

# Modeling and Optimal Design of Planar Linkage Mechanism of Coupled Joint Clearances for Manufacturing

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**Abstract:** The uncertainty of the mechanism motion error is mostly caused by the manufacturing process, so the motion error cannot be effectively predicted at the design phase. The problems of manufacturing complexity and the relationship between design and manufacturing are analyzed, and the influence of dimensional tolerance and fit tolerance on the motion accuracy of the system is considered in the design process. Then based on the Monte Carlo simulation, an optimal design model of planar linkage mechanism is set up. A typical offset slider-crank mechanism is used as an illustrative example to carry out the optimal design. Compared with the result of typical robustness design, the similar variation characteristics of the mean value and the standard deviation can be found, so the proposed method is effective. The method is furthermore applied in the optimization of the schemes with different fit tolerances and the prediction of motion errors in the design phase is achieved. A set of quantitative evaluation system for mechanism optimal design is provided. Finally, a basic strategy is presented to balance the motion precision and manufacturing cost.

**Key words:** manufacturing; planar linkage mechanism; Monte Carlo simulation; dimensional tolerance; fit tolerance; balance

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## 0 Introduction

Planar linkage mechanisms are widely used in machines to ensure their functions. Some motion errors in these machines are inevitable due to tolerance, defects arising from the design and manufacturing process or wearing after a certain working period. Thus, because of these uncertainty factors, the motion error of the mechanism cannot be effectively predicted in the design phase. In the past, the research on the motion error prediction in the presence of uncertainty factors generally can be divided into two kinds of methods.

The first is research on link dimensional errors. Link dimensional error is considered in the design phase, so the motion error of the actual mechanism can be reduced. Dong et al. [1-2] have presented fuzzy reliability analysis of slider-crank

mechanism, so a method of fuzzy reliability design was got. However, it is not enough just to precisely predict the motion error of the post-manufacturing's mechanism.

Moreover, joint clearance has been taken into consideration. Lee and Gilmore<sup>[3]</sup> have presented the joint clearance bar model, and the joint clearance was treated as a massless virtual link with the length equaling to one-half of the clearance. Song et al. [4] have presented the calculation method of the joint clearance bar dimension distribution. Tan et al. [5] have presented an effective model to analyze the offset slider-crank mechanism with joint clearance, and then based on robustness design, the optimal design method was got. Luo and Du<sup>[6-9]</sup> have presented the modified first order second moment based on the first order second moment and the Monte Carlo simu-

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lation method, and used a four-bar mechanism and a slider-crank mechanism to test the proposed method. Aiming at the four-bar mechanism, Gogate et al.<sup>[10]</sup> got three different objective functions. The reliable foundation for the establishment of the model was provided. Zhu and Ting<sup>[11-12]</sup> have presented a general approach for the kinematics of four-bar mechanism with joint clearances by using the assumption of continuous contact model. However, a single DOF four-bar linkage would become a five DOF eight-bar linkage and it will inevitably cause the whole model solves tedious. Thus, Guo et al.<sup>[13-14]</sup> have presented an “effective length model” to replace the “joint clearance bar model”, and verified the precision of the model by an offset slider-crank mechanism example.

In this paper, based on the Monte Carlo simulation method, considering the requirement of motion precision and the fit between hole and shaft, the distribution range of any joint clearances bar size can be calculated. Combined with the dimensional tolerance of the component for parameter optimization, compared with the mean and standard deviations of the motion error, a set of quantitative evaluation system for mechanism optimization design is provided. At the end of this paper, a basic thought of balancing the motion precision and manufacturing cost is presented.

## 1 Multi-objective Optimal Design Model of Planar Linkage Mechanism for Manufacturing

Actual motion error of the mechanism can be described as

$$\Delta = y_{\text{am}} - y_{\text{rm}} \quad (1)$$

where  $y_{\text{rm}}$  and  $y_{\text{am}}$  denote the required motion output and actual motion output, respectively.

Currently, practical mechanism most adopted a serial mode of design to manufacturing. The complexity of the manufacturing process and the relationship between design and manufacturing lead to some inevitable and uncertain factors in

practical mechanism, which increases the uncertainty of the whole motion error<sup>[6-12]</sup>. Thus, the motion error of the actual mechanism is

$$\Delta(\mathbf{X}, \mathbf{Z}) = y'_{\text{am}}(\mathbf{X}, \mathbf{Z}) - y_{\text{rm}} \quad (2)$$

$$\mathbf{X} = x_1, x_2, \dots, x_p; \quad \mathbf{Z} = z_1, z_2, \dots, z_q$$

where  $\mathbf{X}$  denotes a collection of control factors for the mechanism,  $\mathbf{Z}$  a collection of noise factors for the mechanism,  $p$  the number of control factors, and  $q$  the number of noise factors.  $y'_{\text{am}}(\mathbf{X}, \mathbf{Z})$  the actual motion of the mechanism, in which noise factors and other objective factors are considered. Noise factors include environmental factors change, manufacturing parameter change, material aging, abrasion, and processing method change etc.

Optimization of the usual mechanism is trajectory optimization, namely, there are many discrete points in the trajectory and existence multiple point motion errors, so the mechanism's motion error  $\delta$  can be expressed as

$$\delta = \sqrt{\frac{1}{s} \cdot \sum_{n=1}^s (y'_{\text{am}}(\mathbf{X}, \mathbf{Z}) - y_{\text{rm}})^2} \quad (3)$$

where  $s$  is the number of discrete points.

Therefore, the objective function of the optimal design model of the planar linkage mechanism for the manufacturing is

$$\min F(\mathbf{X}, \mathbf{Z}) = \omega_1 \mu_\delta + \omega_2 \sigma_\delta \quad (4)$$

where  $\mu_\delta$  is the mean value of the error  $\delta$ , and  $\sigma_\delta$  the standard deviation.  $\omega_1$ ,  $\omega_2$  are the weight coefficients for mean and standard deviation, respectively<sup>[15]</sup>.

According to different application conditions and the role of the forms diversity caused by these different application conditions, there is a certain relationship between the link of mechanism  $f_i$  (such as rotation characteristics of the whole cycle, the crank existence conditions and the characteristics of minimum transmission angle), and  $f_i$  is defined as the characteristics requirements of the mechanism in Eq. (5).

$$f_i(\mathbf{X}, \mathbf{Z}) - f_{i0} \leq 0$$

$$\mathbf{X} = x_1, x_2, \dots, x_p; \mathbf{Z} = z_1, z_2, \dots, z_q; i = 1, 2, \dots, n \quad (5)$$

where  $n$  is the number of requirements for the mechanism's characteristics.

Other objective factors, such as precision, cost, stability, and dynamic mechanical behavior etc. can furthermore add to a number of corresponding constraints. To filter the data from design result, these constraints are specified in Eq. (6).

$$g_j(\mathbf{X}, \mathbf{Z}) - g_{ij} \leq 0$$

$$\mathbf{X} = x_1, x_2, \dots, x_p; \mathbf{Z} = z_1, z_2, \dots, z_q; j = 1, 2, \dots, m \quad (6)$$

where  $g_{ij}$  are the constraints value on other objective factors (determined by objective requirements), and  $m$  the number of other objective factor constraints.

Finally, an optimal design model of the planar linkage mechanism for manufacturing can be expressed as

$$\min F(\mathbf{X}, \mathbf{Z}) = \omega_1 \mu_\delta + \omega_2 \sigma_\delta$$

$$\text{St. } f_i(\mathbf{X}, \mathbf{Z}) - f_{ii} \leq 0 \quad i = 1, 2, \dots, n$$

$$g_j(\mathbf{X}, \mathbf{Z}) - g_{ij} \leq 0 \quad j = 1, 2, \dots, m$$

$$\mathbf{X} = x_1, x_2, \dots, x_p, \mathbf{X} \in (\mathbf{X}^L, \mathbf{X}^U)^T$$

$$\mathbf{Z} = z_1, z_2, \dots, z_q, \mathbf{Z} \in (\mathbf{Z}^L, \mathbf{Z}^U)^T$$

where  $\mathbf{X}^L$  and  $\mathbf{X}^U$  are collections of control factors' lower and upper bound value, respectively (determined by initial conditions).  $\mathbf{Z}^L$  and  $\mathbf{Z}^U$  are collections of noise factors' lower and upper bound value, respectively (jointly determined by design and processing requirements).

## 2 Modeling of Planar Four-bar Linkage with Joint Clearance and Case Analysis

For planar four-bar mechanism, it is well known that the mechanism performance measure is usually referred to as the ability of reaching the desired position or orientation precisely. Link dimensional tolerances and fit tolerances make the performance of mechanisms worse [11-14]. The collection of control factors includes the length and its range, rotation angle and so on. The collection of noise factors includes fit tolerance and dimensional tolerance, geometric tolerance, roughness, and the deviation between holes etc. The randomness of the tolerance distribution leads to the randomness of the motion error. The correlation between tolerance distribution and the model pose

can be obtained based on Monte Carlo simulation method, then the mathematical model of optimal design is set up. The offset slider-crank mechanism is used as an illustrative example.

### 2.1 Pose description of the model of planar four-bar linkage with joint clearance

The four-bar mechanism is widely used in planar linkage mechanism [16]. According to Lee's "joint clearance bar model" [3], the hole-shaft structure is assumed for each link, namely, the point  $A$  of the frame is the hole, the point  $A$  of the crank is the shaft, the point  $B$  of the crank is the hole, and the point  $B$  of the link is the shaft. The specific pose description of mechanism with joint clearance bar is shown in Fig. 1.

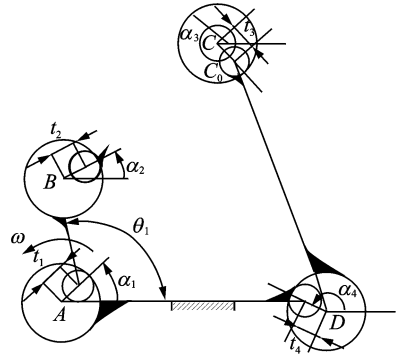


Fig. 1 Pose description with joint clearance bar

Replacing the revolute pair with joint clearance bar that forms a model of an eight-bar linkage mechanism, causing the whole model solves complex. According to Ref. [13], an effective length model was adopted (as shown in Fig. 2, the link  $CD$  in Fig. 1 is used to describe), where  $C_0D + DD_0$  is replaced with  $C_0D$ , that is, replacing the effective length with the sum of joint clearance and the original bar. Finally, a four-bar model with the effective length is generated.

By analysing the relationship between  $C_0D$  and  $C_0D_0$ , the vector equation can be expressed as

$$C_0D_0 e^{i\phi_1} = C_0D e^{i\phi} + t_4 e^{-i\alpha_4} \quad (7)$$

where  $C_0D$  is the length of the original bar,  $t_4$  the length of joint clearance bar of revolute pair  $D$ ,  $\alpha_4$  the azimuth of joint clearance bar, and  $C_0D_0$  the length of the effective bar.

The geometric relation can be expressed as



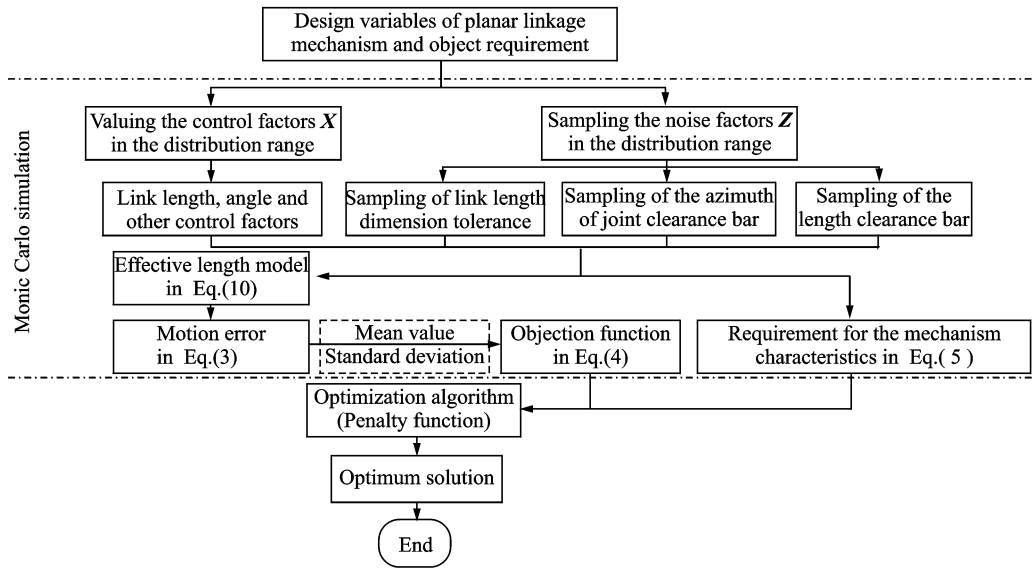


Fig. 3 Flow chart of optimal design for planar four-bar mechanism with joint clearance

**2.3 Optimal design example analyses of offset slider-crank mechanism with joint clearance**

Offset slider-crank mechanism is shown in Fig. 4. For convenient comparison with Ref. [13], the working range of crank is  $[225^\circ, 285^\circ]$ , and the corresponding working range of slider is  $[80 \text{ mm}, 120 \text{ mm}]$ . The relation between the slider displacement  $s_0$  and the crank angle  $\theta$  can be described in Eq. (13), which is the required motion output  $y_{rm}$  in Eq. (3).

$$s_0 = 80 + 40 \times \left(\frac{\theta - 225}{60}\right)^2 \quad (13)$$

Actual motion displacement of the slider can be calculated by Eq. (14), which denotes actual motion output  $y'_{am}$  in Eq. (3).

$$s = l_1 \cos\theta + \sqrt{l_2^2 - (l_3 - l_1 \sin\theta)^2} \quad (14)$$

Thirteen points  $\theta_i$  in the angle range  $[225^\circ, 285^\circ]$  of the crank can be got, in which  $i=1, 2,$

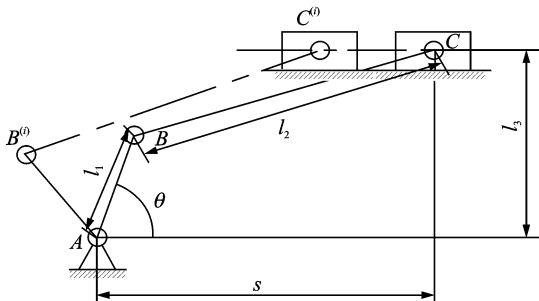


Fig. 4 Offset slider-crank mechanism

..., 13. Therefore, considering the link dimensional tolerance and the revolute pair fit tolerance at the same time, the motion displacement error of the slider can be expressed as

$$\delta = \sqrt{\frac{1}{13} \cdot \sum_{i=1}^{13} (s_0(\theta_i) - s(l_1, l_2, l_3, \theta_i, \Delta l_1, \Delta l_2, \Delta l_3, t_1, t_2, t_3, a_1, a_2, a_3))^2} \quad (15)$$

where  $\mathbf{X} = (l_1, l_2, l_3)^T$  are control factors,  $\mathbf{Z} = (\Delta l_1, \Delta l_2, \Delta l_3, t_1, t_2, t_3, a_1, a_2, a_3)$  are noise factors.  $\Delta l_1, \Delta l_2, \Delta l_3$  are dimensional tolerances of crank, link, and offset distance, respectively.

**2.3.1 Model establishment of offset slider-crank mechanism**

In Ref. [16], supposing that the basic dimension of the inner diameter of the hole and the outer diameter of the shaft range from (10 mm, 18 mm], according to the fit characteristics and application conditions, three fit clearances of  $\frac{H8}{g7}$ ,  $\frac{H7}{f6}$  and  $\frac{H8}{e8}$  are chosen in the order from small to large, respectively. To make the following analysis convenient, 1, 2 and 3 are used to express different fit clearances, such as point A is  $\frac{H8}{g7}$ , point B is  $\frac{H8}{e8}$ , and point C is  $\frac{H7}{f6}$ , that is, each joint point of ABC can be described as scheme 132. According to Ref. [16], the limit deviation value

of hole and shaft can be got, and then, through the method in Section 2. 2. 1, the mean value and the standard deviation of the joint clearance can be obtained. The above results are given in Table 1.

**Table 1 Mean value  $\bar{t}_k$  and standard deviation  $\sigma_{tk}$  of three different joint clearances**

No.	Fit type	$\bar{t}_k$ /mm	$\sigma_{tk}$ /mm
1	H8/g7	0.008 98	0.003 26
2	H7/f6	0.018 39	0.002 58
3	H8/e8	0.023 18	0.002 76

Control factors and noise factors are given in Table 2. Through the principle of  $3\sigma$ <sup>[11-12]</sup>, the standard deviation of the crank, link, and offset distance can be obtained.

**Table 2 The distributed parameter of control factors and noise factors**

Design variable ( $k=1,2,3$ )	Mean value	Standard deviation	Distribution type
$l_k$ /mm	$\bar{l}_k$	$\Delta l_k/3$	Normal
$t_k$ /mm	$\bar{t}_k$	$\sigma_{tk}$	Normal
$\alpha_k$ /( $^\circ$ )	$\bar{\alpha}_k$	$\sigma_{\alpha k}$	Uniform in $[0.2, \pi]$

The optimal design model of slider-crank mechanism with joint clearance can be expressed as

**Table 3 Comparison results of this paper with robustness design**

Design variable	$l_1$ /mm	$l_2$ /mm	$l_3$ /mm	$\Delta l_1$ /mm	$\Delta l_2$ /mm	$\Delta l_3$ /mm
Original scheme <sup>[13]</sup>	58.130	165.480	71.790	0.081	0.072	0.093
Scheme in this paper	58.011	164.787	71.014	0.081	0.072	0.093

$$\begin{aligned} \min F(\mathbf{X}=\mathbf{X}, \mathbf{Z}) &= \omega_1 \mu_\delta + \omega_2 \sigma_\delta \\ \text{s. t. } l_1 + t_1 - l_2 - t_2 + l_3 + t_3 &\leq 0 \quad (16) \\ \mathbf{X}^L &= (50, 160, 65)^T \\ \mathbf{X}^U &= (60, 170, 75)^T \\ \mathbf{Z}^L &= (0.001, 0.001, 0.001)^T \\ \mathbf{Z}^U &= (0.15, 0.15, 0.15)^T \end{aligned}$$

Initial value = (55, 165, 70, 0.07, 0.07, 0.07)<sup>T</sup> where  $\omega_1 = 1, \omega_2 = 1$  is obtained<sup>[15]</sup>. From line position tolerance<sup>[16]</sup>, the range of dimensional tolerances of  $\mathbf{Z}^L$  and  $\mathbf{Z}^U$  can be confirmed.

2. 3. 2 Solution and result analysis of mathematical model

Based on typical robustness optimal design, aiming at offset slider-crank mechanism, an optimal design model with joint clearance was established, and the link dimensional tolerance of the mechanism were got<sup>[13]</sup>. Aiming at the scheme, for contrast in the design effects, the same dimensional and fit tolerance are adopted. The design parameters of Ref. [13] and this paper are given in Table 3.

The mean value and standard deviation of the error are important parameters, from which the motion error of the mechanism can be evaluated<sup>[6-9, 13-14]</sup>. The optimization result is calculated through the design process in Fig. 5, and the mean value and standard deviation of motion error can be obtained, respectively (as shown in Figs. 6, 7).

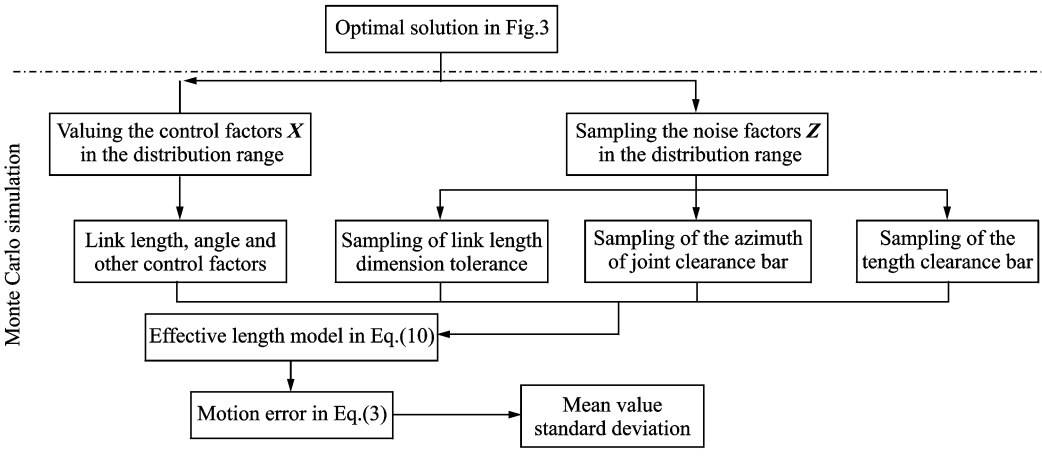


Fig. 5 Calculation flow chart of mean value and standard deviation of motion error with joint clearance

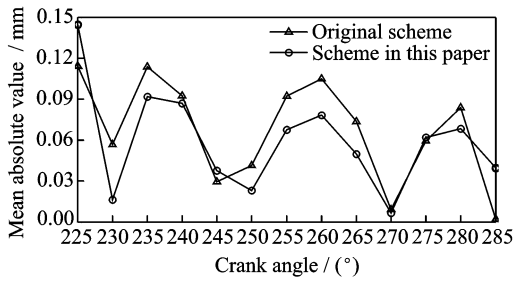


Fig. 6 Mean absolute value of error comparison between original scheme and scheme in this paper

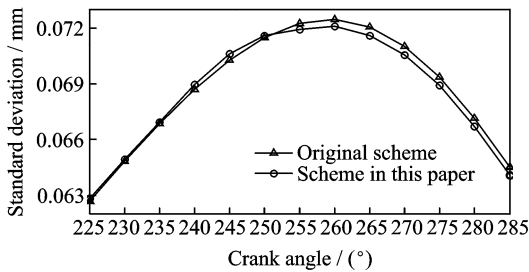


Fig. 7 Standard error deviation comparison between original scheme and scheme in this paper

As shown in Figs. 6, 7, compared with the result of typical robustness design, the similar variation characteristics of mean value and standard deviation can be found, so the method in this paper is effective.

According to the proposed design model the design parameters and the motion error of three schemes (minimum clearance 111, medium clearance 222 and maximum clearance 333) were calculated, through the process in Fig. 3. Results of design parameters for three schemes are given in Table 4, and the mean value and standard deviation of motion error are shown in Figs. 8, 9.

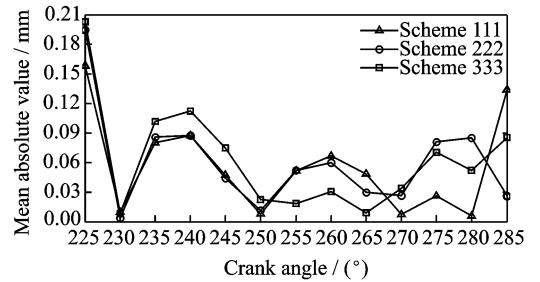


Fig. 8 Mean absolute value of error of comparison of three schemes

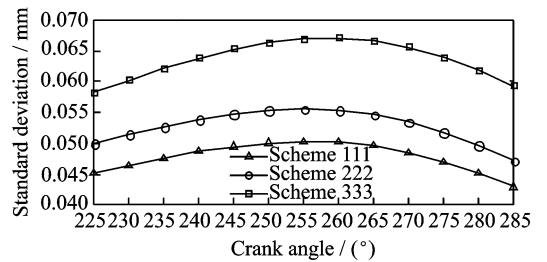


Fig. 9 Error standard deviation comparison of three schemes

Table 4 Optimization results of three schemes

Design variable	$l_1$ /mm	$l_2$ /mm	$l_3$ /mm	$\Delta l_1$ /mm	$\Delta l_2$ /mm	$\Delta l_3$ /mm
Scheme 111	57.798	164.089	70.345	0.068	0.057	0.044
Scheme 222	58.006	164.583	70.742	0.076	0.063	0.045
Scheme 333	57.826	164.071	70.261	0.069	0.051	0.081

### 3 Multi-objective Optimal Design for Motion Precision and Cost Saving

In the engineering, the precision requirement is the precondition of the optimal design, and the cost is one of the main factors to be considered in the optimal design of planar linkage mechanism, but the motion precision of the mechanism and the cost of manufacturing cannot be effectively balanced.

In this paper, the optimal solutions are obtained through the process of Fig. 3, which is on-

ly the minimum value of the objective function. However, when the uncertainty factors increase, the whole precision of the mechanism will be affected<sup>[20-22]</sup>. Hence, it is necessary to evaluate the optimal solutions of the system, and a basis for the following cost requirements is provided.

#### 3.1 Model precision and cost evaluation

In general design, the same fit clearance for each joint point is provided. However, in fact, some points have less impact for the motion precision of the mechanism, if smaller clearances are applied to these points, the unnecessary cost will

be increased. Thus, in this paper, this unnecessary cost is mainly aimed at to be analyzed.

Assuming that each revolute pair of the slider-crank mechanism has three different clearances to be chosen (as shown in Table 1), that is,  $3^3 = 27$  schemes in total. Each scheme is designed by the process in Fig. 3, and 27 optimum solutions will be obtained, which are evaluated according to the following precision evaluation criteria.

Supposing that the object point is  $o_i, i=1, 2, \dots, 13$ , the upper and the lower bound of the object can be expressed as  $o_{ui} = o_i + o_i \cdot p$  and  $o_{li} = o_i - o_i \cdot p$ , respectively.  $p$  is the object-precision. From Figs. (6, 8) it can be found that the mean value at the point of  $225^\circ$  is larger than the other points, hence, 0.2mm is used to define the object-precision, that is,  $0.2/80 = 2.5\%$ , namely the precision  $p=2.5\%$ . In the obtained optimization results,  $\bar{x}_i$  is the mean value of the motion error and  $\sigma_i$  is the standard deviation. Therefore, the upper bound of the optimization result is  $x_{ui} = \bar{x}_i + \sigma_i, i=1, 2, \dots, 13$ , and the lower bound is  $x_{li} = \bar{x}_i - \sigma_i, i=1, 2, \dots, 13^{[6-7]}$ . Thus, the precision of the mechanism fulfills the requirements after optimization when  $x_{ui} \leq o_{ui} \cup x_{li} \geq o_{li}, i=1, 2, \dots, 13$ .

When the optimization results of the schemes fulfill the precision requirements, cost evaluation is then carried out, that is, the biggest sum of the sequence number are chosen (the bigger clearance causes the lower cost). Motion precision and cost evaluation process of the whole mechanism is shown in Fig. 10.

### 3.2 Multi-objective optimal design process and results

According to the process of multi-objective optimal design model in Fig. 10, the 27 schemes are firstly sampled for calculation in turn. Then the motion precision is evaluated and the schemes that fulfill the precision requirement are chosen, that is, 111, 122, 123, 211, 212, 223, 322 and 332. Finally, the biggest clearance scheme is chosen (the lowest cost), which is 332. Thus, the

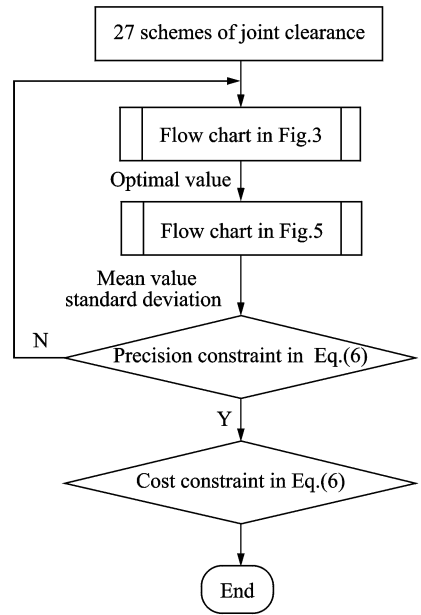


Fig. 10 Process analysis of multi-objective optimal design model

joint point of the actual mechanism is  $\frac{H8}{e8}$  for point A,  $\frac{H8}{e8}$  for point B, and  $\frac{H7}{f6}$  for point C, respectively.

In this way, the lowest manufacturing cost under the motion precision premise of this example can be obtained. Considering there are numerous factors affecting the cost, the basic thought of balancing the precision and the cost is provided.

## 4 Conclusions

(1) In view of the uncertain increases in the mechanism's whole motion error after manufacturing, the motion error is predicted to be difficult at the design phase. This paper analyzes the relationship between design and manufacturing. Based on Monte Carlo simulation method, an optimal design model of planar linkage mechanism with joint clearance is presented.

(2) As shown in Figs. 6, 7, compared with the result of typical robustness design, the similar variation characteristics of the mean value and the standard deviation can be found, so the method in this paper is effective.

(3) As shown in Figs. 8, 9, with the increase



of the joint clearance, the mean value of the motion error fluctuates in a certain range. The standard deviation of the motion error increases, namely, the motion error fluctuation of the actual mechanism increases at the mean value points, so the uncertainty of the mechanism's motion error increases. The model in this paper can be used to quantitatively describe the influence of different fit and dimensional tolerances on the motion error of actual mechanism.

(4) In general, motion precision of the mechanism and the manufacturing cost cannot be effectively balanced. That is, with the decrease of the joint clearance, the motion precision increased, and the manufacturing cost increased. In this paper, aiming at typical offset slider-crank mechanism, three different clearances for each joint point are provided. Applying the proposed model, 27 schemes are calculated, the results are analyzed, and a scheme that meets the requirement of both the precision and the cost can be obtained. Considering there are numerous factors affecting the cost, a basic thought of balancing the precision and the cost is provided.

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