

Partial Improvement of Traditional Grey-Markov Model and Its Application on Fault Prediction

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Abstract: Modeling experiences of traditional grey-Markov show that the prediction results are not accurate when analyzed data are rare and fluctuated. So it is necessary to revise or improve the original modeling procedure of the grey-Markov (GM) model. Therefore, a new idea is brought forward that the Markov theory is used twice, where the first time is to extend the original data and the second to calculate and estimate the residual errors. Then by comparing the original data sequence from a fault prediction case with the simulation sequence produced by the use of GM(1,1) and the new GM method, results are conforming to the original data. Finally, an assumption of GM model is put forward as the future work.

Key words: grey-Markov model; GM(1,1); residual error

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0 Introduction

In 1982, Prof. Deng, a Chinese scholar, created grey system theory to the uncertain system with less data and poor information. Nowadays, grey theory is widely applied to different fields, and accepted as one kind of maturational theory since it shows better accuracy in its application scope.

GM(1,1) is the basic model of grey theory, especially the GM(1,1) mean model, but there are some shortcomings for GM(1,1) model. (1) When the original data of GM(1,1) model are less, fluctuated and random, the prediction accuracy is rough; (2) GM(1,1) model requires that accumulated generating sequence has an exponential feature, so that it can be fitted by differential equations; (3) If the residual value is non-negative or not one accumulated generating operation (1-AGO) and could not increase monotonically, residual sequences are not suitable to be identified and predicted^[1-2].

So grey combination models were put for-

ward, which combine grey theory models and other traditional theory models, and grey-Markov model is one of them. In fact, grey-Markov model overcomes those shortcomings of grey prediction and Markov prediction, and improves the accuracy, thus attaining more reliable, scientific and useful results. Among these, it is common that Markov theory is applied to the GM(1,1) model residuals prediction, where the prediction accuracy is largely improved^[3-6].

These are the procedure of grey-Markov modeling:

- (1) Acquire the simulation sequence;
- (2) Count residuals or relative residuals between simulation sequences and original sequences;
- (3) Divide residuals or relative residuals into several state intervals;
- (4) Count state transferring condition of simulation sequences;
- (5) Build up state transferring probability matrix;
- (6) Predict residuals state of target time;

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(7) Improve prediction values of simulation sequences.

A lot of previous experiences have been acquired since grey-Markov model was widely deployed, featuring the grey-Markov model as follows^[7-9]:

(1) Make sure the advantage of high short-term prediction accuracy and improve the long-term prediction accuracy;

(2) More data, more authentic for the prediction results;

(3) More scientific and practical in predicting big random and fluctuated data.

Although grey-Markov model is widely adopted, it has a shortcoming, that is, when treating less data, it is not good for directly improving the sequence from GM(1,1) model by using the Markov theory^[10-12]. As a matter of fact, the data quantities are also rare after the use of Markov theory, and it is difficult to show the whole regularity of residual sequence^[13-14].

1 Improved Method of Grey-Markov Model

Considering this shortcoming, a new improved grey-Markov model is presented^[15]. Firstly, data application of GM(1,1) model is tested. If data do not conform to the condition of quasi smooth sequence, they need to be improved by average weakening buffer operator (AWBO) to acquire the new sequence $X^{(0)}$. Then $X^{(1)}$ should be calculated through 1-AGO, after that, simulation sequence $\hat{x}^{(1)}(k)$ can be obtained by the use of GM(1,1) mean model and $\hat{x}^{(0)}(k)$ is calculated via accumulated reduction operation. So the relative residuals need to be tested by the formula $\epsilon = \frac{|\hat{x} - x|}{x} \times 100\%$. If the tested results are too big, it will be insignificant to use the Markov theory. However, at this time, the Markov theory would be applied to extending the sequence in order to increase the data quantities of GM(1,1) modeling. Finally, the Markov theory would be

applied to improve the residuals of GM(1,1) model to acquire the final simulation sequence.

It can be easily found that the new improved grey-Markov model adopts the Markov theory twice: the first is to revise original data, and the second is to revise residuals.

The flow diagram of the new model is shown in Fig. 1 with more details.

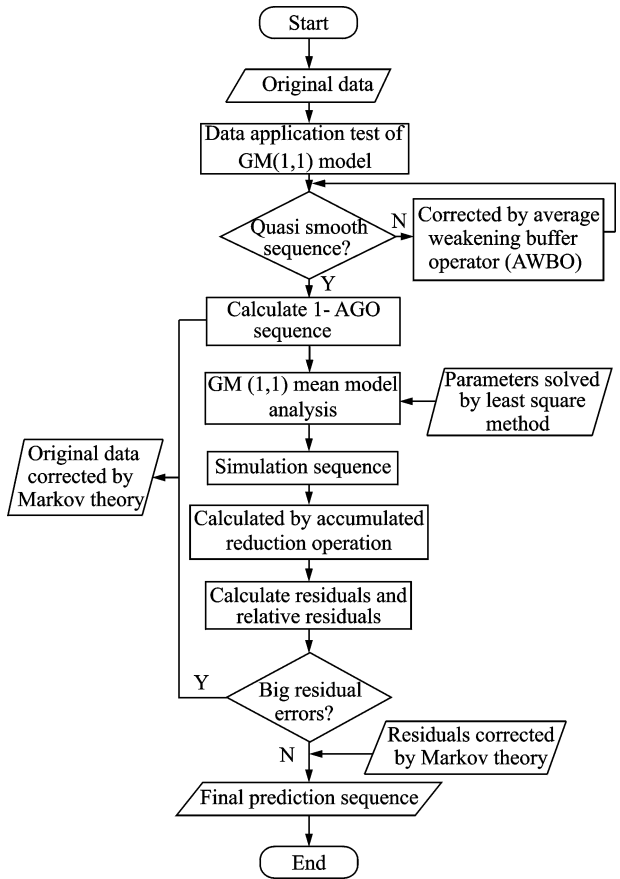


Fig. 1 Flow chart of new improved grey-Markov model

2 Application and Discussion of Improved Grey-Markov model

2.1 Fault data application test of grey-Markov model

With the generators' fault data of B737 aircraft in a certain airlines company fleet between 2004 and 2008, the statistical table of fault numbers is established in Table 1.

Table 1 shows that the fault numbers have a big volatility. However, this paper took the data between the year 2004 and 2007 as the basic data of modeling, and the data of 2008 as prediction accuracy of the model testing.

Table 1 Statistical table of fault numbers of B737's generators

Year	2004(1)	2004(2)	2005(1)	2005(2)	2006(1)	2006(2)	2007(1)	2007(2)	2008(1)	2008(2)
	first half	second half	first half	second half	first half	second half	first half	second half	first half	second half
Fault numbers	8	9	4	9	6	7	11	17	10	13

So setting the fault sequence as $\mathbf{X}^{(0)} = (8, 9, 4, 9, 6, 7, 11, 17)$, the smoothness of the sequence will be tested with the following formula to build a sequence of smoothness.

$$\rho(k) = \frac{x(k)}{\sum_{i=1}^{k-1} x(i)}, k = 2, 3, \dots, n \quad (1)$$

That is,

$$\rho(k) = (1.125, 0.235, 0.429, 0.2, 0.194, 0.256, 0.315).$$

There are three determinant criteria to judge whether the sequence is a quasi smooth sequence.

- (1) $\frac{\rho(k+1)}{\rho(k)} < 1, k = 2, 3, \dots, n-1;$
- (2) $\rho(k) \in [0, \epsilon], k = 3, 4, \dots, n;$
- (3) $\epsilon < 0.5.$

Clearly, $\mathbf{X}^{(0)}$ is not according to the first criteria, so it is not a quasi smooth sequence. Hence, If GM(1,1) model is used with $\mathbf{X}^{(0)}$, the prediction accuracy will not be obtained.

2.2 Fault data revised with a sequence operator

To weaken the volatility of the data sequence, here AWBO is adopted as $x(k)d$.

$$x(k)d = \frac{1}{n-k+1} [x(k) + x(k+1) \dots + x(n)] \quad k = 1, 2, \dots, n \quad (2)$$

By the use of AWBO, the maximum value of shock sequence will decrease, and the minimum value will increase. Overall, the sequence volatility will decrease.

So applying AWBO to the sequence

$$\mathbf{X}^{(0)} = (8, 9, 4, 9, 6, 7, 11, 17)$$

a new sequence is established.

$$\mathbf{X}'^{(0)} = (8.875, 9.9, 10, 10.25, 11.667, 14, 17)$$

As above, applying the smoothness formula to the new sequence, a new smoothness sequence is established as follows

$$\rho(k) = (1.014, 0.503, 0.372, 0.278, 0.248, 0.238, 0.234)$$

Judging that the new sequence conformed to the criteria of quasi smooth sequence (In the smoothness sequence, $0.503 > 0.5$, but the deviation is tiny), the 1-AGO sequence of the data sequence has a quasi exponent law, so it is suitable to the GM(1,1) model.

2.3 GM(1,1) model analysis

The analysis of GM(1,1) model uses the fault data from the year 2004 to 2007 as the data of modeling, then uses the 2008 fault data to test the accuracy of the model. Applying 1-AGO on $\mathbf{X}'^{(0)}$ to obtain 1-AGO sequence $\mathbf{X}'^{(1)}$

$$\mathbf{X}'^{(1)} = (8.875, 17.875, 26.875, 36.875, 47.125, 58.792, 72.792, 89.792)$$

Subsequently, it applies the GM(1,1) mean model to $\mathbf{X}'^{(1)}$

$$\mathbf{X}^{(0)}(k) + a z^{(1)}(k) = b \quad (3)$$

where the parameters of Eq. (3) are required by least square method. Then the parameter vector is established.

$$\hat{\alpha} = [a, b]^T = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{Y} \quad (4)$$

$$\text{where } \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \mathbf{Y}_N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix},$$

$$z^{(1)}(k) = \frac{1}{2} (x^{(1)}(k) + x^{(1)}(k-1)).$$

In this case,

$$\mathbf{Y} = [9 \quad 10 \quad 10.25 \quad 11.667 \quad 14 \quad 17]^T,$$

$$\mathbf{B} =$$

$$\begin{bmatrix} -13.375 & -22.375 & -31.875 & -42 & -52.958 & 5 & -65.792 & -81.292 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T$$

Then, the result showed that

$$\hat{\alpha} = [a, b]^T = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{Y} = \begin{bmatrix} -0.117 & 7 \\ 6.351 & 3 \end{bmatrix}$$

which was applied into the time response formula

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(k-1)} + \frac{b}{a}$$

$$k = 1, 2, \dots, n$$

The solution was

$$\hat{\mathbf{x}}^{(1)}(k) = 62.836\ 767\ 2e^{0.117\ 7(k-1)} - 53.961\ 767\ 2$$

After that, the simulation sequence can be figured out.

$$\hat{\mathbf{x}}^{(1)}(k) = (8.875, 16.724, 25.553, 35.485, \\ 46.657, 59.225, 73.363, 89.267, \\ 107.157, 127.282)$$

With the sequence, the reduction sequence is obtained by applying an inverse accumulation operator.

$$\hat{\mathbf{x}}^{(0)}(k) = (8.875, 7.849, 8.829, 9.932, 11.172, \\ 12.568, 14.138, 15.904, 17.89, 20.125)$$

In contrast to the original sequence $\mathbf{X}^{(0)}$, the model predictive value of the first half of 2008 was 17.89, and the predictive value of the second half of 2008 was 20.125. In fact, the number of faults in the first half of 2008 was 10, and the number in the second half of 2008 was 13.

Hence, it needs to define the relative residuals: $\epsilon = \frac{|\hat{x} - x|}{x} \times 100\%$. Although the relative residuals of the previous eight simulated data are small, compared with the actual data. The relative residuals of the ninth and tenth data are 78.9% and 54.808%. It can be confirmed that the referential value of this simulation sequence is low. At least in this case the effect may last for a while after $k \geq 9$, the simulation sequence loses its predictive accuracy and referential value with two reasons considered:

(1) Simulation sequence and original sequence have higher accuracy and smaller residuals when $k \leq 8$. But after $k \geq 9$, the accuracy drops and the residuals begin to increase suddenly. It cannot be confirmed that the change rate has a steady trend and the following law of residuals will be similar as the law of residuals when $k \leq 8$.

(2) When $k \geq 9$, the simulation sequence loses its predictive accuracy seriously. And the absolute value of the residuals abruptly gets larger. So it could be judged that the simulation sequence loses its referential value.

Obviously, there is no practical significance by applying Markov theory based on residuals to improve the predictive results of the simulation sequence. That means Markov theory should be used to mine information of the original sequence furtherly.

2.4 Improvement of GM (1,1) model based on Markov theory

Starting with the adjacent data in the original sequence, the regularity of original data sequence can be obtained. Since the eighth data of original data sequence, 17, the data of the second half of 2007, have an excessive growth, compared with the seventh data 11, and it exists in the end of the sequence. It is hard to require its transferred state. So the eighth data 17 would be considered as an abnormal data, and it will be deleted flexibly. Based on the above assumption, variation sequence is established.

$$\mathbf{Y}^{(0)} = (+1, -5, +5, -3, +1, +4)$$

where $y^{(0)}(k) = x^{(0)}(k+1) - x^{(0)}(k)$.

According to the observation of the original sequence variation law, the variation states are divided into three states: State 1 with variation range $[+4, +5]$, State 2 with variation range $\{+1\}$, State 3 with variation range $[-5, -3]$.

As above, Markov state chain of the original sequence can be established: $2 \rightarrow 3 \rightarrow 1 \rightarrow 3 \rightarrow 2 \rightarrow 1$. Then the state transition probability matrixes are also built.

$$\mathbf{P}(1) = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}, \quad \mathbf{P}(2) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix}, \\ \mathbf{P}(3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

With those, the original sequence will be extended. When extending, the variation of State 1, State 2, and State 3 is the midpoint of each interval, i. e., 4.5, 1 and -4, respectively. When the state cannot be judged by 1-step state transition probability matrix $\mathbf{P}(1)$, then 2-step and 3-step state transition probability matrixes can be

Table 2 Prediction contrast of GM (1,1) model and grey-Markov model

Serial number	Extended data $X^{(0)}$	GM (1,1) model			grey-Markov model		
		Predictive value $\hat{x}^{(0)}(k)$	Relative error/%	Average relative error/%	Predictive value $\hat{x}^{(0)}(k)'$	Relative error/%	Average relative error/%
1	8	8	0	7.5	6.25		
2	9	6.396	28.93	8.896	1.16		
3	4	6.738	68.45	4.238	5.95		
4	9	7.097	21.14	8.597	4.48		
5	6	7.476	24.6	5.976	2.4		
6	7	7.874	12.49	24.25	7.374	5.34	4.2
7	11	8.296	24.58	10.796	1.85		
8	7	8.738	24.83	7.238	3.4		
9	8	9.204	15.05	8.704	8.8		
10	12.5	9.696	22.43	12.196	2.43		

From Table 2, to $\hat{x}(9)' = 8.704$ and $\hat{x}(10)' = 12.196$ compared with the original value $x(9) = 10$ and $x(10) = 13$, their residuals and relative residuals are $\delta(9) = -1.296$, $\delta(10) = -0.804$, $\epsilon(9) = 12.96\%$, and $\epsilon(10) = 6.185\%$, respectively.

Comparison of simulation results are shown in Fig. 2.

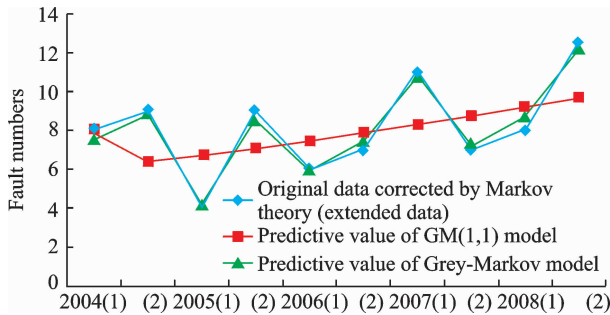


Fig. 2 Comparison of simulation results

From Table 2 and Fig. 2, the predictive value of the grey-Markov model agrees with the original data corrected by the Markov theory better than the predictive value of GM(1,1) model, which clearly shows that, compared with the prediction results of GM(1,1) model, the correction effect of the improved Markov theory is obviously improved with an average relative error of 4.2%.

Here, the grey-Markov model combines GM(1,1) model and the Markov theory has the virtue of high short-term prediction accuracy of GM(1,1) model, and the advantageous of handling high random and fluctuated data as well.

The Markov theory is used twice in this case, namely, the first time to dig sequence rules

and extend the data sequence, and the second time to estimate the residual error state and correct simulation sequence. Finally, the prediction results conform to the real data. However, the data extension must vary with the actual conditions, and the original data could not be groundlessly corrected, because that would make the results distorted.

3 Conclusions

Traditional grey-Markov model is simply revised or improved by a new idea, which is certificated with a case in detail with satisfying results. The new idea for the grey-Markov model mainly includes the following aspects:

(1) When data are rare, the data sequence shows no rule or no obvious regularity, even the sequence is fluctuated, which means that it is difficult to be modeling and acquire good results. To solve this problem, the data quantities are extended with Markov theory.

(2) The residual errors are calculated and estimated with Markov theory after simulation sequences are acquired, which will correct the relevant simulation sequences.

Further, an assumption to the grey-Markov model could be put forward. The original data are divided into several segments, then in every segment, GM(1,1) model is deployed to acquire the simulation sequence so as to show the virtue of GM(1,1) model short-term prediction. Simultaneously, segmental prediction will also transfer

big fluctuated sequences into several weak fluctuated sequences so as to avoid the weakness of GM(1,1) model in the aspect of predicting big fluctuated sequences. However, segmental work will increase the calculating jobs, but it is not a problem with computer technologies now.

In a word, this kind of assumption transfers the complicate sequences into several simple subsequences. And the future work will certificate the assumption.

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