Optimization of Nonlinear Energy Sink for Vibration Suppression of Systems Under Dual-Frequency Excitation

Sun Bin[1,2], *Wu Zhiqiang*[1*]

1. School of Mechanical Engineering, Tianjin Key Laboratory of Nonlinear Dynamics and Chaos Control, Tianjin University, Tianjin 300072, P. R. China; 2. School of Aeronautical Engineering, Civil Aviation University of China, Tianjin 300300, P. R. China

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Abstract: Aiming to decrease the vibration of wing induced by dual-rotor civil turbofan engines, the dynamic models of a single-degree of freedom (DOF) linear main oscillator coupled with single-DOF and two-DOF nonlinear energy sink (NES) are established. According to the related energy criteria for the optimization of the dynamic vibration absorber, focusing on the effects of external excitation on the kinetic energy of the primary mass and total system energy, the vibration suppression effects of single-DOF, two-DOF serial and parallel NES on the main oscillator system are studied. Under the condition that the characteristic parameters of the main oscillator system and additional total mass of the vibration absorber remain unchanged, results show that the two-DOF parallel NES has the best vibration energy suppression effects, which can provide data reference for the optimal design of NES vibration suppression under dual-frequency excitation.

Key words: nonlinear energy sink(NES); civil turbofan engine; dual-frequency; vibration suppression optimization

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0 Introduction

Vibration suppression design of modern civil turbofan engines plays an important role in aeronautical engineering. Related research shows that even for the most advanced civil turbofan aircrafts, the failure problems caused by engine vibration, such as the local structural cracks, pipeline leaks and loosening of fasteners, still occur frequently. At present, most civil turbofan engines adopt a dual-rotor layout, which introduces a typical dual-frequency excitation into the dynamic models[1-3]. Besides, in other engineering areas, the dual-frequency excitation can better predict the real sea conditions and the stability of deep-sea risers, and the establishment of the system dynamic model with certain stochastic excitation characteristics based on dual-frequency excitation has been proved to be feasible[4-5]. The vibration reduction of civil turbofan engines has always been a major issue in the modern aviation field. In order to study the vibration suppression with typical dual-frequency excitation characteristics and the systems, in this paper, the nonlinear energy sink (NES) is introduced into the nonlinear dynamics analysis.

NES has the advantages of wide vibration absorption frequency band and high energy dissipation rate. Compared with traditional dynamic vibration absorber (DVA), the mechanism of energy transfer and dissipation of NES is more complicated due to the influences of nonlinear factors. In recent years, more and more researchers have focused on the theoretical research and applications of NES. Gendelman et al.[6,7] analyzed the attractor problems of tuned forced linear system with NES. The results showed that under certain damping and frequency range settings, NES could have better energy absorption effects than tradi-

* Corresponding author, E-mail address: zhiqwu@tju.edu.cn.

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tional linear DVA, and then a parameter optimization design method of NES was proposed. Starosvetsky et al. [8,9] studied the effects of two-degree of freedom (DOF) linear system NES on the internal resonance problems, and the response mechanism of the system with NES under 1 ∶ 1 ∶ 1 internal resonance quasi-periodic forcing and random excitation. And they proved that the parameters of NES could be tuned to effectively reduce the vibration caused by quasi-periodic and even random excitation.

In Refs. [10-13], initial conditions of target energy transfer (TET) between the nonlinear coupled oscillators in NES were investigated, based on which such typical parameters as cubic stiffness, damping and mass ratio of NES could be designed to obtain better vibration reduction effects. Hubbard et al. [14,15] designed a single-DOF NES on the wingtip for the vibration suppression research by experiments, and described the design process of NES in detail. In the parallel NES optimization design, Boroson et al. [16] considered the uncertainty of the NES efficiency caused by the loading conditions or the disturbances of the design parameters, and proved the effectiveness of the method through comparative analysis. In Refs. [17-18], based on the nonlinear output frequency response function, a vibration transmissibility expression of single- and multi-DOF systems with coupled NES was put forward, which can be employed to analyze the vibration suppression effects of NES.

It can be seen that NES has been widely applied in engineering vibration suppression. However, the vibration suppression effects of different configurations of NES under dual-frequency still remain unknown. The main research goal is to study the vibration suppression effects of single-DOF, two-DOF serial and parallel NES on the main oscillator system using the energy criteria for the DVA optimization.

1 Description of Dynamic Models

In this section, four dynamic models are presented in order to compare the vibration suppression effects of four different absorbers. The wing vibration performs mainly with the first-order torsional mode[19,20]. For example, a type of aircraft has the wing symmetric first-order torsional natural frequency of 21 Hz, on which the engine's dual rotors rotate respectively at about 57 Hz and 270 Hz, in the cruise phase. Therefore, the frequency ratio between the main oscillator system and the two excitations in the dynamic models is set to 1 ∶ 2.67 ∶ 12.66 according to the engineering practice.

1.1 Dynamic model of the system with single-DOF NES

Fig. 1 shows a single linear main oscillator system coupled with single-DOF NES, whose basic physical and dynamic model can be found in Ref. [6], and here the dynamic modeling process takes into account the defined frequency ratio between the main oscillator system and the two excitations. The dynamic equations after time scale transformation can be expressed as

$$\begin{cases} \ddot{x}_1 + x_1 + 2.67\varepsilon\lambda_0(\dot{x}_1 - \dot{x}_2) + 7.14k_{n0}(x_1 - x_2)^3 = \\ \qquad 7.14\varepsilon(A_1\cos(\omega_1 t) + A_2\cos(\omega_2 t)) \\ \varepsilon\ddot{x}_2 + 2.67\varepsilon\lambda_0(\dot{x}_2 - \dot{x}_1) + 7.14k_{n0}(x_2 - x_1)^3 = 0 \end{cases} \tag{1}$$

where the parameters have been treated without dimension. x_1 and x_2 are the displacements of the linear main oscillator and NES, respectively. The mass and natural frequency of linear main oscillator are taken as equal to unity. ω_1 and ω_2 are two excitation frequencies, and the frequency ratio is defined as $\gamma = \omega_2/\omega_1$. ε represents the mass ratio between the NES and the main oscillator being used as the small parameter, and it can also scale the coupling between two oscillators, the damp-

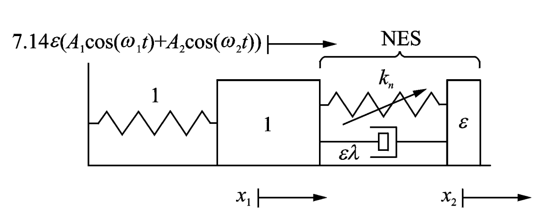


Fig. 1 Schematic diagram of dynamic model of the system coupled with single-DOF NES

ing forces and amplitudes of the dual-frequency excitation. λ_0 is the damping term and $\varepsilon\lambda=2.67\varepsilon\lambda_0$ is the damping coefficient of NES. To simplify the analysis, the stiffness term[6, 21] of NES k_{n0} equals to $4/3\varepsilon$, which means that the nonlinear stiffness is $k_n=7.14k_{n0}$. In addition, $7.14\varepsilon A_1$ ($\varepsilon \ll 1$) and $7.14\varepsilon A_2$ are the amplitudes of the dual-frequency excitation, respectively.

1.2 Dynamic model of the system with single-DOF linear DVA

To analyze the vibration suppression effects of NES, the dynamic model of the single linear main oscillator system coupled with single-DOF linear DVA is established. A DVA oscillator is used to substitute the NES oscillator in Eq. (1), and the related dynamic equations after time scale transformation can be written as

$$\begin{cases} \ddot{x}_1+x_1+2.67\varepsilon\lambda_0(\dot{x}_1-\dot{x}_2)+7.14k_{\mathrm{Lin0}}(x_1-x_2)= \\ \quad 7.14\varepsilon(A_1\cos(\omega_1 t)+A_2\cos(\omega_2 t)) \\ \varepsilon\ddot{x}_2+2.67\varepsilon\lambda_0(\dot{x}_2-\dot{x}_1)+7.14k_{\mathrm{Lin0}}(x_2-x_1)=0 \end{cases} \tag{2}$$

where $k_{\mathrm{Lin}}=7.14k_{\mathrm{Lin0}}$ is the linear stiffness of DVA, and the linear stiffness term k_{Lin0} equals to $4/3\varepsilon$ for comparison. $\varepsilon\lambda=2.67\varepsilon\lambda_0$ is the damping coefficient of DVA, and the other parameters and the dual-frequency excitation are consistent with those in Eq. (1).

1.3 Dynamic model of the system with two-DOF serial NES

Fig. 2 shows a single linear main oscillator system coupled with two-DOF serial NES. The coupling relationships among the main oscillator and the absorbers should be considered in the dynamic modeling. The related dynamic equations after time scale transformation can be expressed as

$$\begin{cases} \ddot{x}_1+x_1+2.67\varepsilon\lambda_0(\dot{x}_1-\dot{x}_2)+7.14k_{n0}(x_1-x_2)^3= \\ \quad 7.14\varepsilon(A_1\cos(\omega_1 t)+A_2\cos(\omega_2 t)) \\ \varepsilon_1\ddot{x}_2+2.67\varepsilon\lambda_0(\dot{x}_2-\dot{x}_1)+2.67\varepsilon\lambda_0(\dot{x}_2-\dot{x}_3)+ \\ \quad 7.14k_{n0}(x_2-x_1)^3+7.14k_{n0}(x_2-x_3)^3=0 \\ \varepsilon_2\ddot{x}_3+2.67\varepsilon\lambda_0(\dot{x}_3-\dot{x}_2)+7.14k_{n0}(x_3-x_2)^3=0 \end{cases} \tag{3}$$

where ε_1 and ε_2 are dimensionless masses of two serial NES, respectively, x_2 and x_3 the displacements of two NES, respectively, and the other characteristic parameters remain the same as those in the system with single-DOF NES.

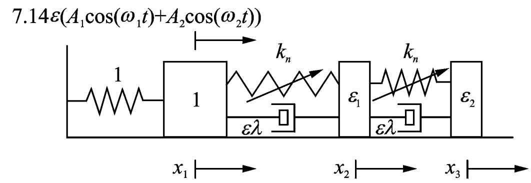


Fig. 2 Schematic diagram of dynamic model of the system coupled with two-DOF serial NES

1.4 Dynamic model of the system with two-DOF parallel NES

Fig. 3 shows a single linear main oscillator system coupled with two-DOF parallel NES. The dynamic modeling should consider the coupling relationship between the main oscillator and each absorber, and the related dynamic equations after time scale transformation can also be expressed as

$$\begin{cases} \ddot{x}_1+x_1+2.67\varepsilon\lambda_0(\dot{x}_1-\dot{x}_2)+7.14k_{n0}(x_1-x_2)^3+ \\ \quad 2.67\varepsilon\lambda_0(\dot{x}_1-\dot{x}_3)+7.14k_{n0}(x_1-x_3)^3= \\ \quad 7.14\varepsilon(A_1\cos(\omega_1 t)+A_2\cos(\omega_2 t)) \\ \varepsilon_1\ddot{x}_2+2.67\varepsilon\lambda_0(\dot{x}_2-\dot{x}_1)+7.14k_{n0}(x_2-x_1)^3=0 \\ \varepsilon_2\ddot{x}_3+2.67\varepsilon\lambda_0(\dot{x}_3-\dot{x}_1)+7.14k_{n0}(x_3-x_1)^3=0 \end{cases} \tag{4}$$

where ε_1 and ε_2 are the dimensionless masses of two parallel NES respectively, x_2 and x_3 the displacements of two NESs, respectively and the other parameters still remain the same as those in the system with single-DOF NES.

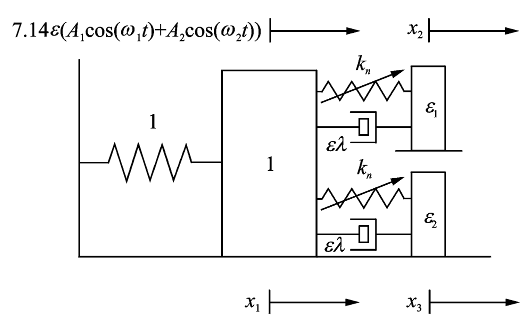


Fig. 3 Schematic diagram of dynamic model of the system coupled with two-DOF parallel NES

To compare the vibration suppression effects of different systems above, according to the related energy criteria proposed in Ref. [21], the vibration suppression optimization of DVA should mainly consider such factors as the main oscillator kinetic energy, total system energy and total area occupied by the total system energy curve. Here the main oscillator kinetic energy of each system can be set as

$$E_{\mathrm{kin}}=\frac{\dot{x}_1^2}{2} \tag{5}$$

Total system energy of the system with single-DOF linear DVA can be expressed as

$$E_{\mathrm{tot\_Lin}}=\frac{\dot{x}_1^2}{2}+\varepsilon\frac{\dot{x}_2^2}{2}+\frac{x_1^2}{2}+k_{\mathrm{Lin}}\frac{(x_1-x_2)^2}{2} \tag{6}$$

In the similar way, total system energy of the system with single-DOF NES, two-DOF serial and parallel NESs can be given as

$$E_{\mathrm{tot\_single\text{-}DOF\ NES}}=\frac{\dot{x}_1^2}{2}+\varepsilon\frac{\dot{x}_2^2}{2}+\frac{x_1^2}{2}+k_n\frac{(x_1-x_2)^4}{4} \tag{7}$$

$$E_{\mathrm{tot\_two\text{-}DOF\ serial\ NES}}=\frac{\dot{x}_1^2}{2}+\varepsilon_1\frac{\dot{x}_2^2}{2}+\varepsilon_2\frac{\dot{x}_3^2}{2}+\frac{x_1^2}{2}+k_n\frac{(x_1-x_2)^4}{4}+k_n\frac{(x_2-x_3)^4}{4} \tag{8}$$

$$E_{\mathrm{tot\_two\text{-}DOF\ parallel\ NES}}=\frac{\dot{x}_1^2}{2}+\varepsilon_1\frac{\dot{x}_2^2}{2}+\varepsilon_2\frac{\dot{x}_3^2}{2}+\frac{x_1^2}{2}+k_n\frac{(x_1-x_2)^4}{4}+k_n\frac{(x_1-x_3)^4}{4} \tag{9}$$

2 Numerical Analysis in Comparison with the Case of Single-DOF Linear DVA

In Eqs. (1) and (2), to compare the vibration suppression effects of the systems with single-DOF NES and linear DVA, we set $\varepsilon=0.01$, $\lambda_0=0.2$, $A_1=1.3$, $A_2=0.1$. The stiffness terms of NES and linear DVA are $k_{n0}=k_{\mathrm{lin0}}=4/3\varepsilon$. Based on the fourth-order Runge-Kutta algorithm, and the initial displacement and velocity of each oscillator are set to zero.

Typical dual-rotor civil turbofan engines are taken as the main engineering backgrounds in this paper, and the maximum frequency ratio between two exciters in the cruise phase is less than 7. Therefore, when ω_1 is fixed to 2.67 as defined in Section 1, the frequency ratio γ is monotonically increased from 1 to 8, making the range of which cover the characteristic frequency corresponding to the target aircraft cruise phase ($\gamma=4.74$). The comparison results of main oscillator kinetic energy and total system energy curves are shown in Fig. 4.

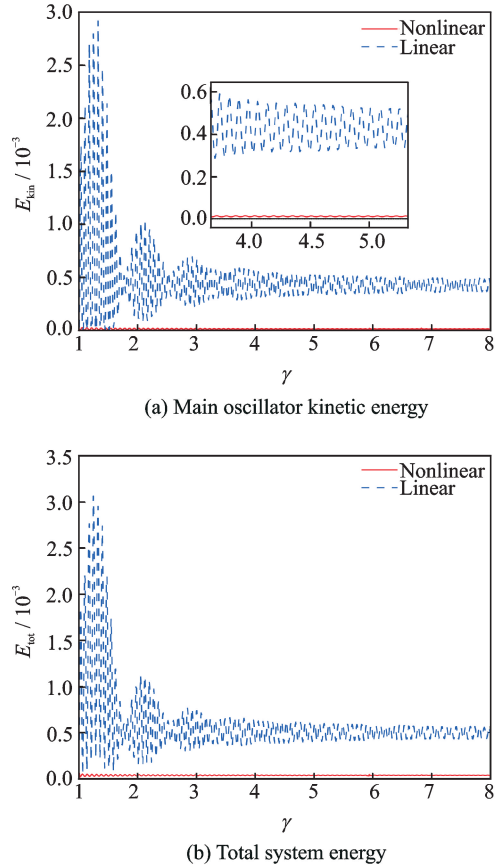


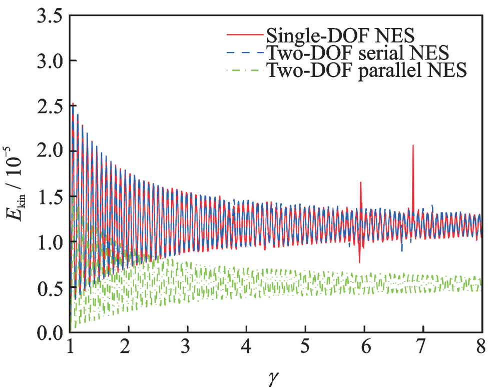
Fig. 4 Comparison of effects of γ on main oscillator kinetic energy and total system energy of the system coupled with linear DVA

The above numeric results indicate that NES has better vibration suppression effects than linear DVA, which is valid when the frequency ratio increases from 1 to 20 after numerical sweeping, and thus NES is promising for the application in civil turbofan engine vibration reduction.

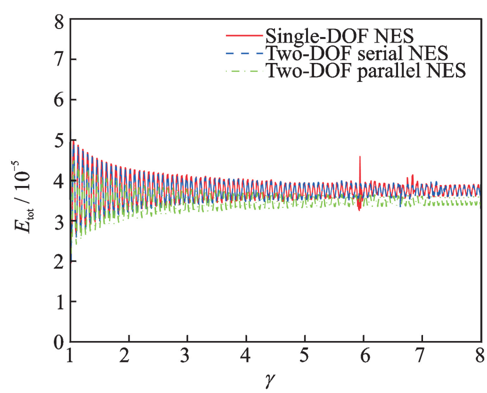
3 Numerical Analysis in Comparison with the Case of Two-DOF Serial/Parallel NES

To achieve the vibration suppression optimization of NES, a comparison with two-DOF serial and parallel NESs is carried out. In Eqs. (3), (4), ε_1 and ε_2 are respectively 0.009 and 0.001, the initial displacement and velocity of each oscillator are 0, and the other characteristic parameters still remain the same as those in Eq. (1).

Here ω_1 equals to 2.67, and the numeric simulation results can be obtained as Fig. 5.



(a) Main oscillator kinetic energy



(b) Total system energy

Fig. 5 Comparison of effects of γ on main oscillator kinetic energy and total system energy of the system coupled with two-DOF serial/parallel NES

The comparison results can show that when the characteristic parameters of the main oscillator system and additional total mass of the vibration absorber remain unchanged, two-DOF parallel NES has the best vibration energy suppression effects. According to the energy criteria described in Section 1, this conclusion is valid when the frequency ratio is increased from 1 to 20 after numerical sweeping.

4 Conclusions

This paper constructed dynamic models of system coupled with different configurations of NES and introduced the modal frequency of the first-order symmetric twist typical state of the wing into the dynamic models. And the numeric results based on the fourth-order Runge-Kutta algorithm indicate that

(1) Compared with the system coupled with linear DVA, the vibration suppression effects of NES are better in a wide frequency range.

(2) With the same characteristic parameters of the main oscillator system and additional total mass of the vibration absorber, compared with the system with single-DOF and two-DOF serial NES, two-DOF parallel NES has the best vibration energy suppression effects.

(3) NES has excellent vibration suppression effect in the aeronautical engineering practice, and the numerical simulation results can provide data reference for vibration suppression design of the dual-rotor turbofan engine.

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Mr. **Sun Bin** received his B. S. degree in Flight Propulsion Engineering from Civil Aviation University of China in 2008 and M. S. degree in Aerospace Mechanics and Avionics from Institut Superieur de L'Aeronautique et de L'Espace (ISAE) of France in 2011, respectively. He is currently a Ph. D. candidate in General and Fundamental Mechanics of Tianjin University, and meanwhile he is a lecturer in Aeronautical Engineering College, Civil Aviation University of China. His research interests are focused on nonlinear dynamics and control, aircraft maintenance, vibration suppression and relevant fields.

Prof. **Wu Zhiqiang** received his M. S. and Ph. D. degrees from Department of Mechanics, Tianjin University in 1993 and 1996, respectively. His research interests are focused on nonlinear dynamics and its applications in engineering, such as vortex-induced vibration (VIV) of pipes, instability of vehicles, and flutter of structures.

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