Simulation Analysis and Experiment on Piezoelectric Cantilever Vibrator

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Abstract: Piezoelectric cantilever bimorph vibrator is the core component of piezoelectric inertial actuators. Piezoelectric cantilever bimorph vibrator with lumped mass at the end is studied, and a dynamic model is constructed by the proposed method of transforming symmetric electrical signal excitation to equivalent harmonic force excitation. Combining the theory and test, the single-degree-of-freedom damp vibration system consisting of the vibrator is analyzed and the full response is provided. Dynamical system modeling and simulation are conducted by software Matlab/Simulink. The output response of deflection, velocity, acceleration, driving force are solved using the input signal of equivalent harmonic force. Based on the data of theory, simulation and experiments, the deflection of vibrator is analyzed and compared under the excitation of sine wave signal. Result shows that the dynamic deflection results of vibrator from theory, simulation and experiments agree well with each other. The proposed modeling and analysis approaches of the single-degree-of-freedom damp vibration system consisting of the vibrator, have some reference significance for further research of piezoelectric inertial actuators.

Key words: piezoelectric cantilever vibrator; vibration; Simulink; dynamic characteristic

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0 Introduction

In recent years, with the rapid development of technology and society, the demand of the automation control technology of micro displacement has increased in the areas of optics, aerospace, electronic technique, mechanical manufacturing, medical engineering, bionics, genetic engineering, and so on[1-3]. The traditional actuators, using electromagnetic, hydraulic and pneumatic driving modes, can not satisfy the higher need of high response, miniaturization and anti-electromagnetic interference in the workplace[4]. As a consequence, the domestic and foreign research institutions have been committed to develop new kinds of precision actuators. The types of precision actuators including artificial muscle, shape memory alloy, magnetostrictive and electrostrictive actuators become the important branches of the mainstream driving types in the research and developing direction, causing wide concern in the scientific and industrial community all over the world[5-7]. Piezoelectric actuators have been widely applied in precision machining, optical focusing and bionic devices because of their advantages of small volume, large stroke, high response, high resolution and large output energy per unit area[8]. The inertial actuator, commonly driven by piezoelectric stack, piezoelectric bimorph or piezoelectric film, is built up by utilizing the reverse piezoelectric effect of piezoelectric ceramic to realize the movement[9].

So far, a lot of research and developing work on piezoelectric driving devices have been conducted by domestic and overseas scholars. A linear

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ultrasonic piezoelectric motor, which could achieve a maximum speed of 55.6 mm/s and a push force of 0.348 N at a preload of 6 N, was designed and tested by Sharp et al[10]. An inchworm-type piezoelectric actuator of large displacement and force for shape control and vibration control of adaptive truss structures was presented by Li et al[11]. Moreover, a novel piezoelectric inertial motor based on inertial motion of the slider with the maximum 20 mm/s velocity and 78.45 mN output force was presented by Mazeika and Vasiljev[12]. Zhang Chunlin et al.[13] had manufactured a rhombic micro-displacement piezoelectric actuator and it could amplify the displacement output from 30 μm to 100 μm by using a rhombic amplification mechanism. Innovative two-degree-of-freedom piezoelectric actuators, which apply piezoelectric buzzers to play as a driving source, were proposed by Chen and Liu[14]. The maximum linear and angular velocity could reach 21 mm/s and 3.72 rad/s, respectively, and the output force of 2.32 mN could be obtained. For the study of the piezoelectric cantilever bimorphs, Ge Yangxiang et al.[15] analyzed the longitudinal oscillation of the piezoelectric bimorph by the general admittance frequency response equation of the longitudinal oscillation, which is obtained from the piezoelectric equations and wave equation. Cheng Guangming et al.[16] studied the effect of clamping length on the dynamic characteristics of a cantilever piezoelectric bimorph vibrator. Ye Huiying and Pu Zhaobang[17] proposed a dynamic parameter testing model of piezoelectric materials based on dynamic admittance matrix. A certain method of fixed frequency and adjustable amplitude to control the actuator was proposed by Zhang Hongzhuang et al[18] according to finite element analysis and experiments of the dynamic characteristics of end loaded piezoelectric cantilever bimorph.

After analyzing the research status of piezoelectric cantilever bimorphs at present, the method of transforming symmetric electrical signal excitation to equivalent harmonic force excitation to construct the dynamic model is proposed. Furthermore, the vibration system mainly consisting of the piezoelectric cantilever bimorph vibrator with lumped mass at the end is analyzed by the theory and experiments.

1 Dynamic Equation of Vibrator

The piezoelectric cantilever bimorph vibrator mainly comprises metal layer and piezoelectric layer, as shown in Fig. 1.

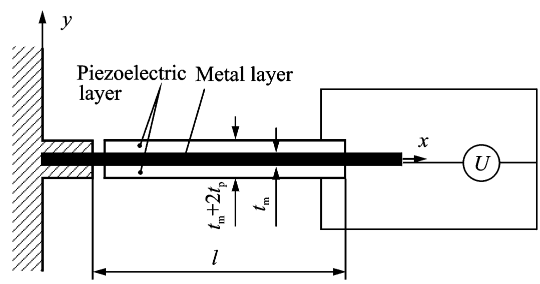


Fig. 1 Structure of piezoelectric cantilever bimorph vibrator

Based on the beam theory, the static tip deflection δ of the piezoelectric cantilever bimorph vibrator under the voltage U is equivalent to the deflection caused by a force F at the end of vibrator. This equivalent force can be written as[19]

$$F=\frac{3w(t_m+2t_p)^2E_p}{8l}\cdot\frac{t_m/t_p+1}{(t_m/2t_p+1)^2}\cdot d_{31}\frac{U}{t_p} \tag{1}$$

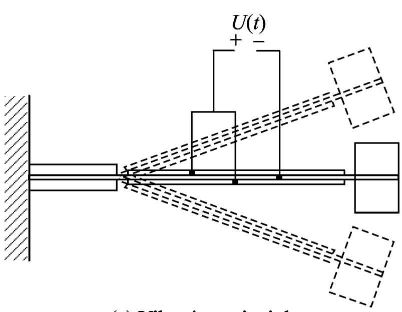
where E_p is the piezoelectric elasticity modulus, d_{31} the piezoelectric constant, t_m the thickness of metal layer, t_p the thickness of piezoelectric layer, w the width, and l the effective length.

Using $C=\frac{3w(t_m+2t_p)^2E_p}{8l}\cdot\frac{t_m/t_p+1}{(t_m/2t_p+1)^2}\cdot\frac{d_{31}}{t_p}$ as defined above, Eq. (1) can be simplified as

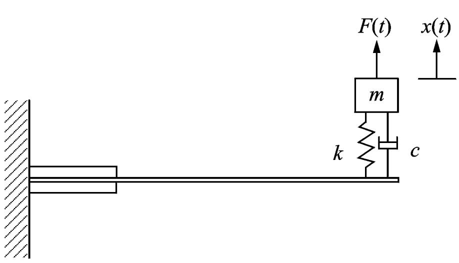
$$F=C\cdot U \tag{2}$$

Fig. 2(a) shows the vibration principle of the piezoelectric cantilever bimorph vibrator which is fixed on the left by the clamping blocks and free on the right. Under exciting signal, the expansion of piezoelectric layer makes the vibrator bend because of the inverse piezoelectric effect, finally leading to the vibration of the bimorph piezoelectric vibrator under the periodic symmetrical electrical signal $U(t)$.

The theoretical analysis of the vibrator indicates that the periodic symmetrical electrical signal $U(t)$ on the piezoelectric layer can be equivalent to the harmonic force $F(t)$ on the end of the vibrator. As long as the conditions of simple harmonic excitation are satisfied, the dynamic model can be constructed for the single-degree-of-freedom damp vibration system consisting of the vibrator, as shown in Fig. 2(b). Where m is the mass of mass block, $x(t)$ the displacement of mass block, k the system stiffness coefficient, and c the system damping coefficient. It should be noted that the distance between the single-degree-of-freedom damp vibration system and the fixed point in the left is about 50 mm and an effective length of the vibrator, which is equivalent to the distance between the geometric center of the mass block and the fixed point.



(a) Vibration principle



(b) Dynamic model

Fig. 2 Vibration principle and dynamic model of piezoelectric cantilever bimorph vibrator

According to the dynamic model, the dynamical equation of the bimorph piezoelectric vibrator can be written as

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=F(t)-kx-c\frac{\mathrm{d}x}{\mathrm{d}t} \tag{3}$$

The equivalent harmonic force of the system is

$$F(t)=C\cdot U(t) \tag{4}$$

Substituting Eq. (4) into Eq. (3) yields the following expression, the dynamical equation can be written as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\frac{c}{m}\frac{\mathrm{d}x}{\mathrm{d}t}+\frac{k}{m}x=\frac{C}{m}U(t) \tag{5}$$

Based on the mechanical vibration theory[20], substituting $c/m=2\zeta\omega_n$, $\omega_n=\sqrt{k/m}$ and $CU(t)=F_0\cos\omega t$ into Eq. (5) leads to the standard motion equation of the damp vibration system under the harmonic force

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+2\zeta\omega_n\frac{\mathrm{d}x}{\mathrm{d}t}+\omega_n^2x=\frac{F_0}{m}\cos\omega t \tag{6}$$

where ζ is the damping ratio, ω_n the undamped natural frequency, F_0 the amplitude, and ω the frequency of the equivalent input harmonic force, respectively.

The full response of the system consists of the transient response and steady-state response

$$x(t)=X_0\mathrm{e}^{-\zeta\omega_n t}\cos(\omega_d t-\varphi_0)+X\cos(\omega t-\varphi) \tag{7}$$

where ω_d is the damped natural frequency, $\omega_d=\sqrt{1-\xi^2}\,\omega_n$

It can be seen from Eq. (5) that the value of the stiffness coefficient k, the damping coefficient c, the mass m and the equivalent harmonic force $F(t)$ should be predetermined before solving the full response of the system under an excitation signal. The above parameters will be got one by one after analyzing the vibration system in Part 2 of the paper.

2 Analysis of Vibration System

It can be easily seen from the theoretical analysis in Part 1 that the bimorph piezoelectric vibrator excited by the equivalent harmonic force constitutes the single-degree-of-freedom damp vibration system. In order to explore the input-output relationship of the system, the theory formula of full response is calculated and the parameters support for the simulation of next stage are provided. The vibration system should be analyzed to solve the value of the stiffness coefficient, damping coefficient, mass and equivalent harmonic force of the system.

2.1 Stiffness analysis

The stiffness coefficient of the system can be calculated from the theoretical equation

$$k=\frac{F}{\delta} \quad (8)$$

where F is the equivalent force at the end of the bimorph piezoelectric vibrator, and δ the static tip deflection. They can be written as

$$F=\frac{3w(t_m+2t_p)^2E_p}{8l}\cdot\frac{2B+1}{(B+1)^2}\cdot d_{31}\frac{U}{t_p} \quad (9)$$

$$\delta=\frac{3l^2}{2(t_m+2t_p)}\cdot\frac{(1+B)\cdot(1+2B)}{AB^3+3B^2+3B+1}\cdot d_{31}E_3 \quad (10)$$

where $A=E_m/E_p$, $B=t_m/2t_p$, E_m and E_p are the elasticity modulus, respectively, t_m and t_p the thickness of the metal layer and piezoelectric layer, respectively.

Substituting Eqs. (9,10) into Eq. (8), the stiffness coefficient k can be written as

$$k=\frac{(E_m t_m^3+6t_m^2E_pt_p+12t_mE_pt_p^2+8E_pt_p^3)w}{4l^3} \quad (11)$$

Table 1 shows the parameter values of the formula above, and it can be got that $k=1.528\times10^3$ N/m after calculation.

Table 1 Parameters for calculating system stiffness coefficient

Parameter	$E_m/(N\cdot m^{-2})$	$E_p/(N\cdot m^{-2})$	t_m/m	t_p/m	l/m	w/m
Value	11.2×10^{10}	8.4×10^{10}	0.2×10^{-3}	0.2×10^{-3}	39×10^{-3}	20×10^{-3}

2.2 Damping analysis

After reviewing massive references, it can be easily seen that the theoretical calculation of the system structure damping is very complicated when the system is made from a variety of materials[21]. The system damping can be usually got by the experimental resonance method. In vibration experiments, the damping ratio ζ can be calculated as

$$\frac{X_{max}}{\delta_{st}}=\frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (12)$$

where X_{max} is the dynamic response maximum amplitude, and δ_{st} the static response amplitude of the vibrator.

In tests, the static response is got when a 20 V DC signal excites the vibrator and finally the static response amplitude is $\delta_{st}=85\times10^{-6}$ m. On the other hand, the dynamic response is analyzed under 20 V, 1—20 Hz sinusoidal signals. Fig. 3 illustrates the amplitude-frequency response curve and it can be got that $X_{max}=177\times10^{-6}$ m. After calculation, the damping ratio is $\zeta=0.247$.

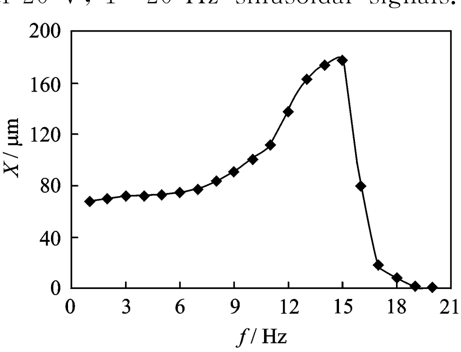


Fig. 3 Amplitude-frequency response of piezoelectric cantilever bimorph vibrator

$$\zeta=\frac{c}{c_c} \quad (13)$$

The damping coefficient c can be figured out by the formula shown above, among which c_c is the critical damping coefficient and $c_c=2\sqrt{mk}$. Finally, the damping coefficient c is equal to 0.60 N/(m·s^{-1}) after calculation.

2.3 Mass and equivalent harmonic force analysis

The precision electronic balance is used to get the mass of the mass block, which is equivalent to the mass of the single-degree-of-freedom damp vibration system constituted by the bimorph piezoelectric vibrator because the mass block occupies most of the mass of the system. Finally, the mass m is equal to 1.29×10^{-2} kg.

The formula Eq. (4) of the equivalent harmonic force is obtained after analyzing the dynamical equation of the bimorph piezoelectric vibrator. Therefore, the input harmonic force equivalent can be figured out from it.

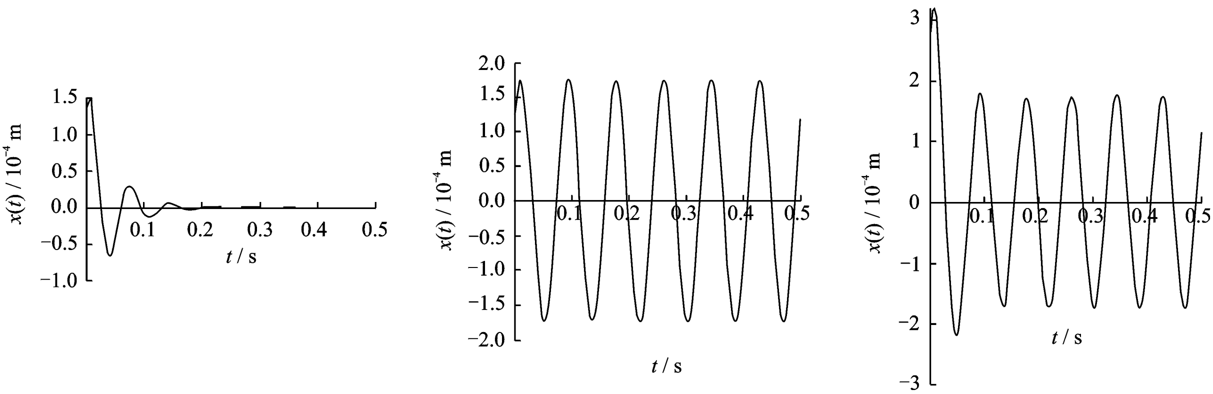
3 Theory, Simulation and Experiment Analysis and Comparison

3.1 Theoretical analysis

After analyzing the single-degree-of-freedom damp vibration system constituted by the vibrator, the stiffness coefficient, damping coefficient, mass and equivalent harmonic force are substituted into the system dynamic equation. Then the result of solving the full response of the vibrator under a 20 V, 12 Hz sinusoidal signal is shown in Eq. (14)

$$x(t)=1.80\times10^{-4}e^{-23.29t}\cos(91.33t-0.86)+1.75\times10^{-4}\cos(75.40t-0.83) \quad (14)$$

Finally, the curves of the transient response, state steady response and full response can be drawn according to Eq. (14), respectively, as shown in Fig. 4.



(a) Transient response (b) Steady state response (c) Full response

Fig. 4 End displacement responses of piezoelectric cantilever bimorph vibrator

3.2 Simulink simulation analysis

The software Matlab/Simulink has the advantage of visual modeling, so it is usually used by researchers to complete the modeling, simulation and analysis by synthesis. In order to reveal and analyze the dynamic characteristics of the vibration system, the Simulink simulation model of the vibrator is established, as shown in Fig. 5.

Table 2 shows the simulation parameters. The input excitation signal $U(t)$ is 20 V, 12 Hz sinusoidal signal. The amplitude and frequency of equivalent harmonic force caused by the signal are $A_{F(t)}$, $f_{F(t)}$, respectively, which can be solved by Eq. (4). Fig. 6 demonstrates the simulation results of the system input and output. The former is the equivalent harmonic force $F(t)$, while the latter are the displacement $x(t)$, velocity $v(t)$, acceleration $a(t)$ and driving force F_d of the vibrator, respectively.

It can be easily seen from the results that the system output is harmonic, which is the same as the input equivalent harmonic force $F(t)$. According to the image above, the output responses of the displacement, velocity, acceleration and driving force all gradually shift from the transient state to the steady state.

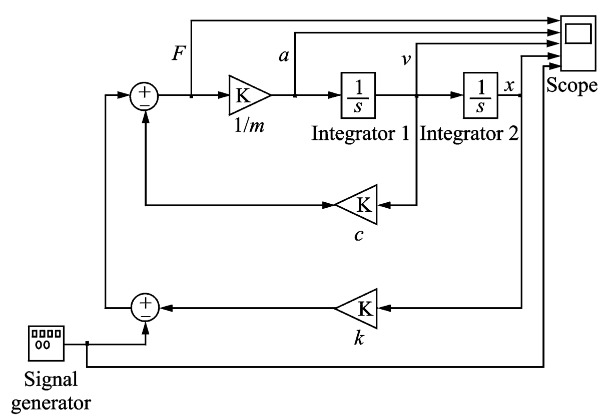


Fig. 5 Simulink simulation model of vibrator

Table 2 Parameters for dynamic characteristic Simulink simulation of vibrator

Parameter	k/(N · m^{-1})	c/(N · m^{-1} · s^{-1})	m/kg	$A_{F(t)}$/N	$f_{F(t)}$/Hz
Value	1.528×10^3	0.601	1.290×10^{-2}	1.450×10^{-1}	12

3.3 Experiment analysis

In order to verify the theoretical and simulation results, the end displacement of the bimorph piezoelectric vibrator is tested in the dynamic re-

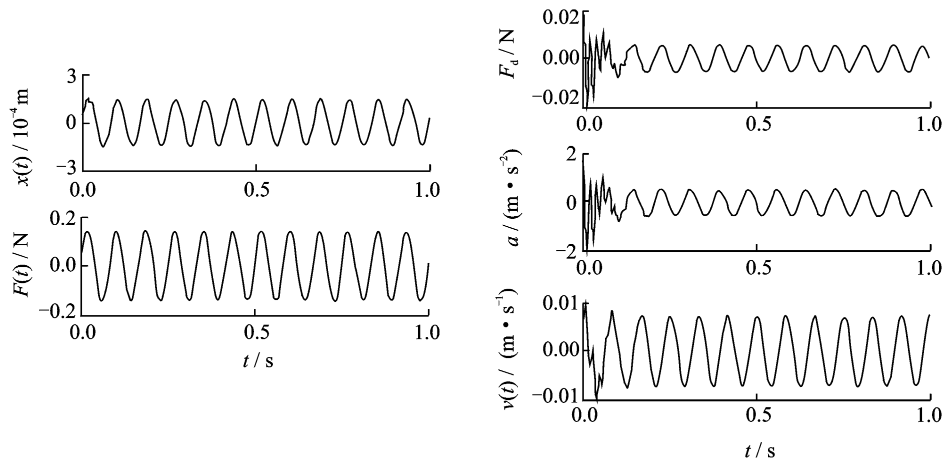


Fig. 6 System input and output of Simulink simulation

sponse experiments. The model of the bimorph of the vibrator is shown in Fig. 7. It consists of two piezoelectric layers and one metal layer. The thickness of each layer is 0. 2 mm.

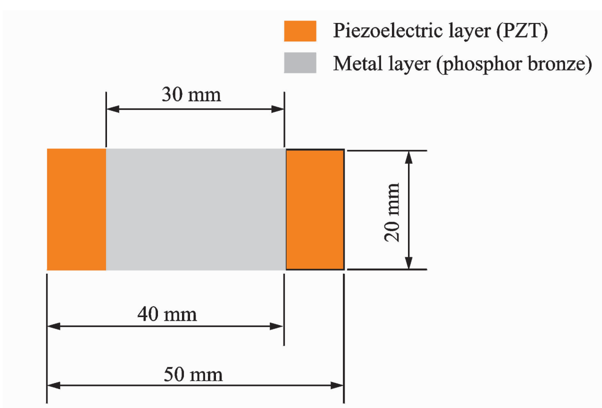


Fig. 7 Model of vibrator

Under the excitation signal, the end displacement of the vibrator is very small, which can even reach micrometer level. To increase the testing precision and guarantee the condition that the displacement testing device has no effect on the movement of vibrator, the non-contact measurement strategy is chosen in experiments.

Fig. 8 provides the experimental system, mainly consisting of a signal generator, power amplifier, laser sensor, laser head with the measuring range and resolution of ±18 mm and 100 nm, and digital storage oscilloscope. Fig. 9 shows the partial enlarged drawing of the piezoelectric cantilever bimorph vibrator measured by the laser head. In experiments, the signal generator emits a sinusoidal signal. Then the signal enlarged by the power amplifier is loaded with the bimorph piezoelectric vibrator. The laser head is used to measure the dynamic end displacement of the vibrator. Finally, all the data from the laser sensor will be gathered, recorded and processed

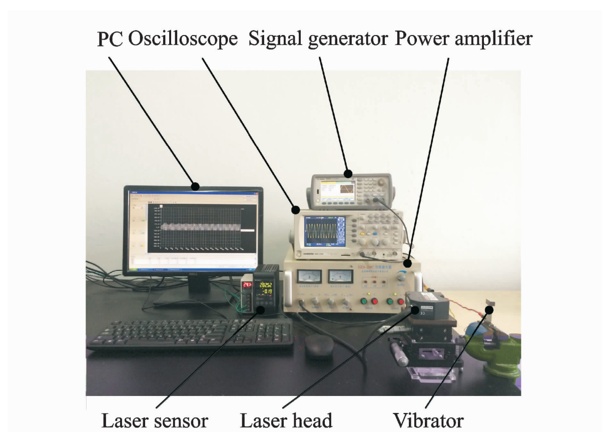


Fig. 8 Experimental system for output end displacement of piezoelectric cantilever bimorph vibrator

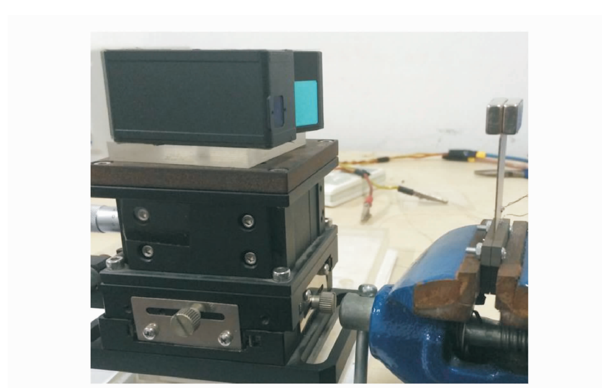


Fig. 9 Partial enlarged drawing of piezoelectric cantilever bimorph vibrator measured by laser head

by the software LK-Navigator on the computer.

The dynamic response testing results of the bimorph piezoelectric vibrator are shown in Fig. 10. The excitation signal is 20 V, 12 Hz sinusoidal signal. It is known from the results that the end displacement output of the vibrator gradually enters into the stage of steady state response after three seconds transient response.

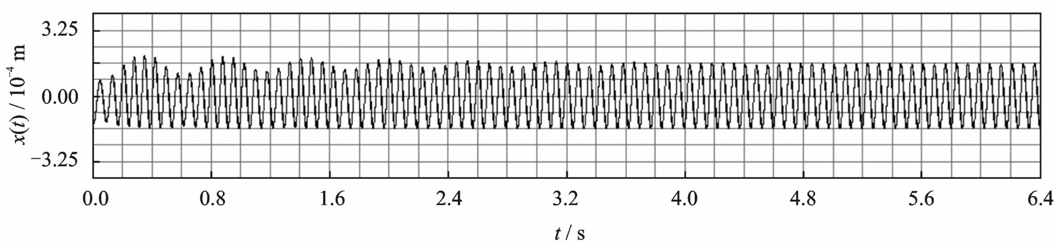


Fig. 10 End displacement of vibrator in dynamic response

Table 3 shows the comparison among the amplitudes of theory, simulation and experiments in steady state response under the same sinusoidal excitation signal (20 V, 12 Hz). Through analyzing the relative error in each case, the experimental results coincide well with the theoretical results and the Simulink simulation results.

Table 3 Comparison among amplitudes of theory, simulation and experiments in steady state response

Item	Theory	Simulation	Experiment
Amplitude/10^{-4} m	1.75	1.58	1.65
Relative error /%	6.06	4.24	—

4 Conclusions

The dynamic model of the single-degree-of-freedom damp vibration system has been built by the proposed strategy of transforming symmetric electrical signal excitation to equivalent harmonic force excitation. After analyzing the system stiffness, damping and other parameters, the dynamic characteristic of the vibrator has been studied and discussed by means of theoretical analysis, simulation and experiments. It is shown that the results of experiments agree well with the simulation and theoretical ones. Finally, a foundation is provided for the further research of piezoelectric inertial actuators.

Acknowledgements

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