Nonlinear Control Method for Hypersonic Vehicle Based on Double Power Reaching Law of Sliding Mode

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(Received 4 July 2018; revised 15 August 2018; accepted 15 October 2018)

Abstract: The intelligence nonlinear control scheme via double power reaching law based sliding mode control method is proposed to solve the problems of model uncertainties and unknown outside disturbances. Firstly, the aerodynamic parameters of the morphing vehicle are replaced with curve-fitted approximation to build the accurate model for control design in the hypersonic flight. Then the nonlinear vehicle model is transformed into the strict feedback multi-input/multi-output nonlinear system by using the input-output feedback linearization approach. At the same time, the disturbance observer is used to approximate the unknown disturbance, and the sliding mode method is used to solve the problem of non-matching and uncertainty. Finally, according to the buffeting problem in sliding mode control, the double power is improved. Simulation results show that the proposed method can ensure the global stability of the closed-loop system, and has good tracking and robust performance.

Key words: hypersonic vehicle; disturbance observer; double power; sliding mode; buffeting

CLC number: TN247 **Document code**: A **Article ID**: 1005-1120(2018)05-0849-09

0 Introduction

The hypersonic vehicle is a multivariable system with strong nonlinearity, strong coupling and fast time-varying[1-2]. Due to the wide range of changes in the flight environment of the aircraft, the dynamic changes of the aerodynamic parameters are intense and there are external aerodynamic disturbances. The control-oriented aircraft model has a certain mismatch with the actual model, that is, the uncertainty of modelling. The above reasons lead to the large uncertainty of hypersonic vehicle, and these uncertainties will have negative effect on the stability and rapidity of hypersonic vehicle control. Therefore, it is very important to study the dynamic characteristic analysis of hypersonic vehicle with model uncertainty.

At present, for the problem of hypersonic aircraft uncertainty, many scholars start from the aspect of controller design directly, regarding the uncertainty as a disturbance, assuming it in a certain range, and introducing it into the design of controller, such as Refs.[3-4]. However, this method does not analyze the physical nature of the uncertain factors, and not specifically reflect the impact of a particularly uncertain factor on the dynamic system, so the analysis of vehicle uncertain parameters does not accurately reflect the dynamic and control characteristics of the aircraft under the real conditions. Conventional vehicle attitude control usually adopts a small disturbance linearization method, which is difficult to apply to highly non-linear hypersonic vehicle. In Refs.[5-7], back stepping control, sliding mode control, robust control and adaptive dynamic surface control

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How to cite this article: Liu Rong, Zhang Heng, Liang Jin. Nonlinear control method for hypersonic vehicle based on double power reaching law of sliding mode[J]. Trans. Nanjing Univ. Aero. Astro., 2018,35(5):849-857.
http://dx.doi.org/10.16356/j.1005-1120.2018.05.849

are used to overcome the influence of external disturbances and uncertainties. However, due to the high requirement of the transient and flight accuracy of the vehicle during the flight, it is difficult to guarantee the transient performance of the system when the external disturbance is aperiodic signal. In Ref. [5], the gain pre-set control was applied to hypersonic vehicle, but the closed-loop performance of the system can not be guaranteed when the nonlinear characteristics of the model change drastically. In Ref. [6], a high-supersonic vehicle height control based on dynamic inverse method was realized. In Ref. [7], a flight control system based on dynamic inverse was designed for a highly nonlinear and complex hypersonic longitudinal model, but it was found that this method does not have robustness to parameter and model changes. Ref. [8] used sliding mode control technique to design a time scale separation flight control system for reusable launch vehicles, but it was prone to buffeting on the sliding surface. Refs. [9-10] analyzed the uncertainty and external disturbances of hypersonic vehicles. In Ref. [11], the sliding mode control method was adopted for the uncertain disturbance of the multi-input and multi-output system, but the buffeting was not improved obviously. In Ref. [12], for the controller design of the near-space vehicle, when there was parameter uncertainty and disturbance, the dynamic inverse control was combined with the sliding mode variable structure control, which greatly improved the ability of the aircraft to coordinate flight. Ref. [13] aimed at the re-entry attitude control of the aircraft, and the control system was designed based on the second-order terminal sliding mode method, which can effectively eliminate buffeting, considering the uncertainty and unknown disturbance during flight. Meanwhile, the adaptive control method was combined with the sliding mode to improve the accuracy of the system. In Ref. [14], a sliding mode control method was used to design a sliding mode control system for the launch and re-entry modes of reusable aircraft (X-33), using high control gain to solve the model uncertainty and external disturbance. In Ref. [15], the adaptive feedback linearization method was applied to the design of the re-entry vehicle (X-33) controller, and the Pseudo-Control Hedging idea was introduced in the neural network to solve the disturbance and actuator saturation in the controller.

In this paper, the sliding mode variable structure controller of hypersonic vehicle is designed by using the double power method based on disturbance observer and the model with parameter uncertainty. In the model of feedback linearization, we add uncertain parameter changes, then design the controller and verify its stability using the Lyapunov function. In order to verify the effectiveness of the method, the nominal model and parameter perturbation model are used in the simulation, and observation results show that the double power sliding mode based on disturbance observer has good command tracking and robustness to the nonlinear control of hypersonic vehicle.

1 Longitudinal Modelling of Hypersonic Vehicle

In this paper, double power reaching law based sliding mode control method and nonlinear disturbance observer are proposed to design the control system of hypersonic vehicle disturbed by uncertainties factors. The system control structure of hypersonic vehicle longitudinal channel is shown in Fig. 1, where h represents the height, h_d the expected height, V the speed, V_e the expected speed, δ_e the elevator rudder output, and β_c the throttle opening output.

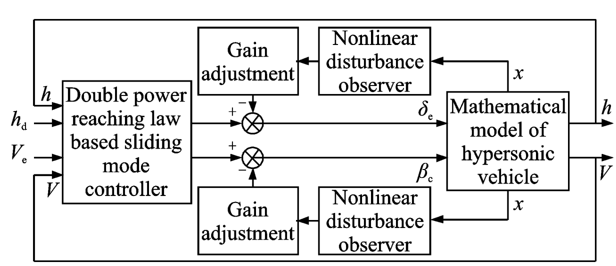

Fig. 1 System control structure of hypersonic vehicle longitudinal channel

1.1 Nonlinear mathematical model

The longitudinal motion model of the vehicle in hypersonic cruise flight conditions is described as[16-17]

$$\dot{V}=(T\cos\alpha-D)/m-\mu\sin\gamma/r^2 \quad (1)$$

$$\dot{\gamma}=(L+T\sin\alpha)/mV-(\mu-V^2 r)\cos\gamma/Vr^2 \quad (2)$$

$$\dot{\alpha}=q-\dot{\gamma} \quad (3)$$

$$\dot{q}=M/I_{yy} \quad (4)$$

$$\dot{h}=V\sin\gamma \quad (5)$$

where V, γ, α, q, h are the indicate the flight speed, the flight-path angle, the angle of attack, the pitch angle rate and the height. L, D, T, M represent the lift, the drag, the thrust and the pitching moment of vehicle, respectively. μ represents the gravitational constant, I_{yy} is the moment of inertia, and r represents the radial distance from Earth's center.

Aerodynamic and aerodynamic torques are expressed as

$$L=\rho V^2 SC_{\mathrm{l}}/2 \quad (6)$$

$$D=\rho V^2 SC_{\mathrm{d}}/2 \quad (7)$$

$$T=\rho V^2 SC_{\mathrm{T}}/2 \quad (8)$$

$$M=\rho V^2 Sc_A C_{\mathrm{m}}/2$$

where S represents the wing penetration area, c_A means the aerodynamic chord, and ρ means the density of air. $C_{\mathrm{l}}, C_{\mathrm{d}}, C_{\mathrm{T}}, C_{\mathrm{m}}$ are the lift coefficient, the drag coefficient, the dynamic thrust coefficient and the pitching moment coefficient, respectively.

The calculation formula of engine thrust is

$$T=0.5\rho V^2 SC_{\mathrm{T}} \quad (9)$$

$$C_{\mathrm{T}}=\begin{cases}0.025\,76\beta & \beta>1\\ 0.022\,4+0.003\,36\beta & \beta<1\end{cases}$$

The dynamic equation of the engine adopts the second order system model

$$\ddot{\beta}=-2\xi\omega_{\mathrm{n}}\dot{\beta}-\omega_{\mathrm{n}}^2\beta+\omega_{\mathrm{n}}^2\beta_{\mathrm{c}}$$

The control input of the model is the engine throttle set value β_{c} and elevator deflection δ_{e}. The output is speed V and height h. $\boldsymbol{y}=[V\quad h]^{\mathrm{T}}$. Damping $\zeta=0.7$, and natural frequency $\omega_{\mathrm{n}}=5$ rad/s.

In the analysis of the hypersonic vehicle model, it is necessary to consider its internal interference and uncertainty, using the assumed rating and adding a variable to represent the uncertainty of the parameter

$$m=m_0+\Delta m, J_y=J_{y0}+\Delta J_y, c_A=c_{A0}+\Delta c_A, S_w=S_{w0}+\Delta S_w, \rho=\rho_0+\Delta\rho$$

where m, J_y, S_w, ρ represent the aircraft mass, moment of inertia, wing infiltration area and atmospheric density, respectively.

1.2 Input-output feedback linearization

Input-output feedback linearization is achieved through accurate state transitions and feedback.

In the paper, a double power sliding mode of hypersonic vehicle is proposed with the control purpose of ensuring that the vehicle flight speed V and height h can be quickly tracked to the specified values V_c and h_c within a given control input vector range. In Eqs. (1)—(5), full-state feedback linearization is used to process the output of flight speed V and flight height h. Differential n and m times for V and h, respectively, until the control input β_{c} or δ_{e} appears in the differential formula. Then, we have

$$\begin{cases}\dot{V}=f_V(\boldsymbol{x})\\ \ddot{V}=\omega_1\dot{\boldsymbol{x}}/m\\ \dddot{V}=(\omega_1\ddot{\boldsymbol{x}}+\dot{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\Omega}_2\dot{\boldsymbol{x}})/m\end{cases} \quad (10)$$

$$\begin{cases}\dot{h}=f_h(\boldsymbol{x})\\ \ddot{h}=\dot{V}\sin\gamma+V\dot{\gamma}\cos\gamma\\ \dddot{h}=\ddot{V}\sin\gamma+2\dot{V}\dot{\gamma}\cos\gamma-V\dot{\gamma}^2\sin\gamma+V\ddot{\gamma}\cos\gamma\\ h^{(4)}=\dddot{V}\sin\gamma+3\ddot{V}\dot{\gamma}\cos\gamma-3\dot{V}\dot{\gamma}^2\sin\gamma+V\ddot{\gamma}\cos\gamma+\\ \quad 3\dot{V}\ddot{\gamma}\cos\gamma-3V\dot{\gamma}\ddot{\gamma}\sin\gamma-\dot{V}\dot{\gamma}^3\cos\gamma+V\ddot{\gamma}\cos\gamma\end{cases} \quad (11)$$

where $\boldsymbol{x}=[V,\gamma,\alpha,q,h]^{\mathrm{T}}$, the five states of the non-linearity of the hypersonic vehicle are the speed, the track angle, the angle of attack, the pitch angle rate, and the height. $\ddot{\boldsymbol{x}}=[\ddot{V}\quad\ddot{\gamma}\quad\ddot{\alpha}\quad\ddot{\beta}\quad\ddot{h}]^{\mathrm{T}}$. $\gamma^{(3)}=\boldsymbol{\pi}_1^{\mathrm{T}}\ddot{\boldsymbol{x}}+\dot{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\Pi}_2\dot{\boldsymbol{x}}$. $\dddot{V}$ and $h^{(4)}$ contain $\ddot{\alpha}$ and $\ddot{\beta}$. $\ddot{\alpha}=\ddot{\alpha}_0+\ddot{\alpha}_{\delta_{\mathrm{e}}}\delta_{\mathrm{e}}+\ddot{\alpha}_{\delta_{\mathrm{a}}}\delta_{\mathrm{a}}$, $\ddot{\alpha}_{\delta_{\mathrm{e}}}=\frac{1}{I_{yy}}\hat{q}cSC_{m,\delta_{\mathrm{e}}}$, $C_{m,\delta_{\mathrm{e}}}=c_{\mathrm{e}}(\delta_{\mathrm{e}}-\alpha)$, $\ddot{\beta}=-2\xi\omega\dot{\beta}-\omega^2\beta+\omega^2\beta_c$. The expressions of $\dddot{V}$ and $h^{(4)}$ contain control input β_{c} and δ_{e}. m means the mass.

The third order differential expression of the output flight speed and the fourth order differentials of the flight height are expressed as

$$\begin{bmatrix}\dddot{V}\\ h^{(4)}\end{bmatrix}=\begin{bmatrix}\dddot{V}_0\\ h_0^{(4)}\end{bmatrix}+\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{bmatrix}\begin{bmatrix}\delta_{\mathrm{e}}\\ \beta_{\mathrm{c}}\end{bmatrix}+$$

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} \tag{12}$$

where

$$\dddot{V}_0 = L_f^3 V(\boldsymbol{x}) = (\omega_1 \ddot{\boldsymbol{x}}_0 + \dot{\boldsymbol{x}}^{\mathrm{T}} \omega_2 \dot{\boldsymbol{x}})/m \tag{13}$$

$$H_0^{(4)} = 3\ddot{V}\dot{\gamma}\cos\gamma - 3\dot{V}\dot{\gamma}^2\sin\gamma + 3\dot{V}\ddot{\gamma}\cos\gamma - 3V\dot{\gamma}\ddot{\gamma}\sin\gamma - V\dot{\gamma}^{(3)}\cos\gamma + \frac{(\omega_1\ddot{\boldsymbol{x}}_0 + \dot{\boldsymbol{x}}^{\mathrm{T}}\Omega_2\dot{\boldsymbol{x}})\sin\gamma}{m} + V\cos\gamma(\boldsymbol{\pi}_1\ddot{\boldsymbol{x}}_0 + \dot{\boldsymbol{x}}^{\mathrm{T}}\Pi_2\dot{\boldsymbol{x}}) \tag{14}$$

$$b_{11} = (\rho V^2 S c_\beta \omega_{\mathrm{n}}^2 / 2m)\cos\alpha \tag{15}$$

$$b_{12} = -(c_{\mathrm{e}}\rho V^2 S\bar{c}/2mI_{yy})(T\sin\alpha + D_\alpha - T_\alpha\cos\alpha) \tag{16}$$

$$b_{21} = (\rho V^2 S c_\beta \omega_{\mathrm{n}}^2 / 2m)\sin(\alpha+\gamma) \tag{17}$$

$$b_{22} = \left(\frac{c_{\mathrm{e}}\rho V^2 S\bar{c}}{2mI_{yy}}\right)[T\cos(\alpha+\gamma) + L_\alpha\cos\gamma + T_\alpha\sin(\alpha+\gamma) - D_\alpha\sin\gamma] \tag{18}$$

and $d_1(t)$, $d_2(t)$ represent uncertainty factors.

Assumed that ratings add a variable to represent the uncertain output of parameters. The third order differentials of the output flight speed and the fourth order differentials of the flight height are expressed as

$$\begin{bmatrix} \dddot{V} \\ h^{(4)} \end{bmatrix} = \begin{bmatrix} \dddot{V}_0 \\ h_0^{(4)} \end{bmatrix} + \boldsymbol{B}\begin{bmatrix} \delta_{\mathrm{e}} \\ \beta_{\mathrm{c}} \end{bmatrix} + \boldsymbol{B}\begin{bmatrix} \Delta F_V \\ \Delta F_h \end{bmatrix} \tag{19}$$

where $\boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, f_V and f_h are the short-hand expressions of the right-hand side of Eqs. (1), (2), respectively, and $\omega_1 = \frac{\partial f_V(\boldsymbol{x})}{\partial \boldsymbol{x}}$, $\Omega_2 = \frac{\partial \omega_1}{\partial \boldsymbol{x}}$, $\boldsymbol{\pi}_1 = \frac{\partial f_h(\boldsymbol{x})}{\partial \boldsymbol{x}}$, $\Pi_2 = \frac{\partial \boldsymbol{\pi}_1}{\partial \boldsymbol{x}}$. ΔF_V, ΔF_h are uncertain errors.

1.3 Analysis of open-loop characteristics of hypersonic vehicle

In high-speed cruise flight, the zero-input response of the hypersonic vehicle nonlinear model and the motion state quantity curve of the aircraft are obtained, and the open-loop characteristic of hypersonic vehicle is analyzed. Set its initial state to be $V = 4\ 590$ m/s, $h = 33\ 528$ m, $\delta_{\mathrm{e}} = 0$, other states are zero, and the response curve of each state variable is shown in Fig. 2.

As can be seen from Fig. 2, when the hypersonic vehicle is cruising, all the motion parameters cannot be kept stable under the zero input response of the aircraft. On the one hand, the flight speed of the aircraft is constantly falling and the pitch angle rate keeps fluctuating. On the other hand, the flight angle of attack shows a trend of increasing, which makes the flight height continuously rise, the entire aircraft lose stability and cannot maintain normal flight. Therefore,

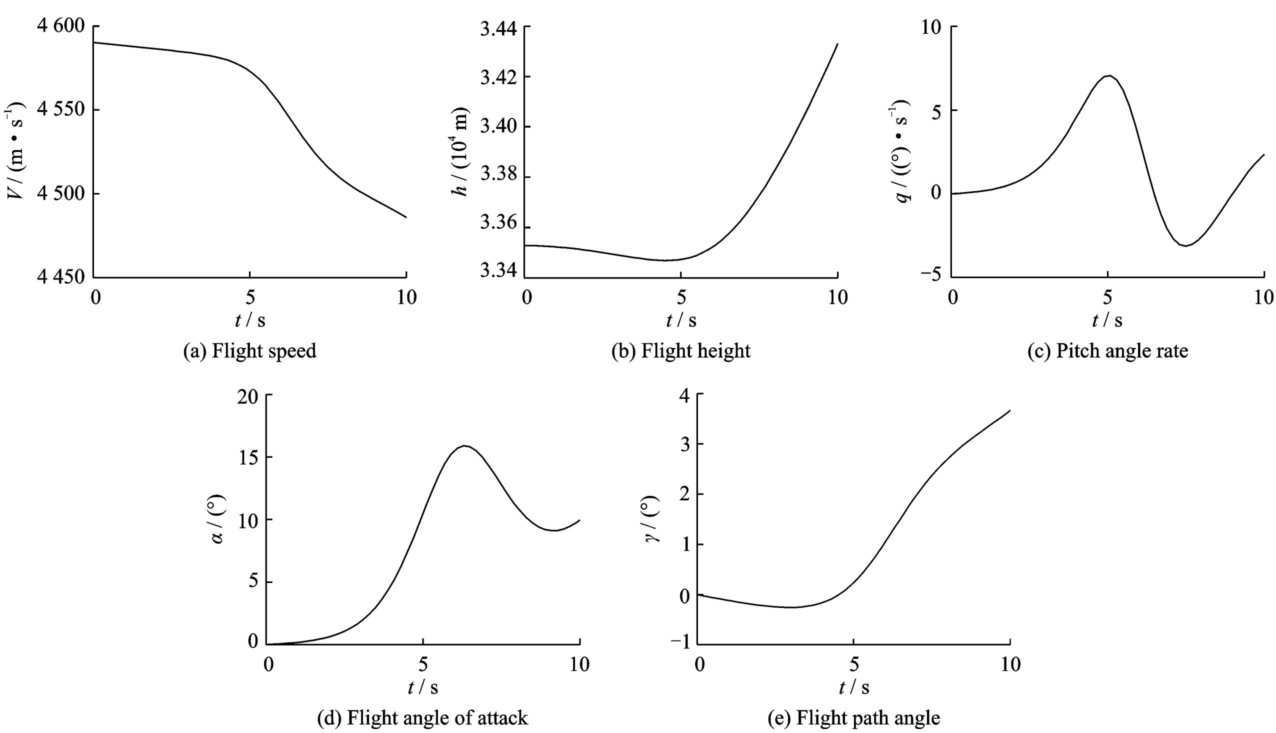

Fig. 2 Output response of hypersonic vehicle in zero input state

the intelligence-nonlinear control scheme via double power reaching law based sliding mode control method is proposed to solve the stable tracking control of the hypersonic morphing vehicle considering compound disturbances caused by the morphing vehicle, as well as the influence of model uncertainties and unknown outside disturbances.

2 Design of Double Power Sliding Mode Controller Based on Interference Observer

During flight, due to its own elastic deformation and uncertainty disturbances caused by other factors, hypersonic vehicle will make flight control worse. In this paper, the nonlinear disturbance observer and double power sliding mode dynamic surface control method are applied to the design of hypersonic vehicle systems with unknown disturbances.

2.1 Controller design

The controller adopts double power sliding mode method, which can ensure that the system can approach the stability at a faster speed and guarantee the smaller control gain of the system, which can greatly reduce the buffeting in sliding mode control.

Assumption 1 The determinism of the parameters is bounded, $|\Delta F_V| < M_V$, $|\Delta F_h| < M_h (M_V > 0, M_h > 0)$.

Assumption 2 $\boldsymbol{BB}^{\mathrm{T}}$ is a non-singular matrix, and $(\boldsymbol{BB}^{\mathrm{T}})^{-1}$ is existent.

In the all flight envelope, track angle $\gamma \neq \pm \frac{\pi}{2}$, flight track is not perpendicular to horizontal plane, thus matrix $\boldsymbol{B}$ is reversible[18], and Assumption 2 is tenable.

According to the model of hypersonic vehicle, the sliding surface is chosen as

$$\boldsymbol{S} = \begin{bmatrix} S_V \\ S_h \end{bmatrix} = \begin{bmatrix} (\mathrm{d}/\mathrm{d}t + \lambda_V)^3 \int e_V(\tau)\,\mathrm{d}\tau \\ (\mathrm{d}/\mathrm{d}t + \lambda_h)^4 \int e_h(\tau)\,\mathrm{d}\tau \end{bmatrix} \tag{20}$$

where λ_V, λ_h are constant values. $e_V(t) = V - V_{\mathrm{d}}$ and $e_h(t) = h - h_{\mathrm{d}}$ are the tracking errors of speed and height. The integral term is used to eliminate the static error.

Then, we have

$$\dot{\boldsymbol{S}} = \begin{bmatrix} \dot{S}_V \\ \dot{S}_h \end{bmatrix} = \begin{bmatrix} -\bar{V}_{\mathrm{d}} + 3\lambda_V \ddot{e}_V + 3\lambda_V^2 \dot{e}_V + \lambda_V^3 e_V \\ -h_{\mathrm{d}}^{(4)} + 4\lambda_h \dddot{e}_h + 6\lambda_h^2 \ddot{e}_h + 4\lambda_h^3 \dot{e}_h + \lambda_h^4 e_h \end{bmatrix} \tag{21}$$

The double power design is

$$u_s = \begin{pmatrix} -k_{V1} |S_V|^{\beta_V} \mathrm{sgn}(S_V) - k_{V2} |S_V|^{\alpha_V} \mathrm{sgn}(S_V) \\ -k_{h1} |S_h|^{\beta_h} \mathrm{sgn}(S_h) - k_{h2} |S_h|^{\alpha_h} \mathrm{sgn}(S_h) \end{pmatrix} \tag{22}$$

where $k_{V1}, k_{V2}, k_{h1}, k_{h2} > 0, 0 < \alpha_V, \alpha_h < 1, \beta_V, \beta_h > 1$, $\mathrm{sgn}(S_V), \mathrm{sgn}(S_h)$ are sign functions.

$$u = \boldsymbol{B}^{\mathrm{T}} (\boldsymbol{BB}^{\mathrm{T}})^{-1} \left(- \begin{bmatrix} \bar{V}_0 \\ h_0^{(4)} \end{bmatrix} + \begin{bmatrix} \bar{V}_{\mathrm{d}} \\ h_{\mathrm{d}}^{(4)} \end{bmatrix} - \begin{bmatrix} F_V \\ F_h \end{bmatrix} \right) + \begin{pmatrix} -k_{V1} |S_V|^{\beta_V} \mathrm{sgn}(S_V) - k_{V2} |S_V|^{\alpha_V} \mathrm{sgn}(S_V) \\ -k_{h1} |S_h|^{\beta_h} \mathrm{sgn}(S_h) - k_{h2} |S_h|^{\alpha_h} \mathrm{sgn}(S_h) \end{pmatrix} \tag{23}$$

where $\begin{bmatrix} F_V \\ F_h \end{bmatrix} = \begin{bmatrix} 3\lambda_V \ddot{e}_V + 3\lambda_V^2 \dot{e}_V + \lambda_V^3 e_V \\ 4\lambda_h \dddot{e}_h + 6\lambda_h^2 \ddot{e}_h + 4\lambda_h^3 \dot{e}_h + \lambda_h^4 e_h \end{bmatrix}$

Bring Eq. (22) into Eq. (21), we have

$$\dot{\boldsymbol{S}} = \begin{bmatrix} \Delta F_V - k_{V1} |S_V|^{\beta_V} \mathrm{sgn}(S_V) - k_{V2} |S_V|^{\alpha_V} \mathrm{sgn}(S_V) \\ \Delta F_h - k_{h1} |S_h|^{\beta_h} \mathrm{sgn}(S_h) - k_{h2} |S_h|^{\alpha_h} \mathrm{sgn}(S_h) \end{bmatrix} \tag{24}$$

2.2 Design of nonlinear disturbance observer

To eliminate the influence of uncertainty and unknown disturbance on the performance of the system, the disturbance observer is firstly used to estimate the system disturbance, and the unobserved part of the disturbance is compensated by using sliding mode control.

Considering the external interference and uncertainty of the system, we have

$$x_1 = v, x_2 = \dot{v}, x_3 = \ddot{v}, x_4 = h, x_5 = \dot{h}, x_6 = \ddot{h}, x_7 = \dddot{h} \tag{25}$$

Make $[\delta_{\mathrm{e}}, \beta_{\mathrm{c}}] = [u_1, u_2]$, the system is

$$\begin{cases} \dot{x}_3 = f_1(x) + b_{11} \cdot u_1 + b_{12} \cdot u_2 + d_3 \\ \dot{x}_7 = f_2(x) + b_{21} \cdot u_1 + b_{22} \cdot u_2 + d_7 \end{cases} \tag{26}$$

Considering the nonlinear control of speed tracking, the following state of the disturbance observer is designed

$$z_1 = d_3^* - p_1(\boldsymbol{x}) \tag{27}$$

where d_3^* is the estimation of d_3, $p_1(\boldsymbol{x})$ is the nonlinear function to be designed, and it should satisfy $\dot{p}_1(\boldsymbol{x}) = L_1(\boldsymbol{x}) \cdot \dot{x}_3$.

The design of the speed observer is

$$\begin{cases} z_1 = d_3^* - p_1(\boldsymbol{x}) \\ \dot{z}_1 = -L_1(\boldsymbol{x}) z_1 + L_1(\boldsymbol{x})(-p_1(\boldsymbol{x}) - b_{11}\delta_e - \\ \qquad b_{12}\beta_c - f_1) \end{cases} \tag{28}$$

Similarly, the height of the nonlinear observer is

$$\begin{cases} z_2 = d_8^* - p_2(\boldsymbol{x}) \\ \dot{z}_2 = -L_2(\boldsymbol{x}) z_2 + L_2(\boldsymbol{x})(-p_2(\boldsymbol{x}) - b_{21}\delta_e - \\ \qquad b_{22}\beta_c - f_2) \end{cases} \tag{29}$$

Observing unknown linear disturbances so that the height and speed can be well controlled in the presence of interference, which can improve the robustness of the system and reduce the impact of external uncertainties on the vehicle, thus obtaining the improved stability of the vehicle.

Lemma[19] The system disturbed by uncertainty factors is given, and its extended state space form is

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u + \boldsymbol{E}d$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{30}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{E}$, $\boldsymbol{C}$ are the matrices of coefficient of the state space expression. d represents uncertainty factor. Thus, the disturbance observer of this system can be designed as

$$\dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}u + \boldsymbol{L}(\boldsymbol{y} - \hat{\boldsymbol{y}})$$
$$\hat{\boldsymbol{y}} = \boldsymbol{C}\boldsymbol{z} \tag{31}$$

Errors of disturbance observer can be defined as: $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{z}, \dot{\boldsymbol{e}} = \boldsymbol{A}_e\boldsymbol{e} + r$, $\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{L}\boldsymbol{C}$, $\boldsymbol{r} = \boldsymbol{E}d$.

If the suitable gain of disturbance observer $\boldsymbol{L}$ is selected, matrix $\boldsymbol{A}_e$ is Hurwitz, so arbitrary bounded disturbance d can make the error of disturbance observer $\boldsymbol{e}$ be bounded.

According to Eq.(12), uncertainty factors of hypersonic vehicle system D_1 and D_2 can be described as

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{32}$$

The error of height disturbance observer $\dot{\boldsymbol{e}}_1$ and the error of speed disturbance observer $\dot{\boldsymbol{e}}_2$ are described as

$$\begin{cases} \dot{\boldsymbol{e}}_1 = (A_1 - L_1C_1)\boldsymbol{e}_1 + E_1f_1 = A_{e_1}\boldsymbol{e}_1 + E_1f_1 \\ \dot{\boldsymbol{e}}_2 = (A_2 - L_2C_2)\boldsymbol{e}_2 + E_2f_2 = A_{e_2}\boldsymbol{e}_2 + E_2f_2 \end{cases} \tag{33}$$

where $\boldsymbol{e}_1 = [e_{11} \quad e_{12} \quad e_{13}]^{\mathrm{T}}$; $\boldsymbol{e}_{1j} = x_{1,j} - \widehat{x}_{1,j}, j = 1, \cdots, 3$; $\boldsymbol{e}_2 = [e_{21} \quad e_{22} \quad e_{23} \quad e_{24}]^{\mathrm{T}}$; $\boldsymbol{e}_{2k} = x_{2,k} - \widehat{x}_{2,k}$, $k = 1, \cdots, 4$; $\boldsymbol{E}_1 = [0 \quad 0 \quad 1]^{\mathrm{T}}$, $\boldsymbol{E}_2 = [0 \quad 0 \quad 0 \quad 1]^{\mathrm{T}}$, $\boldsymbol{C}_1 = [1\ 0\ 0]^{\mathrm{T}}$, $\boldsymbol{C}_2 = [1\ 0\ 0\ 0]^{\mathrm{T}}$,

$$\boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \ \boldsymbol{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

According to Lemma 1 and Assumption 2, A_{e_1} and A_{e_2} are Hurwitz, so arbitrary bounded disturbances D_1 and D_2 can make the errors of disturbance observers $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ be bounded. If gain of disturbance observer is adjusted suitably, estimating error can be arbitrarily small, and the accuracy estimation of disturbance can be realized.

2.3 Stability analysis

Select the system Lyapunov function

$$V = 0.5\boldsymbol{S}^2 \tag{34}$$

The derivation of Eq.(34) is

$$\begin{gathered} \dot{V} = \boldsymbol{S}^{\mathrm{T}}\dot{\boldsymbol{S}} = S_V\dot{S}_V + S_h\dot{S}_h = \\ S_V(\Delta F_V - k_{V1}|S_V|^{\beta_V}\mathrm{sgn}(S_V) - k_{V2}|S_V|^{\alpha_V}\mathrm{sgn}(S_V)) + \\ S_h(\Delta F_h - k_{h1}|S_h|^{\beta_h}\mathrm{sgn}(S_h) - k_{h2}|S_h|^{\alpha_h}\mathrm{sgn}(S_h)) = \\ S_V\Delta F_V - k_{V1}|S_V|^{1+\beta_V} - k_{V2}|S_V|^{1+\alpha_V} + S_h\Delta F_h - \\ k_{h1}|S_h|^{1+\beta_h} - k_{h2}|S_h|^{1+\alpha_h} \end{gathered} \tag{35}$$

Then, we have

$$\begin{gathered} \dot{V} \leqslant -k_{V1}|S_V|^{1+\beta_V} - k_{V2}|S_V|^{1+\alpha_V} - k_{h_1}|S_h|^{1+\beta_h} - \\ k_{h2}|S_h|^{1+\alpha_h} + |S_h||\Delta F_h| + |S_V||\Delta F_V| = \\ -k_{V1}|S_V|^{1+\beta_V} - k_{h1}|S_h|^{1+\beta_h} - \\ |S_V|(k_{V2}|S_V|^{\alpha_V} - |\Delta F_V|) - \\ |S_h|(k_{h2}|S_h|^{\alpha_h} - |\Delta F_h|) \end{gathered} \tag{36}$$

Eq.(36) is not negative, if $k_{V2}|S_V|^{\alpha_V} - |\Delta F_V| \geqslant 0$, $k_{h2}|S_h|^{\alpha_h} - |\Delta F_h| \geqslant 0$. Then Eq.(35) can be expressed as $\dot{V} \leqslant -k_{V1}|S_V|^{1+\beta_V} - k_{h1}|S_h|^{1+\beta_h}$, and it is negative.

Finally, according to Assumption 1, if $|S_V| \leqslant \left(\frac{M_V}{k_{V1}}\right)^{\frac{1}{\beta_V}}$ and $|S_h| \leqslant \left(\frac{M_h}{k_{h1}}\right)^{\frac{1}{\beta_h}}$, the system is stable.

In addition, Eq.(36) can be expressed as

$$\dot{V} \leqslant -k_{V1}|S_V|^{1+\beta_V} - k_{V2}|S_V|^{1+\alpha_V} - k_{h_1}|S_h|^{1+\beta_h} -$$

$$k_{h2}|S_h|^{1+\alpha_h}+|S_h||\Delta F_h|+|S_V||\Delta F_V|= -k_{V2}|S_V|^{1+\alpha_V}-k_{h2}|S_h|^{1+\alpha_h}- |S_V|(k_{V1}|S_V|^{\beta_V}-|\Delta F_V|)- |S_h|(k_{h1}|S_h|^{\beta_h}-|\Delta F_h|) \quad (37)$$

The inequality of the right side of Eq. (37) is not negative, but if $k_{V1}|S_V|^{\beta_V}-|\Delta F_V|\geqslant 0$ and $k_{h1}|S_h|^{\beta_h}-|\Delta F_h|\geqslant 0$, we can get $\dot{V}\leqslant -k_{V2}|S_V|^{1+\alpha_V}-k_{h2}|S_h|^{1+\alpha_h}$, and it is negative.

2.4 Simulation

To verify the feasibility of the double power sliding mode based on the disturbance observer, the simulation of the hypersonic vehicle is carried out with MATLAB/Simulink, and the balance conditions of hypersonic vehicle are $V_0=4\ 590.3$ m/s, $h_0=335\ 28$ m, $\gamma_0=0°$, $q_0=0$ °/s. Simulation parameters are selected as $\lambda_V=0.5$, $\lambda_h=0.5$, $k_{V1}=0.8$, $k_{V2}=0.2$, $k_{h1}=2.8$, $k_{h2}=0.2$, $b_V=1.5$, $a_V=0.5$, $b_h=1.5$, $a_h=0.5$.

To verify the robustness of the method, the maximum uncertainty parameter change is selected as $|\Delta m|/m_0=0.03$, $|\Delta J_y|/J_{y0}=0.02$, $|\Delta c_A|/c_{A0}=0.02$, $|\Delta S_w|/S_{w0}=0.03$, $|\Delta\rho|/\rho_0=0.03$.

The simulation results are shown in Figs. 3—8.

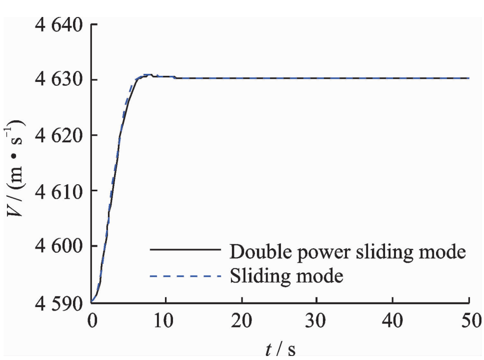

Fig. 3 Response to speed between double power sliding mode and sliding mode

Figs. 3, 4 show that both the sliding mode controller and the double power sliding mode nonlinear controller can well track the desired trajectory. The tracking trajectory is fast and the overshoot is small, but the double power sliding mode controller is obviously superior to the sliding mode controller in the tracking speed and accuracy, so the double power sliding mode controller has better speed and stability.

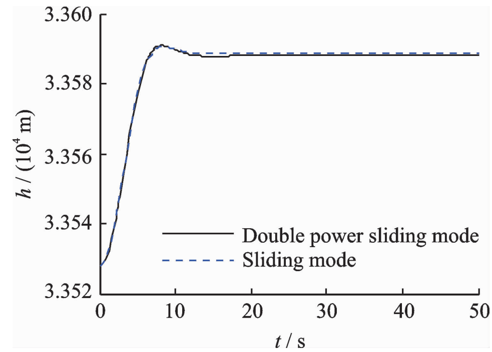

Fig. 4 Response to height between double power sliding mode and sliding mode

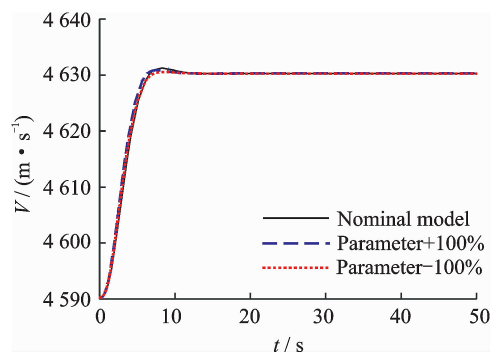

Fig. 5 Response of nominal model and uncertainty model to speed

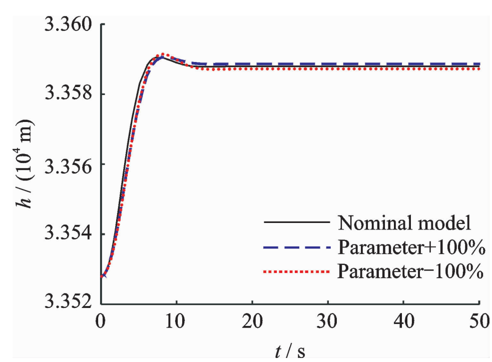

Fig. 6 Response of nominal model and uncertainty model to height

From Figs. 5, 6, it can be seen that there is a small overshoot in the speed tracking when the parameter perturbation occurs, and the fine tracking error occurs in the height tracking. Therefore, the speed and height are well controlled.

The hypersonic control input elevator and the control valve have very little buffeting, and from the track point of view, the parameter perturbation has little effect on the trajectory change,

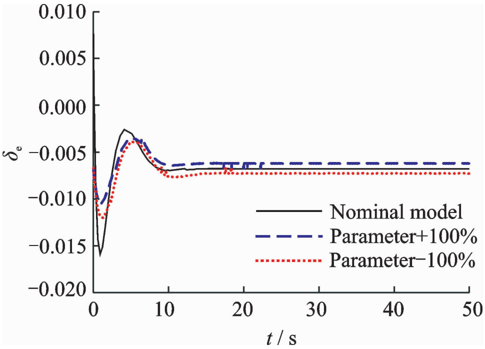

Fig. 7 Response of nominal model and uncertainty model to elevator

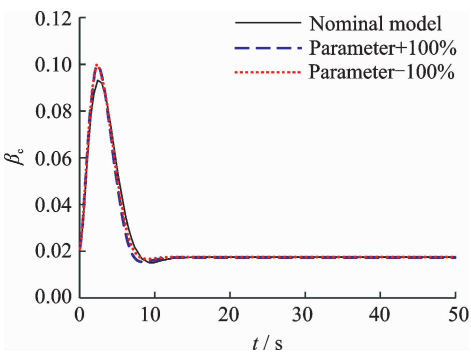

Fig. 8 Response of nominal model and uncertainty model to control valve

which proves that the method has good robustness. The simulation results show that the double power sliding mode based on the disturbance observer has better tracking and robustness to the hypersonic vehicle.

3 Conclusions

A double power sliding mode nonlinear controller based on disturbance observer is designed for the outer ring stability tracking control of hypersonic vehicle with strong nonlinearity and uncertainty. The controller utilizes the unique advantages of the sliding mode method in dealing with nonlinear problems and uses the disturbance observer to solve the nonlinear interference. The simulation results show that the controller designed in this paper realizes the stability control of the outer ring of the hypersonic vehicle, improves the flight control performance under the internal parameter perturbation and external unknown disturbance, and has good tracking and robust performance.

Acknowledgements

This work was supported by the Advanced Research Foundation of Ministry of Army Equipment Development in 13th Five-Year (No. 30102080101), the Aviation Science Funds (No. 20165852052), and the Fundamental Research Funds for the Central Universities (No. NS2018059).

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(Production Editor: Sun Jing)