Analysis on Output Power for Multi-direction Piezoelectric Vibration Energy Harvester

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Abstract: To predict the performance of multi-direction piezoelectric vibration energy harvester, an equation for calculating its output power is obtained based on elastic mechanics theory and piezoelectricity theory. Experiments are performed to verify theoretical analysis. When the excitation direction is along Y direction, a maximal output power about 0.139 mW can be harvested at a resistive load of 65 kΩ and an excitation frequency of 136 Hz. Theoretical analysis agrees well with experimental results. Furthermore, the performance of multi-direction vibration energy harvester is experimentally tested. The results show that the multi-direction vibration energy harvester can harvest perfect energy as the excitation direction changes in XY plane, YZ plane, XZ plane and body diagonal plane of the harvester.

Key words: multi-direction; vibration energy harvesting; piezoelectric transducer; output power

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1 Introduction

The use of wireless sensors and wearable electronics has been growing fast over the past decades. Providing efficient and clean power for these devices is a challenge. In most cases, all these electronics have relied on electrochemical batteries. However, the reduction of battery lifespan limits the functionality of the devices. Methods of obtaining electrical energy from the ambient energy surrounding the device have been investigated to extend its life. Potential energy sources available in the operating environment of microdevices include solar, thermal, acoustic vibrations, temperature gradient, mechanical, electrical, and some combination thereof. More and more researchers have been interested in harvesting ambient vibration energy for its widespread and high density[1]. However, it is difficult to harvest multi-direction vibration energy.

After reviewing and compairing different methods of scavenging vibration energy, we find that piezoelectric harvester[2-7] is very promising, because it has significantly higher efficiency than other potential power scavenging technologies, such as electrostatic harvesters[8], electromagnetic harvesters[9-10], and so on. Furthermore, the piezoelectric harvesters require no external voltage source and are particularly attractive for micro-electro mechanical system (MEMS). As a result, piezoelectric materials for scavenging energy from ambient vibration sources have dramatically risen including resonant piezoelectric-based structures of cantilever beam configuration, piezoelectric 'cymbal' transducers, piezoelectric 'drum' transducers, and piezoelectric windmill for generating electric energy from wind energy[11-13].

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However, these piezoelectric devices work only in one direction and present low efficiency when surrounding with random vibration. To harvest multi-direction ambient vibration energy, a new multi-direction piezoelectric vibration energy harvester was proposed[14].

In order to predict the performance of the multi-direction piezoelectric vibration energy harvester, an equation for calculating the output power of the harvester is obtained based on elastic mechanics theory and piezoelectricity theory. Experiments are performed to verify the theoretical analysis and to test the performance of the multi-direction piezoelectric vibration energy harvester.

2 Theoretical Model and Numerical Simulations

Fig. 1 shows a schematic diagram of the multi-direction piezoelectric vibration energy harvester. It can be seen that the harvester includes cubic shape metal framework, metal mass, and eight identical rainbow shape piezoelectric transducers. In order to harvest multi-direction ambient vibration energy, the universal flexure hinge is applied to connect rainbow shape piezoelectric transducers with cubic shape metal framework and metal mass. In applications, the multi-direction piezoelectric vibration energy harvester can be fixed on the bridges or vehicles to harvest ambient vibration energy. Of course, the vibration directions of the vibration sources are random.

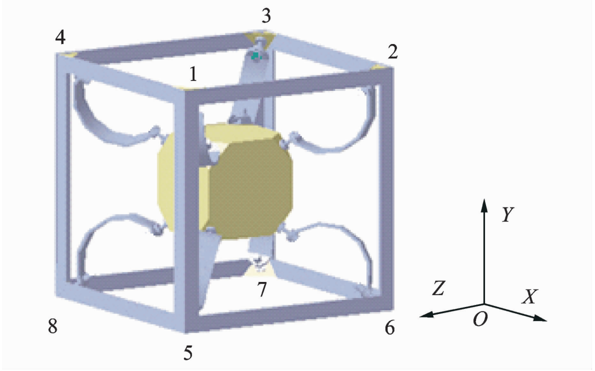

Fig. 1 Multi-direction piezoelectric vibration energy harvester

Fig. 2 shows that the rainbow shape piezoelectric transducer consists of a metal flexible substrate, two piezoelectric films and four electrodes. The metal flexible substrate is sandwiched between two piezoelectric films and the piezoelectric film is sandwiched between two electrodes. The rainbow shape piezoelectric transducer can be deformated as the multi-direction piezoelectric vibration energy harvester vibrates. Therefore, the strain and the stress of the piezoelectric films change at the same time. According to piezoelectricity theory, electric charge can be generated on the surface of the piezoelectric films as the strain and the stress change.

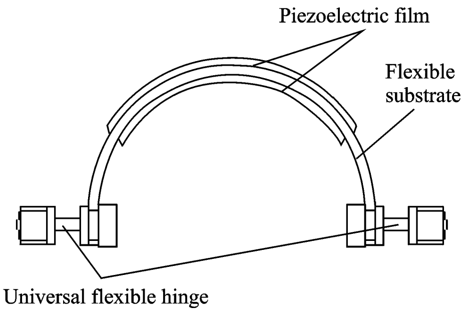

Fig. 2 Rainbow shape piezoelectric transducer

Assume the multi-direction piezoelectric vibration energy harvester is excited by a excitation $u\sin(\omega t)$ and the intersection angles between the excitation and the three axes (X, Y and Z) are α, β and γ. In order to obtain the response of the metal mass, the excitation can be decomposed along three axes. Based on vibration theory, the vibration component along three axes can be expressed by

$$u_{Xj} = u\cos\alpha\sin(\omega t) \tag{1}$$

$$u_{Yj} = u\cos\beta\sin(\omega t) \tag{2}$$

$$u_{Zj} = u\cos\gamma\sin(\omega t) \tag{3}$$

When the multi-direction piezoelectric vibration energy harvester is excited by u_{Xj}, u_{Yj} and u_{Zj} respectively, the kinetic equation of the harvester can be obtained as

$$m\ddot{u}_X + c_{\mathrm{h}}\dot{u}_X + k_{\mathrm{h}}u_X = -m\ddot{u}_{Xj} \tag{4}$$

$$m\ddot{u}_Y + c_{\mathrm{h}}\dot{u}_Y + k_{\mathrm{h}}u_Y = -m\ddot{u}_{Yj} \tag{5}$$

$$m\ddot{u}_Z + c_{\mathrm{h}}\dot{u}_Z + k_{\mathrm{h}}u_Z = -m\ddot{u}_{Zj} \tag{6}$$

where m is the mass of metal mass; c_h the damping factor of harvester; k_h the equivalent stiffness of harvester; and u_X, u_Y and u_Z are the displacement component along X, Y and Z.

If the excitation force applied on the Rainbow shape piezoelectric transducer is $F_k(k=1,2,\cdots,8)$ and R is the initial curvature radius of the transducer. b_p and l_p and t_p are the width, the length and the thickness of the piezoelectric film, respectively. b_m, l_m and t_m are the width, the length and the thickness of the metal flexible substrate, respectively. E_p and E_m are the elastic modulus of the piezoelectric film and the metal flexible substrate, respectively. e_{31} is the piezoelectric stress constant. Based on the calculation formulas of piezoelectric materials potential energy and the mechanical analysis of the multi-direction vibration energy harvester, the charge generated of every piezoelectric film can be expressed as ($n=1$ denotes arc inside piezoelectric film and $n=2$ denotes arc outside piezoelectric film)[14]

$$Q_{\mathrm{p},k,n}=\frac{E_\mathrm{p}R^2e_{31}b_\mathrm{p}^2F_k}{2(h^2-ae)^2}\left[-\varphi_{4,n}+(-1)^n\frac{1}{2}(t_\mathrm{p}+t_\mathrm{m})\varphi_{5,n}\right] \tag{7}$$

where

$$\varphi_{4,n}=-2[p_{1,n}p_4e^2-p_{1,n}p_5he-2p_{2,n}p_4he+p_{2,n}p_5(h^2+ae)+p_{3,n}p_4h^2-p_{3,n}p_5ah]$$

$$\varphi_{5,n}=-2[-p_{1,n}p_4he+p_{1,n}p_5h^2+p_{2,n}p_4(h^2+ae)-2p_{2,n}p_5ah-p_{3,n}p_4ah+p_{3,n}p_5a^2]$$

$$a=2E_\mathrm{p}b_\mathrm{p}t_\mathrm{p}+E_\mathrm{m}b_\mathrm{m}t_\mathrm{m}$$

$$h=\left(\frac{1}{2}t_\mathrm{m}^2+t_\mathrm{m}t_\mathrm{p}+\frac{2}{3}t_\mathrm{p}^2\right)\frac{E_\mathrm{p}b_\mathrm{p}t_\mathrm{p}}{R}+\frac{1}{12R}E_\mathrm{m}b_\mathrm{m}t_\mathrm{m}^3$$

$$e=Rh$$

$$p_{1,n}=4t_\mathrm{p}/\{[2R+(-1)^n2t_\mathrm{p}+(-1)^nt_\mathrm{m}][2R+(-1)^nt_\mathrm{m}]\}$$

$$p_{2,n}=Rp_{1,n}-(-1)^n\ln[1+(-1)^n2t_\mathrm{p}/(2R+(-1)^nt_\mathrm{m})]$$

$$p_{3,n}=t_\mathrm{p}-R^2p_{1,n}+2Rp_{2,n}$$

$$p_4=-2R\sin\left(\frac{l_\mathrm{p}}{2R}\right)$$

$$p_5=R\left[l_\mathrm{p}\cos\left(\frac{l_\mathrm{m}}{2R}\right)-2R\sin\left(\frac{l_\mathrm{p}}{2R}\right)\right]$$

If the piezoelectric strain constant is d_{31} and the relative dielectric constant is ε_{33}, then the equivalent capacitor of the piezoelectric film is represented by[14]

$$C_{\mathrm{p},k,n}=\frac{E_\mathrm{p}R^2e_{31}^2b_\mathrm{p}^3l_\mathrm{p}}{2(h^2-ae)^2}\left[2\varphi_{1,n}+\frac{1}{2}(t_\mathrm{p}+t_\mathrm{m})^2\varphi_{2,n}+(-1)^{n+1}(t_\mathrm{p}+t_\mathrm{m})\varphi_{3,n}\right]+\frac{l_\mathrm{p}b_\mathrm{p}}{t_\mathrm{p}}(\varepsilon_{33}-e_{31}d_{31}) \tag{8}$$

where

$$\varphi_{1,n}=p_{1,n}e^2-2hep_{2,n}+p_{3,n}h^2$$

$$\varphi_{2,n}=p_{1,n}h^2-2ahp_{2,n}+p_{3,n}a^2$$

$$\varphi_{3,n}=-2p_{1,n}he+2p_{2,n}(h^2+ae)-2p_{3,n}ah$$

Figs. 3, 4 show the equivalent circuit of the piezoelectric film with a resistive load. Based on the definition of current and diversion current principle of the circuit, we have

$$i_{1,k,n}=\frac{\mathrm{d}Q_{\mathrm{p},k,n}}{\mathrm{d}t}-C_{\mathrm{p},k,n}\frac{\mathrm{d}V_{\mathrm{p},k,n}}{\mathrm{d}t} \tag{9}$$

$$i_{2,1,1}+i_{2,1,2}+\cdots+i_{2,8,2}=\frac{V_\mathrm{r}}{r}+C_\mathrm{f}\frac{\mathrm{d}V_\mathrm{r}}{\mathrm{d}t} \tag{10}$$

where V_r is the resistive voltage, r the resistive load, C_f the capacitor, $i_{1,k,n}$ the current of the piezoelectric film, $V_{\mathrm{p},k,n}$ the voltage of the piezoelectric film, $i_{2,1,1}$, $i_{2,1,2}$, $\cdots$, $i_{2,8,2}$ are rectification circuit current.

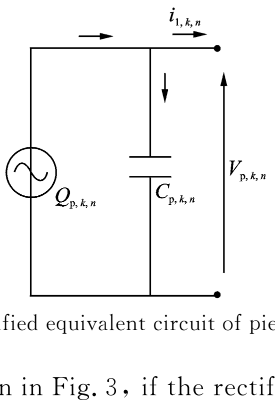

Fig. 3 Simplified equivalent circuit of piezoelectric film

As shown in Fig. 3, if the rectification circuit has no effect on the charge wave form generated in the harvester, we have

$$\int_{t_\mathrm{ini}}^{t_\mathrm{fin}}i_{1,1,1}\mathrm{d}t+\int_{t_\mathrm{ini}}^{t_\mathrm{fin}}i_{1,1,2}\mathrm{d}t+\cdots+\int_{t_\mathrm{ini}}^{t_\mathrm{fin}}i_{1,8,2}\mathrm{d}t=\frac{TV_\mathrm{r}}{2r} \tag{11}$$

where $t_\mathrm{ini}-t_\mathrm{fin}=T/2$ and $T=2\pi/\omega$.

By Eqs(9−11), the load voltage and output power of the multi-direction piezoelectric vibration energy harvester can be obtained as

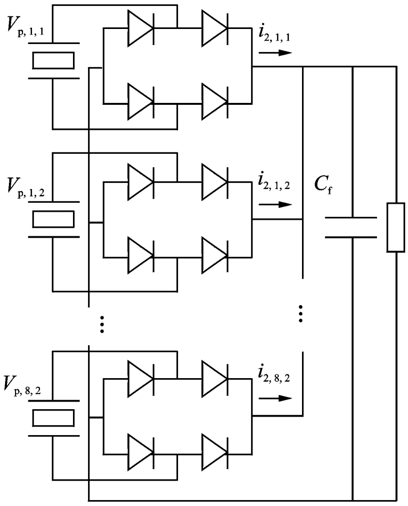

Fig. 4 Passive electronic interfaces for multiple piezoelectric films

$$P_r = [\omega^2 r(V_{0.1.1}C_{P,1,1} + V_{0,1,2}C_{P,1,2} + \cdots + V_{0,8,2}C_{P,8,2})^2]/[\omega r(C_{P,1,1} + C_{P,1,2} + \cdots + C_{P.8.2}) + 0.5\pi]^2 \quad (12)$$

where P_r is the output power of the harvester.

Some numerical results are presented for illustration in the following. In the calculations, we use polyvinylidene fluoride (PVDF) as our piezoelectric material and use beryllium bronze as our flexible substrate material. Table 1 gives some material properties and structural parameters of the multi-direction piezoelectric vibration energy harvester. Others are given as below

$$u = 0.2\ \text{mm},\ c_h = 0.026$$

$$\boldsymbol{E} = \begin{bmatrix} 0 & 0 & 0.0104 \\ 0 & 0 & -0.0164 \\ 0 & 0 & -0.065 \\ 0 & 0 & 0 \\ -0.0388 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{c} = \begin{bmatrix} 8.1 & 4.84 & 4.84 & 0 & 0 & 0 \\ 4.84 & 6.92 & 4.38 & 0 & 0 & 0 \\ 4.84 & 4.38 & 6.92 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.38 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.38 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.38 \end{bmatrix}$$

where the unit of $\boldsymbol{E}$ is C/m^2 and the unit of $\boldsymbol{c}$ is 10^9 N/m^2.

Table 1 Dimensions and material properties of the harvester

Parameter	PVDF	Beryllium bronze	Metal mass
Density/(kg · m^3)	1 780	8 290	7 850
Elastic modulus/GPa	2	131	
Poisson ratio	0.3	0.35	
Thickness/mm	0.2	0.1	
Width/mm	4	4	
Length/mm	12	15.7	
Initial curvature radius/mm	5	5	
Relative dielectric constant mass/g	12		33

Fig. 5 shows the output power of the harvester as the excitation direction is along Y direction. A maximal power of about 0.17 mW can be harvested at an excitation frequency of 132 Hz and a resistive load of 65 kΩ. It can be found that the output power initially increases with the resistive load, whereas it decreases when the resistive load is further increased. The output power reaches maximum at an optimum resistive load, which is equal to the equivalent impedance of the harvester for each frequency.

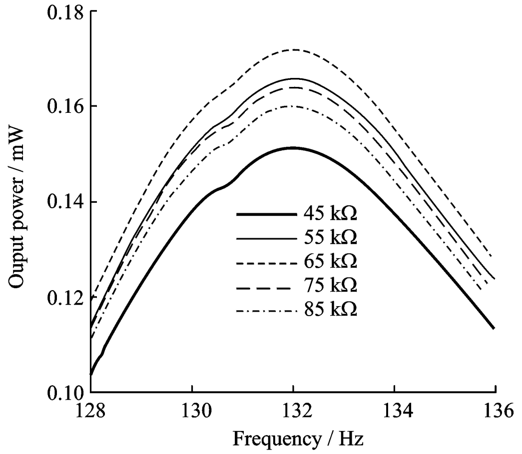

Fig. 5 Effect of resistive load and frequency on output power of harvester

3 Experimental Results and Discussion

To verify theoretical analysis results and to test the performance of the multi-direction piezoelectric vibration energy harvester, experiments are conducted. Fig. 6 shows the multi-direction vibration energy harvester. A mechanical shaker controlled by a high-power amplifier and a function generator was used to measure the response of the multi-direction vibration energy harvester. This shaker can apply a maximal force of 50 N in a frequency band of 5—3 000 Hz. The output voltage from the multi-direction vibration energy harvester across a rectification circuit was measured using a digital oscilloscope.

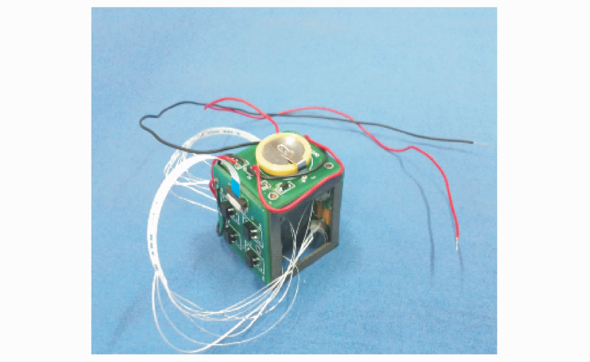

Fig. 6 Multi-direction piezoelectric vibration energy harvester

Fig. 7 shows the experimental results of output power of the harvester as the excitation direction is along Y direction. A maximal power of about 0.139 mW can be harvested at an excitation frequency of 136 Hz and a resistive load of 65 kΩ. It can be found that the theoretical results agree well with the experimental ones. The variation of the power with frequency and resistive load follows the similar trend with that of the predicted model.

In order to investigate the effect of the excitation direction on the output power of the harvester, experiments are launched with an excitation frequency of 136 Hz and resistive load of 65 kΩ. Fig. 8 shows the output power of the harvester as the excitation direction changes in XY plane. The results show that the output power of the harvester reaches the maximum as the excitation direction is along plane diagonal direction of the harvester and the maximal value is 0.27 mW. The minimal output power is 0.14 mW as the excitation direction is along X direction and Y direction. Because of the symmetry, the multi-direction piezoelectric vibration energy harvester has the similar results as the excitation direction changes in YZ plane and XZ plane.

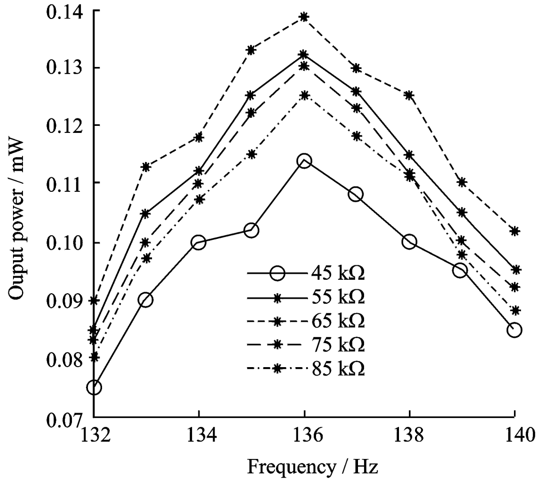

Fig. 7 Effect of resistive load and frequency on output power of harvester

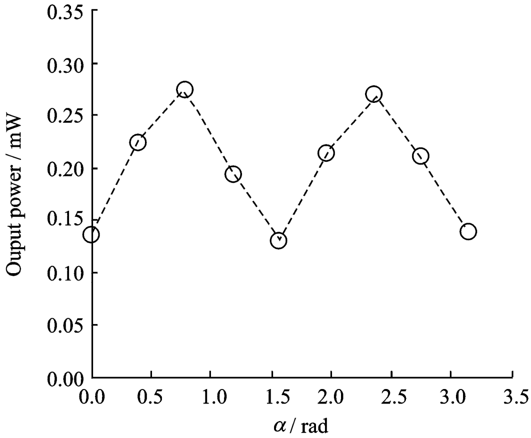

Fig. 8 Output power of harvester with excitation direction changing in XY plane

Fig. 9 shows the output power of the harvester as the excitation direction changes in body diagonal plane of the harvester. The results show that the output power of the harvester reaches the maximum as the excitation direction is along body diagonal direction of the harvester and the maximal output power is 0.28 mW. The minimal output power is 0.14 mW as the excitation direction is along Y direction.

As shown in Figs. 8, 9, the ratio of the maximal output power and the minimal output power of the multi-direction piezoelectric vibration energy harvester is 2. Thus, the harvester can har-

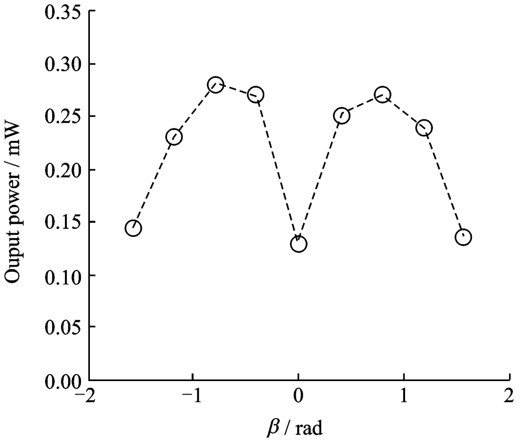

Fig. 9 Output power of harvester with excitation direction changing in body diagonal plane

vest perfect energy as the excitation direction changes in XY plane, YZ plane, XZ plane and body diagonal plane.

Energy conversion efficiency of the harvester is computed from the ratio of input mechanical energy to the output electrical energy. An estimation of the energy conversion efficiency is carried out.

During the experiments, the maximal displacement applied from the shaker is 0. 2 mm with a payload of 0. 69 kg at a frequency of 136 Hz. Then, the average input power can be calculated by[15]

$$P_{\mathrm{in}}=\frac{2}{T}\int_{0}^{\frac{T}{2}}\frac{1}{2}Mu^{2}\omega^{2}[\cos(\omega t)]^{2}\mathrm{d}t=5.03\ \mathrm{mW} \tag{13}$$

where M is the payload of harvester; P_{in} the average input power of harvester; and T the vibration period of harvester.

Based on the experimental results, the maximal output power is 0. 28 mW at a frequency of 136 Hz with a resistive load of 65 kΩ. Hence, the maximal energy conversion efficiency can be calculated by

$$\eta_{\max}=\frac{P_{\mathrm{out}}}{P_{\mathrm{in}}}=5.57\% \tag{14}$$

The calculation results show that the energy conversion efficiency is low because of two points: (1) The piezoelectric constant of PVDF is smaller than the piezoelectric ceramics; (2) the efficiency of the energy storage circuit is low.

4 Conclusions

(1) An equation for calculating the output power of the harvester is obtained to predict the performance of the harvester.

(2) Theoretical results agree well with experimental ones. When the excitation direction is along Y direction, a maximal output power of around 0. 139 mW can be harvested at an excitation frequency of 136 Hz and a resistive load of 65 kΩ. When the excitation direction changes in XY plane, the maximal output power of is 0. 27 mW as the excitation direction is along plane diagonal direction of the harvester. When the excitation direction changes in body diagonal plane, a maximal output power of around 0. 28 mW can be harvested as the excitation direction is along body diagonal direction. The ratio of the maximal output power and the minimal output power is 2, thus, the multi-direction piezoelectric vibration energy harvester can harvest perfect energy as the excitation direction changes in XY plane, YZ plane, XZ plane and body diagonal plane of the harvester.

(3) It can be found that the maximal energy conversion efficiency of the harvester is low because: ① The piezoelectric constant of PVDF is smaller than the piezoelectric ceramics; ② the efficiency of the energy storage circuit is low. Hence, in the subsequent research, we may consider to design transducer using piezoelectric ceramics materials and design some new energy storage circuits.

References:

[1] Roundy S, Wright P K, Pister K S. Micro-electrostatic vibration to electricity converters [C]//Proceedings of ASME International Mechanical Engineering Congress & Exposition. New Orleans, Louisiana: ASME, 2002:1-10.

[2] Chew Z J, Li L J. Design and characterization of a piezoelectric scavenging device with multiple resonant frequencies[J]. Sensors and Actuators A, 2010,162 (1):82-92.

[3] Kan J W, Wang S Y, Peng S F, et al. Output per-

formance of piezoelectric generators with multi-vibrators[J]. Optics and Precision Engineering, 2011,19(9):2108-2116. (in Chinese)

[4] Wang S Y, Kan J W, Wang H Y, et al. Piezoelectric energy generator based on deflection-limiting circular arc[J]. Optics and Precision Engineering, 2013,21(2):342-348. (in Chinese)

[5] Liu X J, Chen R W, Zhu L Y. Energy conversion efficiency of rainbow shape piezoelectric transducer[J]. Chinese Journal of Aeronautics, 2012,25(5):691-697.

[6] Erturk A, Renno J M, Inman D J. Piezoelectric energy harvesting from a L-shaped beam-mass structure with an application to UAVs[J]. Journal of Intelligent Material Systems and Structures, 2009,20(5):529-544.

[7] He X F, Du Z G, Zhao X Q, et al. Modeling and experimental verification for cantilevered piezoelectric vibration energy harvester[J]. Optics and Precision Engineering, 2011,19(8):1771-1778. (in Chinese)

[8] Mitcheson P D, Miao P, Stark B H, et al. MEMS electrostatic micropower generator for low frequency operation[J]. Sensors and Actuators A, 2004,115(2/3):523-529.

[9] Foisal A R M, Chinsuk H, Chung G S. Multi-frequency electromagnetic energy harvester using a magnetic spring cantilever[J]. Sensors and Actuators A, 2012,182(4):106-113.

[10] Owens B A M, Mann B P. Linear and nonlinear electromagnetic coupling models in vibration-based energy harvesting[J]. Journal of Sound and Vibration, 2012,331(4):922-937.

[11] Lu Y G, Yan Z F. Finite element analysis on energy harvesting with cymbal transducer[J]. Journal of Vibration and Shock, 2013,32(6):157-162. (in Chinese)

[12] Yuan J B, Xie T, Shan X B, et al. Resonant frequencies of a piezoelectric drum transducer[J]. Journal of Zhejiang University-Science A: Applied Physics & Engineering, 2009,10(9):1313-1319.

[13] Priya S, Chen C T, Fye D, et al. Piezoelectric windmill: a novel solution to remote sensing[J]. Japanese Journal of Applied Physics, 2005,44(3):104-107.

[14] Liu X J, Chen R W. Analysis of the load voltage and output power for Rainbow shape piezoelectric monomorph energy transferring elements[J]. Acta Aeronautica et Astronautica Sinica, 2011,32(3):561-570. (in Chinese)

[15] Yuan J B, Shan X B, Xie T, et al. Modeling and improvement of a cymbal transducer in energy harvesting[J]. Journal of Intelligent Material Systems and Structures, 2010,21(8):765-771.

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