna, and it is assumed to be

constant over a frame ( $T$  symbols), i. e., it experiences quasi-static flat fading. The channel gains are modeled as independent complex Gaussian random variables with zero-mean and variance 0.5 per real dimension, i. e.,  $h_{ji} \sim CN(0, 1)$ . A complex orthogonal STBC is represented by a  $T \times N_t$  transmission matrix  $\mathbf{X}$ . The matrix  $\mathbf{X}$  is a linear combination of  $L$  input symbols satisfying the complex orthogonality:  $\mathbf{X}^H \mathbf{X} = \epsilon (|x_1|^2 + \dots + |x_L|^2) \mathbf{I}_{N_t}$ , where  $\{x_l, l=1, \dots, L\}$  are the  $L$  input symbols, and  $\epsilon$  is a constant which depends on the space-time coded transmission matrix<sup>[12]</sup>. Therefore, the transmission rate of STBC is  $R = L/T$ .

In the paper, we assume that CSI is perfectly known at the receiver, but there is delay in the CSI feedback from the receiver to the transmitter. Considering the feedback delay, the relation between the actual channel  $\mathbf{H}$  and its delayed version can be expressed as<sup>[20]</sup>

$$\mathbf{H} = \rho \hat{\mathbf{H}} + \sqrt{1 - \rho^2} \mathbf{E} \quad (1)$$

where  $\hat{\mathbf{H}}$  is the  $\tau$  time-delayed version of  $\mathbf{H}$ , and their entries are independent zero mean complex Gaussian random variables (r. v. s.) of unit variance. The entries  $\{\hat{h}_{ji}\}$  are correlated with  $\{h_{ji}\}$  which has correlation coefficient  $\rho = J_0(2\pi f_d \tau)$ , where  $J_0(\cdot)$  is the zero-order Bessel function of the first kind<sup>[21]</sup> and  $f_d$  is the maximum Doppler frequency<sup>[22]</sup>.  $\mathbf{E}$  is the estimation error matrix independent of  $\hat{\mathbf{H}}$ , and the entries of  $\mathbf{E}$  are independent complex Gaussian r. v. s with zero mean and unit variance. The coefficient  $\rho$  reflects the quality of channel information feedback, when  $\rho$  equals one, the channel is perfectly known.

The input-output relationship of the system is given by

$$\mathbf{Y} = \hat{\mathbf{H}} \mathbf{U} \mathbf{P} \mathbf{X} + \mathbf{Z} = \bar{\mathbf{H}} \mathbf{P} \mathbf{X} + \mathbf{Z} \quad (2)$$

where  $\bar{\mathbf{H}} = \hat{\mathbf{H}} \mathbf{U}$ ,  $\mathbf{U}$  is a  $N_t \times N_t$  beamforming matrix which contains the  $N_t$ -eigenvectors of  $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$  with eigenvalues  $\{\hat{\lambda}_i\}$  sorted in decreasing order, and it is a unitary matrix.  $\mathbf{Y}$  is the  $N_r \times N_t$  received signal matrix, and  $\mathbf{Z}$  is the  $N_r \times N_t$  noise matrix whose entries are independent identically distributed (i. i. d) zero-mean complex Gaussian

with variance  $\sigma_n^2$ .  $\mathbf{P} = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \dots, \sqrt{P_{N_t}})$  is the diagonal matrix, where  $\{P_i, i=1, \dots, N_t\}$  is the power allocation to the  $N_t$  eigen-beams with the following power constraint

$$\sum_{i=1}^{N_t} P_i = 1 \quad (3)$$

$$P_i \geq 0, i=1, \dots, N_t \quad (4)$$

The instantaneous SNR per symbol after STBC decoding can be expressed as<sup>[12]</sup>

$$\gamma = \|\bar{\mathbf{H}} \mathbf{P}\|_F^2 \bar{\gamma} / R = \sum_{i=1}^{N_t} P_i \alpha_i \bar{\gamma} / R \quad (5)$$

where  $\|\bar{\mathbf{H}} \mathbf{P}\|_F^2$  is the Frobenius norm defined as

$$\|\bar{\mathbf{H}} \mathbf{P}\|_F^2 = \sum_{i=1}^{N_t} P_i \sum_{j=1}^{N_r} |\bar{h}_{ji}|^2, \bar{\gamma} \text{ is the average SNR, and } \alpha_i \text{ is defined as}$$

$$\alpha_i = \sum_{j=1}^{N_r} |\bar{h}_{ji}|^2 = \sum_{j=1}^{N_r} \left| \sum_{k=1}^{N_t} h_{jk} \hat{u}_{ki} \right|^2 \quad (6)$$

In the following, we re-label the indices of the value of  $\{\alpha_i\}$  in a descending order with  $\alpha_1$  be the largest value for the sake of convenience.

With Eq. (1), using the Bayesian linear model and Theorem 10.3 in Ref. [23], the mean and covariance matrix of  $\mathbf{H}$  conditioned on  $\hat{\mathbf{H}}$ , can be expressed as

$$E\{\mathbf{H} | \hat{\mathbf{H}}\} = \rho \hat{\mathbf{H}}, \quad \mathbf{C}_{h|h} = (1 - \rho^2) \mathbf{I}_{N_t N_r} \quad (7)$$

where  $\mathbf{h}$  and  $\hat{\mathbf{h}}$  are the vectorized versions of  $\mathbf{H}$  and  $\hat{\mathbf{H}}$ , respectively. Therefore, the elements of  $\mathbf{H}$ ,  $\{h_{ji}\}$ , conditioned on  $\hat{\mathbf{H}}$ , become complex Gaussian random variables with mean  $\rho \hat{h}_{ji}$  and variance  $\sigma_e^2 = (1 - \rho^2)$ . Thus with Eq. (7) and  $\bar{\mathbf{H}} = \hat{\mathbf{H}} \mathbf{U}$ , the mean and covariance matrix of  $\bar{\mathbf{H}}$  conditioned on  $\hat{\mathbf{H}}$  can be written as

$$E\{\bar{\mathbf{H}} | \hat{\mathbf{H}}\} = \rho \hat{\mathbf{H}} \mathbf{U}, \quad \mathbf{C}_{h|h} = \sigma_e^2 \mathbf{I}_{N_t N_r} \quad (8)$$

Therefore, conditioned on  $\hat{\mathbf{H}}$ , the elements  $\{\bar{h}_{ji}\}$  of  $\bar{\mathbf{H}}$  become complex Gaussian random variables

with mean  $\rho \sum_{k=1}^{N_t} \hat{h}_{jk} \hat{v}_{ki}$  and variance  $\sigma_e^2$ . With these independent Gaussian distributed  $\{\bar{h}_{ji}\}$ ,  $\{\alpha_i\}$  will be independent noncentral chi-square distributed.

Utilizing Eq. (2.1-118) in Ref. [22], the probability density function (PDF) of  $\alpha_i$  conditioned on  $\hat{\mathbf{H}}$  can be expressed as

$$f(\alpha_i | \hat{\mathbf{H}}) = \frac{1}{\sigma_e^2} \left( \frac{\alpha_i}{\sigma_e^2} \right)^{(N_r - 1)/2} \exp\left(-\frac{\tilde{\alpha}_i + \alpha_i}{\sigma_e^2}\right) \times$$

$$I_{N_r-1} \left( \frac{2\sqrt{\alpha_i \alpha_i}}{\sigma_e^2} \right) \quad (9)$$

where  $\tilde{\alpha}_i = \rho^2 \sum_{j=1}^{N_r} \left| \sum_{k=1}^{N_t} \hat{h}_{jk} \hat{u}_{ki} \right|^2 = \rho^2 \hat{\alpha}_i$ , and  $\hat{\alpha}_i = \sum_{j=1}^{N_r} \left| \sum_{k=1}^{N_t} \hat{h}_{jk} \hat{u}_{ki} \right|^2 = \hat{\lambda}_i$ .  $I_v(x)$  is the  $v$ th-order modified Bessel function of the first kind<sup>[21]</sup>, which can be expanded as

$$I_v(x) = \sum_{u=0}^{\infty} \frac{(x/2)^{v+2u}}{[u! \Gamma(v+u+1)]} \quad (10)$$

### 3 Cross-Layer Design with Power Allocation Scheme for MIMO

In this section, we will give a suboptimal power allocation scheme for MIMO system with CLD and STBC by maximizing average SE of the system. The system is referred as CLD-STBC, and square  $M$ -ary quadrature amplitude modulation (MQAM) is considered for modulation in the system. For discrete-rate MQAM, the constellation size  $M_n$  is defined as  $\{M_0=0, M_1=2, \text{ and } M_n=2^{2n-2}, n=2, \dots, N\}$ , where  $M_0=0$  means no data transmission. The instantaneous SNR range is divided into  $N$  fading regions with switching thresholds  $\{\gamma_0, \gamma_1, \dots, \gamma_N, \gamma_{N+1}; \gamma_0=0, \gamma_{N+1} = +\infty\}$ . MQAM of constellation size  $M_n$  is used for modulation when  $\gamma$  falls in the  $n$ th region  $[\gamma_n, \gamma_{n+1})$ . Therefore, the data rate is  $b_n = \log_2 M_n$  with  $b_0=0$ .

According to Eq. [2], PER of MQAM with two-dimensional Gray code over additive white Gaussian noise (AWGN) channel for the received SNR  $\gamma$  and constellation size  $M_n$  is approximately given by

$$\text{PER}_n(\gamma) \approx \begin{cases} 1 & \gamma < \gamma_{pn} \\ a_n \exp(-g_n \gamma) & \gamma \geq \gamma_{pn} \end{cases} \quad (11)$$

where  $\{a_n, g_n, \gamma_{pn}\}$  are constellation and packet-size dependent constants, and they can be obtained by fitting Eq. (11) to the exact PER. Specifically, their values can be found in Table 1 in Ref. [2].

In our CLD scheme, adaptive modulation at the physical layer and the truncated ARQ protocol at the data link layer are employed for cross-layer design. We first define the target packet loss rate

(PLR) for the data link layer as  $P_{\text{loss}}$ . Since truncated ARQ is used at the data link layer, the packets in error may be retransmitted up to  $N_r^{\text{max}}$  (maximum number of retransmissions). Therefore, the target PER is  $P_o = P_{\text{loss}} \frac{1}{(N_r^{\text{max}}+1)}$  at the physical layer, which is generally limited as  $P_o < 1$ . The switching thresholds  $\{\gamma_n\}$  can be set to be the required SNR to achieve the target PER,  $P_o$ , over an AWGN channel. By inverting the  $P_o$  in Eq. (11), we can obtain the switching threshold values as follows

$$\gamma_n = \begin{cases} 0 & n=0 \\ B_n/g_n & n=1, \dots, N \\ +\infty & n=N+1 \end{cases} \quad (12)$$

where  $B_n = -\ln(P_o/a_n)$  is a factor dependent on the choosing of the modulation mode  $n$ . With the above switching thresholds, PER and SE performance of CLD-STBC system will be effectively evaluated. Moreover, the system will operate with PER below target PER  $P_o$ .

In what follows, we will derive the power control algorithm to maximize the average SE of the system subject to the fixed power constraint Eq. (3). According to Eqs. [2,3], the overall average SE of CLD-STBC can be expressed as

$$\overline{\text{Se}} = \frac{\overline{\text{Se}}_{\text{phy}}}{\overline{N}_a} \quad (13)$$

where  $\overline{\text{Se}}_{\text{phy}}$  is the average SE at the physical layer,  $\overline{N}_a$  the average number of transmissions per packet, and can be calculated as<sup>[2]</sup>

$$\overline{N}_a = 1 + \overline{\text{Per}} + \overline{\text{Per}}^2 + \dots + \overline{\text{Per}}^{N_r^{\text{max}}} = \frac{1 - \overline{\text{Per}}^{N_r^{\text{max}}+1}}{1 - \overline{\text{Per}}} \quad (14)$$

where  $\overline{\text{Per}}$  is the average PER of the system.

Substituting Eq. (14) into Eq. (13) yields

$$\overline{\text{Se}} = \frac{\overline{\text{Se}}_{\text{phy}} (1 - \overline{\text{Per}})}{1 - \overline{\text{Per}}^{N_r^{\text{max}}+1}} \quad (15)$$

Considering that  $\overline{\text{Per}}$  is usually less than 1,  $\overline{\text{Per}}^{N_r^{\text{max}}+1}$  will be much smaller. Thus, with Eq. (13), the relationship between average SE and average PER can be approximately given by

$$\overline{\text{Se}} \approx (1 - \overline{\text{Per}}) \cdot \overline{\text{Se}}_{\text{phy}} \quad (16)$$

where  $\overline{\text{Se}}_{\text{phy}}$  and  $\overline{\text{Per}}$  are respectively defined as

$$\overline{\text{Se}}_{\text{phy}} = R \sum_{n=1}^N b_n \int_{\gamma_n}^{\gamma_{n+1}} p_{\tilde{\gamma}}(\tilde{\gamma}) d\tilde{\gamma} \quad (17)$$

$$\overline{Per} = R \sum_{n=1}^N b_n \int_{\gamma_n}^{\gamma_{n+1}} \widetilde{Per}_n \cdot p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} / \overline{Se}_{\text{phy}} \quad (18)$$

where  $p_{\hat{\gamma}}(\hat{\gamma})$  is PDF of  $\hat{\gamma} = \sum_{i=1}^{N_t} P_i \hat{\alpha}_i \bar{\gamma} / R$ ,  $\widetilde{Per}_n$  the conditional average PER for constellation  $M_n$  under the imperfect CSI  $\hat{\gamma}$ .

Substituting Eqs. (17,18) into Eq. (16) gives

$$\overline{Se} \approx$$

$$R \sum_{n=1}^N b_n \left[ \int_{\gamma_n}^{\gamma_{n+1}} p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} - \int_{\gamma_n}^{\gamma_{n+1}} \widetilde{Per}_n \cdot p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \right] = R \sum_{n=1}^N b_n \left[ \int_{\gamma_n}^{\gamma_{n+1}} (1 - \widetilde{Per}_n) \cdot p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma} \right] \quad (19)$$

From Eq. (19), it is found that maximizing average SE is equivalent to the minimization of  $\widetilde{Per}_n$ .

With Eqs. (5, 11), the approximate PER with power allocation given the delayed channel  $\hat{\mathbf{H}}$  can be written as

$$Per_n(\bar{\mathbf{H}} | \hat{\mathbf{H}}) \approx a_n \exp\left(-g_n \sum_{i=1}^{N_t} P_i \alpha_i \bar{\gamma} / R\right) = a_n \exp\left(-\zeta_n \sum_{i=1}^{N_t} P_i \alpha_i\right) \quad (20)$$

where  $\zeta_n = g_n \bar{\gamma} / R$ , and the threshold  $\gamma_{pn}$  is neglected for analysis simplicity. Using Eq. (20) and the conditional PDF of  $\alpha_i$  shown as Eq. (9), the conditional average PER given  $\hat{\mathbf{H}}$  can be written as

$$\widetilde{Per}_n \approx a_n \prod_{i=1}^{N_t} \int_0^{\infty} \exp(-\zeta_n P_i \alpha_i) f(\alpha_i) d\alpha_i = a_n \prod_{i=1}^{N_t} \frac{1}{(1 + \zeta_n \sigma_e^2 P_i)^{n_R}} \exp\left(-\frac{\zeta_n \rho^2 \hat{\alpha}_i P_i}{1 + \zeta_n \sigma_e^2 P_i}\right) \quad (21)$$

In the above derivation, the independence of  $\{\alpha_i\}$  and Eq. (6.643) in Ref. [21] are utilized.

Through minimizing Eq. (21), the suboptimal power allocation  $\{P_i\}$  can be obtained. In order to facilitate analysis, we will take the logarithm of Eq. (21) as the optimized objective, i. e.

$$L(P_i) = \ln(\widetilde{Per}_n / a_n) = - \sum_{i=1}^{N_t} \left[ N_r \ln(1 + \zeta_n \sigma_e^2 P_i) + \frac{\zeta_n \rho^2 \hat{\alpha}_i P_i}{1 + \zeta_n \sigma_e^2 P_i} \right] \quad (22)$$

It is well known that if the channel information is perfect, only the maximum eigenmode should be employed to maximize the received SNR, resulting in one-directional beamforming (1D-BF), namely  $P_1 = 1$  and  $P_n = 0$ ,  $n = 2, \dots, N$ . However,

when CSI is imperfect, using only the maximum eigenmode will cause the performance sensitive to the feedback delay. Since the imperfect feedback information may bring about uncertainty onto the received SNR at the transmitter, we define the following compressed SNR to take this uncertainty into account

$$\gamma_v = (\bar{\gamma} / r) \sum_{i=1}^{N_t} (P_i \bar{\alpha}_i)^v \quad 0 \leq v \leq 1 \quad (23)$$

where  $\bar{\alpha}_i = E(\alpha_i | \hat{\mathbf{H}}) = N_r \sigma_e^2 + \bar{\alpha}_i = N_r \sigma_e^2 + \rho^2 \hat{\alpha}_i$ , is the expectation of  $\alpha_i$  conditioned on  $\hat{\mathbf{H}}$ .  $v$  is the compressed factor which will be used to optimize  $\gamma_v$ . Eq. (23) is defined based on the compression function  $f(x) = x^v$  ( $x > 0$ ,  $0 \leq v \leq 1$ ), where its slope decreases as  $x$  increases. By using the compressed function, we can adjust the contributions from the eigen-beams to the received SNR. Thus, the definition of Eq. (23) is to de-emphasize the largest eigen-beam and place appropriate emphasis to other beams in accordant with the feedback channel information.

In the following, we will derive the power control to maximize  $\gamma_v$  in Eq. (23) subject to the power constraint in Eq. (3). An auxiliary objective function is defined as follows

$$L_v(P_1, \dots, P_{N_t}) = \sum_{i=1}^{N_t} (P_i \bar{\alpha}_i)^v + \theta \left(1 - \sum_{i=1}^{N_t} P_i\right) \quad (24)$$

where  $\theta$  is a Lagrange multiplier. Taking partial derivatives of the objective function with respect to  $\{P_n, n = 1, \dots, N_t\}$  and equating the partial derivatives to zero gives

$$P_i = \theta^{-\frac{1}{v-1}} [\bar{\alpha}_i^v]^{-\frac{1}{v-1}} \quad i = 1, \dots, N_t \quad (25)$$

Substituting Eq. (25) into Eq. (3) yields

$$(\theta / \nu)^{\frac{1}{v-1}} = \frac{1}{\sum_{i=1}^{N_t} \bar{\alpha}_i^{\nu/(1-\nu)}} \quad (26)$$

Substituting Eq. (26) into Eq. (25) gives

$$P_i = \frac{\bar{\alpha}_i^{\nu/(1-\nu)}}{\sum_{k=1}^{N_t} \bar{\alpha}_k^{\nu/(1-\nu)}} = \frac{\bar{\alpha}_i^\mu}{\sum_{k=1}^{N_t} \bar{\alpha}_k^\mu} \quad i = 1, \dots, N_t \quad (27)$$

where  $\mu = \nu / (1 - \nu)$ . With  $0 < \nu < 1$ ,  $\mu \in (0, +\infty)$ . The power allocation scheme of Eq. (27) is general enough to cover from equal power allocation ( $\mu \rightarrow 0$ ) to 1-D BF ( $\mu \rightarrow \infty$ ).

Note that  $P_i$  in Eq. (27) is always nonnega-

tive, automatically satisfying the requirement of positive power (Eq. (4)). The corresponding power calculation becomes straightforward, and thus the iterative calculation of the existing optimal PA scheme in MIMO system is avoided. Moreover, the PA scheme can be used to generalize some existing schemes by setting  $\mu$  to different values. For example, setting  $\mu=0$ , Eq. (27) becomes the equal power allocation. When  $\mu \rightarrow \infty$  and  $\hat{\mathbf{U}}=\mathbf{I}_{N_t}$ , it gives the antenna selection scheme in Ref. [24]. Besides, with perfect CSI and  $\hat{\mathbf{U}}=\mathbf{I}_{N_t}$ ,  $P_i=\alpha_i^r/\sum_{k=1}^{N_t}\alpha_k^r$ , it is identical to the adaptive power control scheme in Ref. [17]. The derivation of the power control and the way to obtain optimal  $\mu$  are both not provided.

Substituting Eq. (27) into Eq. (22) yields the function  $L(\mu)$  with respect to  $\mu$  as follows

$$L(\mu) = - \sum_{i=1}^{N_t} \left[ N_r \ln \left( 1 + \zeta_n \sigma_e^2 \frac{\bar{\alpha}_i^r}{\sum_{k=1}^{N_t} \bar{\alpha}_k^r} \right) + \frac{\zeta_n \rho^2 \hat{\alpha}_i^r}{\zeta_n \sigma_e^2 \bar{\alpha}_i^r + \sum_{k=1}^{N_t} \bar{\alpha}_k^r} \right] \quad (28)$$

It can be shown that the objective is a quasi-convex function with respect to  $\mu$ . Although numerical method can be used to calculate  $\mu$  [25], we focus on the derivation of a closed-form solution of  $\mu$ . According to the asymptotic analysis of the spatial PA with imperfect CSI in Ref. [14], more power will be allocated to the eigenbeam with the largest eigenvalue in low SNR, which means that  $\mu$  is large for low SNR. On the other hand, all eigenbeams are almost allocated equal power in very high SNR, which means that  $\mu$  is close to zero. Based on this, we may use Taylor's series expansion at  $\mu=0$  to approximate  $L(\mu)$  with small  $\mu$  as

$$L(\mu) \approx L(0) + L'(0)\mu + L''(0)\mu^2 \quad (29)$$

where  $L(0) = - \left[ N_t N_r \ln \left( \frac{N_t + \zeta_n \sigma_e^2}{N_t} \right) + \frac{\zeta_n \rho^2}{N_t + \zeta_n \sigma_e^2} \cdot \sum_{i=1}^{N_t} \hat{\alpha}_i \right]$ , the first derivatives of  $L(\mu)$  with respect to  $\mu$  at  $\mu=0$ , can be obtained as

$$L'(0) = \left. \frac{\partial L(\mu)}{\partial \mu} \right|_{\mu=0} = \frac{N_t \zeta_n \rho^2}{(N_t + \zeta_n \sigma_e^2)^2} \sum_{i=1}^{N_t} \hat{\alpha}_i d_i \quad (30)$$

where  $d_i = \ln \bar{\alpha}_i - (1/N_t) \sum_{k=1}^{N_t} \ln \bar{\alpha}_k$ , and  $\sum_{i=1}^{N_t} d_i = 0$ .

The second derivatives of  $L(\mu)$  with respect to at  $\mu=0$  can be obtained as

$$L''(0) = \frac{N_t \zeta_n^2 \sigma_e^4}{(N_t + \zeta_n \sigma_e^2)^2} \sum_{i=1}^{N_t} d_i^2 + \frac{2N_t \zeta_n^2 \sigma_e^2 \rho^2}{(N_t + \zeta_n \sigma_e^2)^3} \sum_{i=1}^{N_t} \hat{\alpha}_i d_i^2 \quad (31)$$

From  $L(0) < 0$ ,  $L'(0) > 0$  and  $L''(0) > 0$  shown in Eqs. (29–31),  $L(\mu)$  is the convex function of  $\mu$ . With Eq. (29), by setting  $\partial L(\mu)/\partial \mu = 0$ ,  $\mu$  can be obtained by

$$\mu = N_t \rho^2 \sum_{i=1}^{N_t} \hat{\alpha}_i d_i / \left[ N_r \zeta_n \sigma_e^4 \sum_{i=1}^{N_t} d_i^2 + \left( \frac{2N_t \zeta_n \sigma_e^2}{N_t + \zeta_n \sigma_e^2} \right) \sum_{i=1}^{N_t} \rho^2 \hat{\alpha}_i d_i^2 \right] \quad (32)$$

It can be shown that  $\mu$  calculated by Eq. (32) is always positive. Thus, we obtain the closed-form solution of  $\mu$  and the resultant closed-form expressions of power allocation. From Eq. (32), we can see that when average SNR  $\bar{\gamma}$  is low, the corresponding  $\mu$  will be large. Thus, more power will be allocated to the eigenbeam with the largest  $\hat{\alpha}_n$  in terms of Eq. (27). Whereas for large  $\bar{\gamma}$ ,  $\zeta_n$  will become large, and corresponding  $\mu$  will be close to zero. Thus, equal power is allocated for each eigenbeam according to Eq. (27). The above results accord with the previous asymptotic analysis, which means that the derived  $\mu$  is valid.

## 4 Simulation Results

In this section, we will evaluate the performance of the developed CLD scheme with power allocation in space-time coded MIMO systems by computer simulation. In the simulation, the channel is assumed to be quasi-static flat Rayleigh fading. The Gray code is used to map the data bits to symbol constellations. The set of MQAM constellations is  $\{M_n\} = \{0, 2, 4, 16, 64\}$ ,  $n=0, 1, \dots, 4$ . The target packet loss rate at the data link layer,  $P_{\text{loss}} = 10^{-3}$ , and the data packet contains 1 080 bit. Different space-time block codes, such as  $G_2$ ,  $G_3$ , and  $G_4$  codes [12–13] are adopted for evaluation and comparison.

Fig. 1 gives the average SE of the cross-layer design with different power allocation schemes for  $1 \times 2$  system, where JSO represents that CLD

with the existing optimal PA scheme (i. e., the PA scheme in Ref. [14] is applied to CLD-STBC for comparison consistency). In Fig. 1,  $G_2$  is used for STBC,  $f_d\tau=0.05$ ,  $N_r^{\max}=0,1,2,3$  is considered. As shown in Fig. 1, CLD with the developed power allocation scheme has higher SE than that with equal power allocation scheme under the same  $N_r^{\max}$ , and it gives about 0.8 (bit/s)/Hz gain over the latter. The above result means that the developed PA scheme is valid for improving the SE performance of CLD. Moreover, due to the better approximation, the proposed suboptimal PA scheme has almost the same performance as the optimal JSO scheme. More importantly, the former has closed-form power control, and thus it can simplify the optimal power allocation procedure greatly. In addition, the average SE can improve as  $N_r^{\max}$  increases, but it improves less when  $N_r^{\max}$  becomes larger. Based on this, in the following simulation, the maximum number of retransmissions,  $N_r^{\max}$  is set equal to 2.

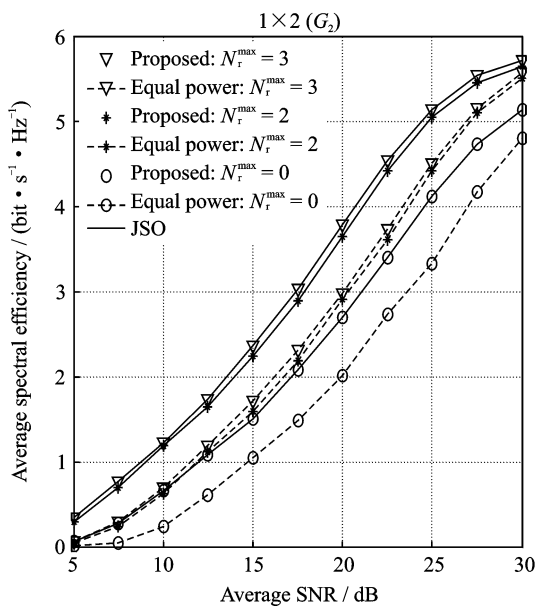


Fig. 1 ASE of CLD-STBC with two transmit antennas and one receive antenna ( $f_d\tau=0.05$ )

Fig. 2 gives the average SE of the cross-layer design with different time delay for  $1\times 2$  system using  $G_2$  code, where  $N_r^{\max}=2$ , the three PA schemes: the presented scheme, equal power scheme, 1-D beamforming scheme are used for comparison. From Fig. 2, we can see that ASE of CLD with the proposed scheme has better SE per-

formance than that with the equal PA scheme or 1-D beamforming scheme. It has almost the same ASE performance as CLD with equal PA scheme for large SNR, which accords with the asymptotic analysis at high SNR in section 2. Moreover, when SNR is low, it has almost the same ASE performance as CLD with 1-D beamforming scheme, which is also consistent with the asymptotic analysis at low SNR in section 2. Besides, when the feedback delay is small (e. g.  $f_d\tau=0.01$ ), its performance is very close to the performance of CLD with 1-D beamforming (which is optimal for perfect CSI). The above results show that the proposed suboptimal scheme is a practical method with near-optimal performance.

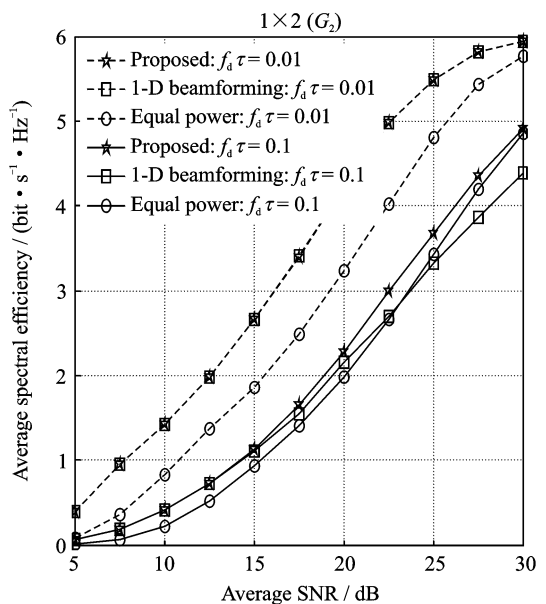


Fig. 2 ASE of CLD-STBC with two transmit antennas and one receive antenna ( $N_r^{\max}=2$ )

In Fig. 3, we plot the average BER vs. normalized time delay for CLD-STBC with two transmit antennas and one receive antenna. The average SNR is set equal to 15 dB. The developed scheme and equal power scheme are used for performance comparison. The results show that CLD-STBC with the above two PA schemes can tolerate the normalized time-delay up to about 0.01 with a slight degradation in the average SE. But when the normalized time-delay  $f_d\tau$  increases beyond 0.01, the ASE performance will degrade increasingly. It is found that CLD with the developed scheme is still superior to CLD scheme with equal PA scheme over a wide range of feedback

delays due to the application of power adaptation, especially for small time-delay. However, with the large feedback time-delay, their SE will be close to each other. This is because when the time-delay increases, the feedback becomes less reliable yielding a non-adaptive power allocation without regarding the feedback. As a result, the proposed scheme will tend to use the equal power allocation, and correspondingly, the two schemes will have the same performance.

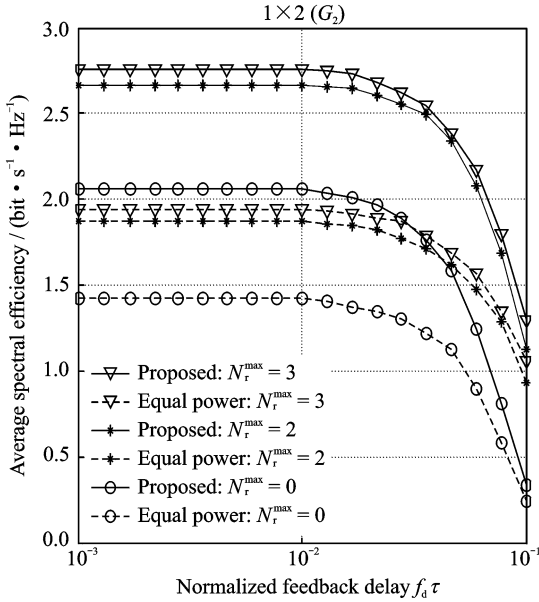


Fig. 3 Effect of normalized time-delay on ASE of CLD-STBC with two transmit antennas and one receive antenna

Fig. 4 gives the average SE of CLD-STBC with different transmit antennas and receive antennas, where the developed PA scheme is used for performance evaluation. In Fig. 4,  $xTyR$  denotes a MIMO system with  $x$  transmit antenna and  $y$  receive antennas. The normalized time-delay ( $f_d \tau$ ) is set as 0.05, and  $N_r^{\max} = 2$ . As shown in Fig. 4, the system ASE will increase as the receive antenna increases under the same number of transmit antenna, e.g. the  $2 \times 2$  system can give about 1.5 (bit/s)/Hz gain over the  $1 \times 2$  system. Moreover, for the same number of receive antenna, ASE of CLD-STBC with  $G_2$  code is higher than that of CLD-STBC with single transmit antenna, ASE of CLD-STBC with  $G_4$  code is higher than that of CLD-STBC with  $G_3$  code. Besides, ASE of CLD-STBC with  $G_2$  code is higher than that of with  $G_3$  and  $G_4$  codes. It is because the  $G_2$

code is a full rate code while  $G_3$  and  $G_4$  codes are having half code rate.

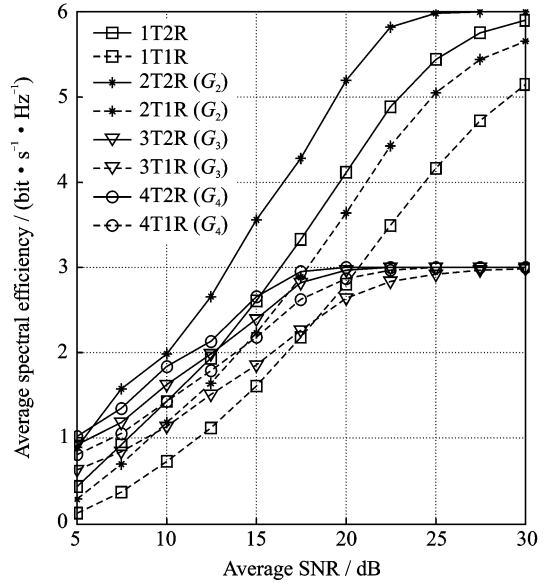


Fig. 4 ASE of CLD-STBC with different transmit antennas and receive antennas ( $N_r^{\max} = 2$ ,  $f_d \tau = 0.05$ )

## 5 Conclusions

The performance of CLD-STBC system with imperfect CSI in Rayleigh fading channels is analyzed. The optimal power allocation for maximizing the system ASE is derived based on imperfect feedback information. According to the relationship between ASE and average PER, the optimal problem is converted to minimize the average PER of the system for simplifying the optimization calculation. By utilizing a compressed SNR criterion, a closed-form compressed factor is derived to minimize PER. As a result, a closed-form suboptimal solution provides not only a direct computation of the compressed factor but also a closed-form power allocation. Moreover, the suboptimal PA scheme can always be positive, and thus it avoids the numerical search and iteration of the existing optimal PA scheme to find the positive power allocation. Simulation results show that CLD-STBC system with PA can effectively utilize imperfect CSI to improve ASE performance, and the system can tolerate the normalized time-delay up to about 0.01 with a slight degradation in ASE. The results indicate that CLD-STBC with PA has higher ASE than that with equal power, and can obtain the performance



close to CLD with the optimal PA.

## References:

- [1] Cohen R, Grebla G, Katzir L. Cross-layer hybrid FEC/ARQ reliable multicast with adaptive modulation and coding in broadband wireless networks[J]. *IEEE/ACM Transactions Networking*, 2010, 18(6): 1908-1920.
- [2] Liu Q W, Zhou S, Giannakis G B. Cross-layer combining of adaptive modulation and coding with truncated ARQ over wireless links[J]. *IEEE Transactions Wireless Communication*, 2004, 3(5): 1746-1755.
- [3] Maaref A, Aissa S. A cross-layer design for MIMO Rayleigh fading channels[C]// *Proceedings of Canadian Conference on Electrical and Computer Engineering (CCECE'2004)*. Canada: IEEE, 2004, 4: 2247-2250.
- [4] Lu Xiaofeng, Zhu Guangxi, Liu Gan, et al. A cross-layer design over MIMO Rayleigh fading channels[C] // *Proceedings of IEEE International Conference on Wireless Communications, Networking and Mobile Computing (WiCOM' 2005)*. Wuhan, China: IEEE, 2005, 1: 30-33.
- [5] Zaidi S A R, Hafeez M. Cross layer design for orthogonal space-time block coded optical MIMO systems[C]// *Proceedings of IEEE Fifth International Conference on Wireless and Optical Communications Networks (WOCN '08)*. Surabaya: IEEE, 2008: 1-5.
- [6] Zhu Xuemei, Yuan Dongfeng. Cross-layer design for MIMO correlated Nakagami fading channels[C] // *Proceedings of IEEE Proceedings of the International Workshop on Cross Layer Design*. Jinan, China: IEEE, 2007: 50-54.
- [7] Yu Xiangbin, Chen Xiaomin, Zhu Qiuming. Cross-layer design for MIMO systems with antenna selection over Nakagami-m fading channels[C] // *Proceedings of IEEE International Conference on Communication Systems (ICCS'12)*. Singapore: IEEE, 2012: 349-343.
- [8] Qi Jian, Aissa S. Cross-layer design for multiuser MIMO MRC systems with feedback constraints[J]. *IEEE Transactions on Vehicular Technology*, 2009, 58(7): 3347-3360.
- [9] Chae C B, Forenza A, Heath R W, et al. Adaptive MIMO transmission techniques for broadband wireless communication systems[J]. *IEEE Communication Magazine*, 2010, 48(5): 112-118.
- [10] Tang Lei, Yang Fengfan, Zhang Shunwai, et al. V-blast based LDPC-coded relay cooperation [J]. *Transactions of Nanjing University of Aeronautics & Astronautics*, 2013, 30(4): 374-382.
- [11] Yu Xiangbin, Shi Huibin. Performance analysis of variable-power adaptive modulation in space-time block coded MIMO diversity systems[J]. *Science China: Information Sciences*, 2010, 53(10): 2106-2115.
- [12] Shin H, Lee J H. Performance analysis of space-time block codes over keyhole Nakagami-m fading channels[J]. *IEEE Transactions on Vehicular Technology*, 2004, 53(2): 351-362.
- [13] Yu Xiangbin, Leung S H, Chen Xiaomin. Performance analysis of space-time block-coded MIMO systems with imperfect channel information over Rician fading channels[J]. *IEEE Transactions on Vehicular Technology*, 2011, 60(9): 4450-4461.
- [14] Jongren G, Skoglund M, Ottersten B. Combining beamforming and orthogonal space-time block coding [J]. *IEEE Transactions on Information Theory*, 2002, 48(10): 611- 627.
- [15] Huang J W, Au E K S, Lau V K N. Precoder design for space-time coded MIMO systems with imperfect channel state information[J]. *IEEE Transactions on Wireless Communications*, 2008, 7(6): 1977-1981.
- [16] Zhou Shengli, Giannakis G B. Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback[J]. *IEEE Transactions on Signal Processing*, 2002, 50(10): 2599-2613.
- [17] Xian Liang, Liu Huaping. An adaptive power allocation scheme for space-time block coded MIMO systems[C]// *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC' 05)*. [S. l.]: IEEE, 2005: 504-508.
- [18] Li Xiao, Jin Shi, Gao Xiqi, et al. Near-optimal power allocation for MIMO channels with mean or covariance feedback [J]. *IEEE Transactions on Communications*, 2010, 58(1): 289-300.
- [19] Vu M, Paulraj A. Optimal linear precoders for MIMO wireless correlated channels with nonzero mean in space-time coded systems[J]. *IEEE Transactions on Signal Processing*, 2006, 54(6): 2318-2332.
- [20] Sharma V, Premkumar K, Swamy R N. Exponential diversity achieving spatio-temporal power allocation scheme for fading channels [J]. *IEEE Transactions on Information Theory*, 2008, 54(1): 188-208.
- [21] Gradshteyn I S, Ryzhik I M. *Table of integrals, series, and products* [M]. 7th Ed. San Diego, CA: Academic, 2007.
- [22] Proakis J G. *Digital communications*[M]. 5th Ed. New York: McGraw-Hill, 2007.
- [23] Kay S M. *Fundamentals of statistical signal processing; Estimation theory*[M]. NJ: Englewood Cliffs Prentice-Hall, 1993.
- [24] Ganesan G, Stoica P, Larsson E G. Orthogonal space-time block codes with feedback[J]. *Wireless Pers Communications*, 2004, 28(4): 287-312.
- [25] Boyd S, Vandenberghe L. *Convex optimization*[M]. U. K. : Cambridge University Press, 2004.

