Runge-Kutta Multi-resolution Time-Domain Method for Modeling 3D Dielectric Curved Objects

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Abstract: A conformal Runge-Kutta multi-resolution time-domain (C-RKMRTD) method is present and applied to model and analyze curved objects. Compared with the non-conformal method, the proposed method is more accurate. The scattering analyses of the cylinder and ellipsoid are presented to validate the proposed method. The numerical results demonstrate that the proposed scheme perform better than the MRTD method and other higher order methods with a higher accuracy.

Key words: conformality; Runge-Kutta; multi-resolution time-domain(MRTD); scattering

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1 Introduction

The finite-difference time-domain (FDTD) method has been widely used in the field of computational electromagnetics due to its simple implementation and a capability to address complex targets[1]. It is known that the FDTD method has two primary drawbacks. One is that the numerical dispersion is the dominate limitation to the accuracy of the FDTD method. The other is that it is not able to accurately model curved surfaces and material discontinuities by using the stair-casing approach with structured grids. In the past decades, numerous efforts have been made to improve the traditional FDTD method such as the high-order methods. The multi-resolution time-domain (MRTD) method has been proposed to improve numerical dispersion properties[2-6]. The Runge-Kutta multi-resolution time-domain (RK-MRTD) has been proposed by Cao[7-8] to improve the dispersion and convergence in both time and spatial domains. However, these methods also have shortcomings to deal with curved objects. The conformal FDTD technique is one of candidates to circumvent this problem. Nowadays, more attentions are focused on how to modeling curved objects. Locally conformal FDTD (CFDTD) method was proposed by Dey, et al[9] to accurately model the curved metallic objects, and it is more accurate than the FDTD method. Stefan, et al[10] proposed a new conformal perfect electric conductor (PEC) algorithm, of the FDTD method, which only needed to change two field-updated coefficients. It could privilege either speed or accuracy when choosing a time step reduction. Some other papers investigated how to accurately model curved dielectric objects using the CFDTD method[11-13].

However, few papers discuss the conformal RK-MRTD (C-RKMRTD) method to deal with the curved dielectric objects. In this paper, the C-RKMRTD method is derived and presented. Besides, numerical examples are also given to verify the proposed method[14].

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2 C-RKMRTD Method

2.1 RK-MRTD method

For simplicity ($\sigma=0$) and without loss of generality, in three-dimensional (3D) one of the RK-MRTD[7] update equations can be written as

$$\varepsilon\frac{\partial E^{x}_{i+1/2,j,k}(t)}{\partial t}=\sum_{v=1}^{m}a(v)\left[\frac{1}{\Delta y}(H^{z}_{i+1/2,j-1/2+v,k}(t)-H^{z}_{i+1/2,j+1/2-v,k}(t))-\frac{1}{\Delta z}(H^{y}_{i+1/2,j,k-1/2+v}(t)-H^{y}_{i+1/2,j,k+1/2-v}(t))\right] \quad (1)$$

where m is the spatial stencil size. Parameters ε, Δt, Δx, Δy, and Δz are the permittivity, temporal step size, and spatial step sizes along x-, y- and z-directions, respectively. The coefficients $a(v)$ is the same as defined in Ref. [7].

2.2 C-RKMRTD method

In order to derive the general update equations of the C-RKMRTD method with the spatial step size $\Delta x=\Delta y=\Delta z$, Eq. (1) can be rewritten in another form as

$$\begin{aligned}\varepsilon\frac{\partial E^{x}_{i+1/2,j,k}(t)}{\partial t}=&\frac{a(1)}{\Delta x}[(H^{z}_{i+1/2,j+1/2,k}(t)-H^{z}_{i+1/2,j-1/2,k}(t))-(H^{y}_{i+1/2,j,k+1/2}(t)-H^{y}_{i+1/2,j,k-1/2}(t))]+\frac{3a(2)}{3\Delta x}[(H^{z}_{i+1/2,j+3/2,k}(t)-\\&H^{z}_{i+1/2,j-3/2,k}(t))-(H^{y}_{i+1/2,j,k+3/2}(t)-H^{y}_{i+1/2,j,k-3/2}(t))]+\cdots+\\&\frac{(2v-1)a(v)}{(2v-1)\Delta x}[(H^{z}_{i+1/2,j-1/2+v,k}(t)-H^{z}_{i+1/2,j+1/2-v,k}(t))-(H^{y}_{i+1/2,j,k-1/2+v}(t)-H^{y}_{i+1/2,j,k+1/2-v}(t))]\end{aligned} \quad (2)$$

From Ref. [13], we know that $\sum_{v=1}^{m}a(v)(2v-1)=1$, Eq. (2) can be decomposed into $(2v-1)$ sub-equations as follows

$$a(1)\cdot\varepsilon(1)(E^{x,n+1}_{i+0.5,j,k}-E^{x}_{i+0.5,j,k})=a(1)\frac{\Delta t}{\Delta x}\cdot(H^{z,n+0.5}_{i+0.5,j+0.5,k}-H^{z,n+0.5}_{i+0.5,j-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+0.5}+H^{y,n+0.5}_{i+0.5,j,k-0.5}) \quad (3)$$

$$3a(2)\cdot\varepsilon(2)(E^{x,n+1}_{i+0.5,j,k}-E^{x}_{i+0.5,j,k})=3a(2)\frac{\Delta t}{3\Delta x}\cdot(H^{z,n+0.5}_{i+0.5,j+1.5,k}-H^{z,n+0.5}_{i+0.5,j-1.5,k}-H^{y,n+0.5}_{i+0.5,j,k+1.5}+H^{y,n+0.5}_{i+0.5,j,k-1.5}) \quad (4)$$

$$\vdots$$

$$(2v-1)a(v)\cdot\varepsilon(2v-1)(E^{x,n+1}_{i+0.5,j,k}-E^{x}_{i+0.5,j,k})=+(2v-1)\frac{a(v)\Delta t}{(2v-1)\Delta x}(H^{z,n+0.5}_{i+0.5,j+v-0.5,k}-H^{z,n+0.5}_{i+0.5,j-v-0.5,k}-H^{y,n+0.5}_{i+0.5,j,k+v-0.5}+H^{y,n+0.5}_{i+0.5,j,k-v+0.5}) \quad (5)$$

where coefficients $\varepsilon(v)$ ($v=1, 2, \cdots, 2v-1$) are the permittivities corresponding to the cell size Δx, $3\Delta x$, $\cdots$, and $(2v-1)\Delta x$, respectively. It is clear that for a given Δx, Eqs. (4,5) can thus be treated as the intervals $3\Delta x$ and $(2v-1)\Delta x$ in the FDTD update equations. The multi-region decomposition of electric field E is shown in Fig. 1.

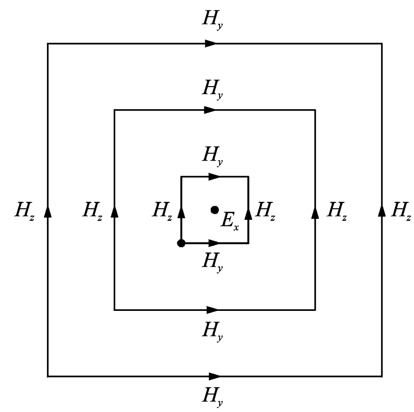


Fig. 1 E field multi-region decomposition for conformal high-order FDTD method

Adding Eqs. (2−5), the update equation of the C-RKMRTD method is expressed as

$$\sum_{v=1}^{m}(2v-1)a(v)\varepsilon(v)\frac{\partial E^{x}_{i+1/2,j,k}(t)}{\partial t}=\sum_{v=1}^{m}a(v)\frac{\Delta t}{\Delta x}\cdot(H^{z,n+0.5}_{i+0.5,j+v-0.5,k}-H^{z,n+0.5}_{i+0.5,j-v+0.5,k})-H^{y,n+0.5}_{i+0.5,j,k+v-0.5}+H^{y,n+0.5}_{i+0.5,j,k-v+0.5}) \quad (6)$$

From Eq. (6), it is easy to see that the update equation of C-RKMRTD method is constituted by $(2v-1)$ normal FDTD method with cell sizes of Δx, $3\Delta x$, $\cdots$, $(2v-1)\Delta x$.

Comparing Eqs. (1) and (6), it is clearly found that the effective dielectric constant ε^{eff} is

$$\varepsilon^{\text{eff}}=\sum_{v=1}^{m}(2v-1)a(v)\varepsilon(v) \quad (7)$$

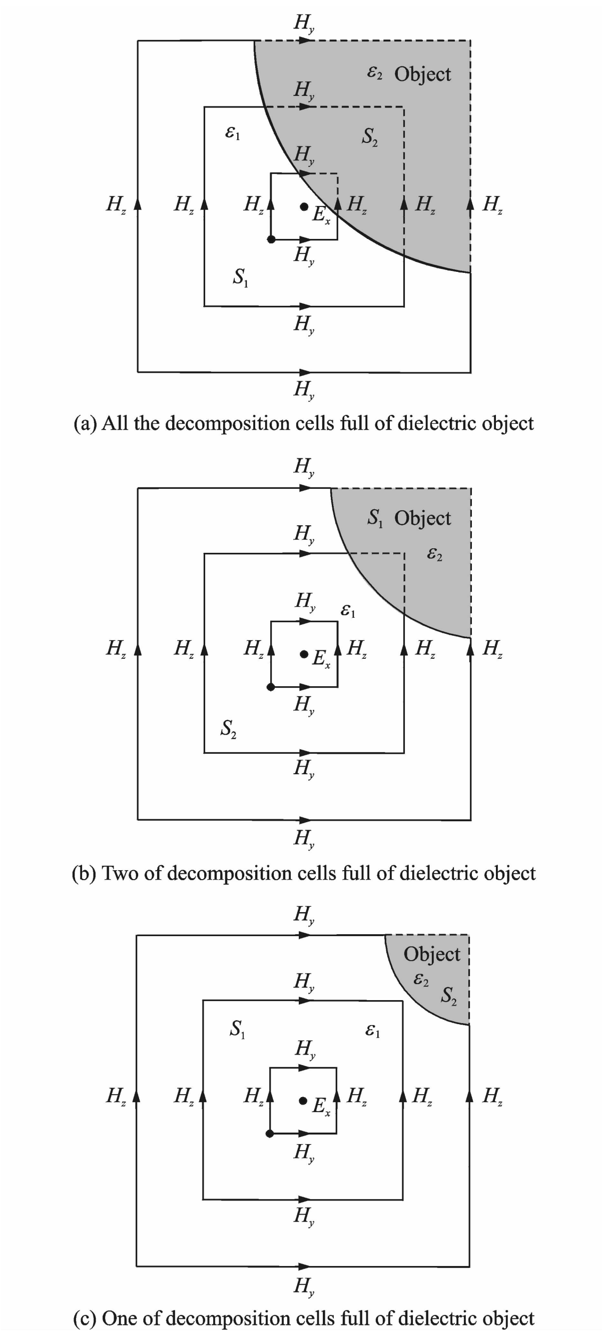
The weighting area[15] is used to obtain $\varepsilon(v)$ as

$$\varepsilon(v)=\frac{S_1}{S_1+S_2}\cdot\varepsilon_1+\frac{S_2}{S_1+S_2}\cdot\varepsilon_2 \quad (8)$$

Three different conditions for the objects interface are shown in Fig. 2.

3 Numerical Experiments

The numerical simulations are presented to validate the C-RKMRTD method. The two simu-



(a) All the decomposition cells full of dielectric object

(b) Two of decomposition cells full of dielectric object

(c) One of decomposition cells full of dielectric object

Fig. 2 Different conditions for curved object interface

lations both take 10 cells per wavelength. The number of Courant, Friedrichs, Lewy (CFL) is 0.3, and an eight-layer of anistropic perfectly matched layer (APML) is used to truncate the computational domain. All computational simulations are based on a computer of Pentium with a dual-core 2.8 GHz CPU and 1.87 G memory.

3.1 Dielectric cylinder

The dielectric cylinder with a radius of 0.06 m, height of 0.015 m, the relative permittivity ε_r of 4, and relative permeability μ_r of 1.0. The cylinder is illuminated by an incident plane wave coming from the z-direction with a polarization in the x-direction at 10 GHz. The total computational volume is discretized into $82\times82\times82$ cells. The bistatic radar cross sections (RCSs) in E-plane obtained from different methods, i. e., method of moments (MoM), MRTD and C-RK-MRTD, are shown in Fig. 3, where θ is the incident angle. The C-RKMRTD method agrees with the MoM method better than the MRTD method. Table 1 lists the magnitudes of the spatial discretization, temporal discretization, total computational domain, total time steps and CPU time. Fig. 4 shows the difference between the C-RKMRTD/MRTD and the MoM method.

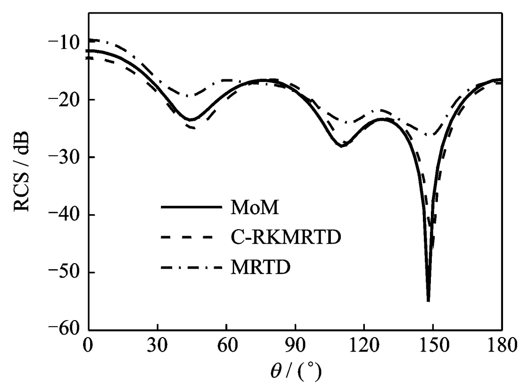


Fig. 3 Bistatic RCS in E-plane of the dielectric cylinder obtained by different methods

Table 1 Comparison for different methods

Method	MRTD	C-RKMRTD
Δx	0.003	0.003
Δt	3	3
Cell	$82\times82\times82$	$82\times82\times82$
Total time	2 000	2 000
CPU time /s	1 645.43	3 989.53

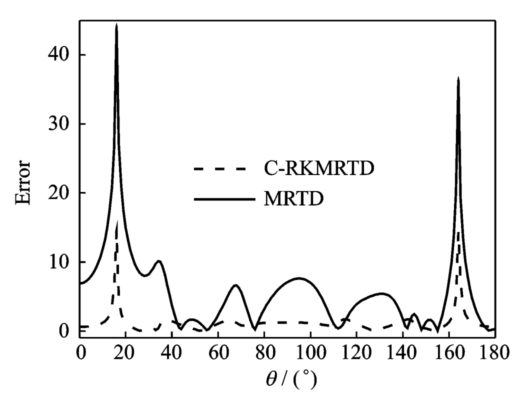


Fig. 4 Difference between C-RKMRTD (MRTD) and MoM method

3.2 Dielectric ellipsoid

The structure of dielectric ellipsoid with the radii of 0.6, 0.6 and 0.3 m in the x-, y-, z- directions, respectively. The relative permittivity ε_r is 4, the relative permeability μ_r is 1, the polarization of the electric field is in the x-direction, and the wavelength is 0.3 m.

Backward scattering bistatic RCSs obtained by different methods are shown in Figs. 5, 6. It is found that the results of C-RKMRTD method is consistent with those of the MoM method and its performance is better than that of the non-conformal methods. The comparisons of the computational cost of different methods are displayed in Table 2 . Fig. 7 shows the difference between C-RKMRTD/MRTD and the MoM methods. Fig. 8 shows the difference between C-RKMRTD/FDTD and the MoM method.

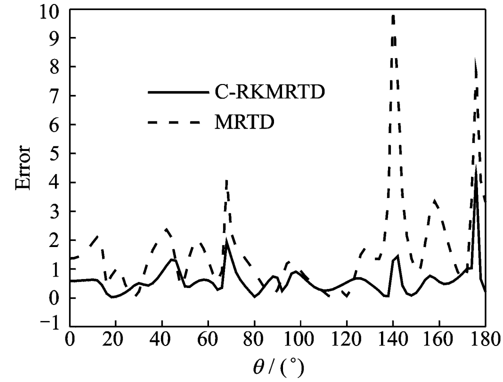


Fig. 7 Difference between C-RKMRTD/MRTD and MoM method

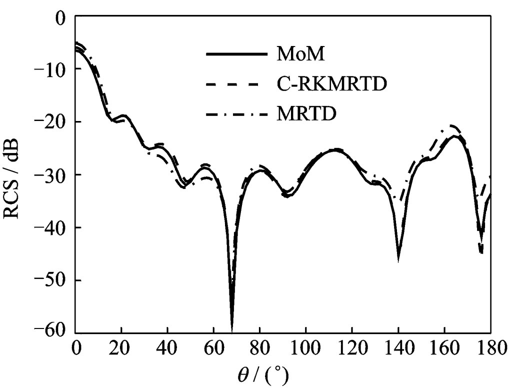


Fig. 5 Dielectric RCS in E-plane of the ellipsoid obtained by different methods

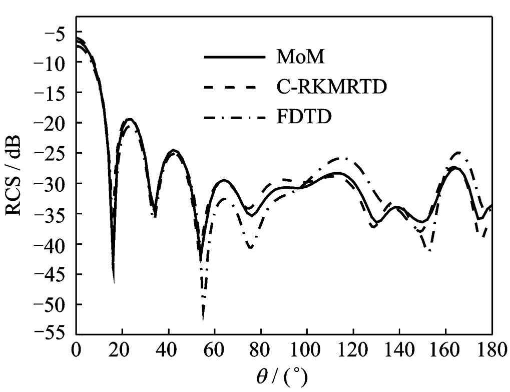


Fig. 6 Bistatic RCS in H-plane of the dielectric ellipsoid using different methods

Table 2 Comparison for different methods

Method	MRTD	C-RKMRTD
Δx	0.03	0.03
Δt	30	30
Cell	118×118×92	118×118×92
Total time	2 000	2 000
CPU time /s	4 125.37	8 046.79

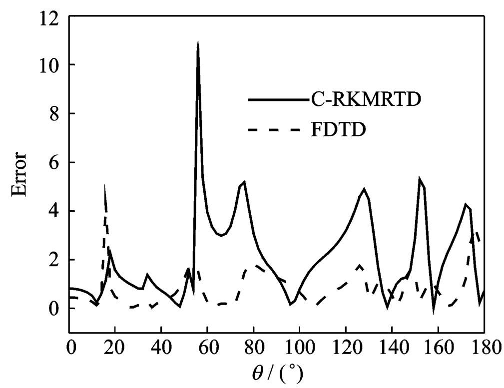


Fig. 8 Difference between FDTD/C-RKMRTD and the MoM method

4 Conclusion

An efficient approach that combines the conformal technique and RK-MRTD method is implemented to model the curved objects for the scattering problems. Numerical results demonstrate the higher accuracy and efficiency of the proposed method, compared with non-conformal methods including the MRTD and FDTD methods.

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