Performance Analysis of AF Relaying Aided Space Shift Keying System with Imperfect Channel Estimation

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(Received 2 January 2018; revised 12 September 2018; accepted 15 October 2018)

Abstract: Space shift keying (SSK) is a spectrally efficient and low-complexity technique that only uses antenna index to convey information. Combining SSK with cooperative communication, the transmission reliability of SSK system can be improved effectively. In this paper, considering imperfect channel information, the performance of cooperative SSK system with amplify-and-forward (AF) relaying protocol is investigated, and the effect of estimation error on the performance is analyzed. According to the performance analysis, the probability density function and moment generating function of effective signal-to-noise ratio are derived, respectively. Using these results, the closed-form expression of average bit error rate (BER) can be achieved. Meanwhile, the asymptotically approximated BER and the corresponding diversity order analysis are presented for the performance evaluation. By computer simulations, the validness of the presented theoretical analysis is verified, and the theoretical BERs with different estimation errors are shown to be close to those of the corresponding simulations.

Key words: space shift keying; amplify-and-forward; channel-estimation error; bit error rate; diversity order **CLC number:** TN919.3 **Document code:** A **Article ID:** 1005-1120(2018)06-1038-09

0 Introduction

Cooperative relaying, as a promising technology to improve spectral efficiency and link reliability, has attracted much attentions [1,2]. In a cooperative communication system, the relay aids to the transmission of two nodes with different protocols and brings about diversity gain. Amplify-and-forward (AF) is one of the relaying protocols, where the relay amplifies and transfers the signals sent by the source to the destination. For its simple implementation, AF has been widely employed in cooperative communication.

Space shift keying (SSK) activates only one antenna to transmit information at any time slot, so that the need for transmit antenna synchronization and inter-channel interference can be eliminated^[3,4]. In SSK, the active transmit antenna index is the unique way to convey information.

Thereby, the optimal detection at the receiver only needs to detect antenna indices. For lack of the symbol detection, the receiver complexity of SSK is decreased [5].

Considering the low complexity of SSK and the diversity gain brought by cooperative relaying, the combination of SSK and cooperative communication has been proposed to further enhance the performance [6-9]. In Ref. [6], the cooperative SSK system with AF and decode-and-forward relaying was presented, and the average bit error rate (BER) expressions are derived. Based on the conventional AF protocol, Refs. [6, 7] proposed an opportunistic AF relaying scheme for the cooperative SSK system, where the best relay is chosen to transmit signals. Spatial modulation (SM) is an extension of SSK that simultaneously exploits antenna indices and constellation symbols to convey information [8]. In Refs. [9,

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How to cite this article: Pan Qing, Yu Xiangbin, Hu Yaping, et al. Performance analysis of AF relaying aided space shift keying system with imperfect channel estimation[J]. Trans. Nanjing Univ. Aero. Astro., 2018,35(6):1038-1046. http://dx.doi.org/10.16356/j.1005-1120.2018.06.1038

10, SM is introduced into the cooperative system, and the corresponding BER performance is analyzed. The above works all assume that the channel state information (CSI) is accurately estimated at the receiver, while in particular the influence of channel-estimation error needs to be considered. The performance of cooperative DF and cognitive radio system with SM are studied under the assumption that the imperfect CSI is available at the relay and destination[11,12]. However, these literatures focus on the SM system with DF and only consider single antenna at the destination for convenient analysis. There are few works addressing the study on the performance of the AF aided SSK (AF-SSK) system, especially in the presence of imperfect channel estimation.

Therefore, in this paper, we will study the performance of the AF-SSK system with multiple receive antennas under imperfect channel estimation, and analyse the effect of channel-estimation error on the performance of AF-SSK system. With the performance analysis, the probability density functions (PDF) and the moment generating functions (MGF) of effective signal-to-noise ratio (SNR), as well as the pairwise error probability (PEP) are derived. Thereby the closedform expression of average BER is achieved by a union upper bound. Moreover, the asymptotically approximated BER at high SNR is also analyzed. Using this approximation, the diversity gain of this system is further derived for the performance evaluation. Simulation shows that the theoretical BER results can match the simulated values well for different channel-estimation errors, which verifies the correctness of theoretical analysis, and can provide the effective evaluation for the system performance.

1 System Model

The AF-SSK system combining cooperative AF relaying and SSK is shown as Fig. 1, which consists of a source with $N_{\rm t}$ transmit antennas, a single-antenna relay and a destination with $N_{\rm r}$ receive antennas. At the source, the SSK mapper

utilizes $\log_2 N_t$ bits to determine the active transmit antenna index $i, i \in [1, N_t]$. At the destination, with the channel estimate, the optimal detection algorithm based on the maximum likelihood (ML) principle is employed. The output of SSK mapper is given by [13]

$$\boldsymbol{x}_i = \begin{bmatrix} 0 & 0 & \cdots & 1 & \cdots & 0 \end{bmatrix}^{\mathrm{T}} \tag{1}$$

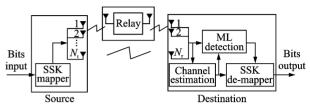


Fig. 1 Cooperative AF-SSK system model

where x_i is an N_t -dimensional symbol vector, and the unique nonzero element is in the *i*-th row, which means that the *i*-th antenna is activated. Based on the basic idea of cooperative communication, the transmission process is divided into two phases. In the first phase, the source transmits the signal to the relay and the destination. The received signals of the relay and the destination are respectively given by

$$y_{\rm sr} = \sqrt{P_{\rm s}} \, \boldsymbol{h}_{\rm sr}^{\rm T} \boldsymbol{x}_i + n_{\rm sr} \tag{2}$$

$$\mathbf{y}_{\mathrm{sd}} = \sqrt{P_{\mathrm{s}}} \mathbf{H}_{\mathrm{sd}} \mathbf{x}_{i} + \mathbf{n}_{\mathrm{sd}} \tag{3}$$

where P_s is the transmission power of the source, \mathbf{h}_{sr} a $N_{t} \times 1$ channel vector of source-to-relay link, and \mathbf{H}_{sd} a $N_{r} \times N_{t}$ channel matrix of source-to-destination link. In the second phase, the relay amplifies the received signal y_{sr} and transfers it to the destination, thus the received signal at the destination is expressed as

$$\mathbf{y}_{\mathrm{rd}} = \mathbf{h}_{\mathrm{rd}} (A \mathbf{y}_{\mathrm{sr}}) + \mathbf{n}_{\mathrm{rd}} \tag{4}$$

where $\mathbf{h}_{\rm rd}$ is a $N_{\rm r} \times 1$ channel vector of relay-to-destination link. The entries of $\mathbf{h}_{\rm sr}$, $\mathbf{H}_{\rm sd}$ and $\mathbf{h}_{\rm rd}$ are zero-mean complex Gaussian random variables with variance of $\delta_{\rm sr}^2$, $\delta_{\rm sd}^2$ and $\delta_{\rm rd}^2$, respectively. The variance is defined as $\delta^2 = d^{-a}$, where d is the distance between two nodes and a is path-loss exponent. $\mathbf{n}_{\rm sr}$, $\mathbf{n}_{\rm sd}$ and $\mathbf{n}_{\rm rd}$ are zero-mean complex Gaussian noises with the variance of N_0 . A is the amplification factor and can be written as A=

$$\sqrt{rac{P_{ ext{r}}}{P_{ ext{s}}\delta_{ ext{sr}}^2+N_0}}$$
 , where $P_{ ext{r}}$ is the transmit power at

the relay. Let $P_{\rm t}$ be the total transmit power, $P_{\rm s}+P_{\rm r}=P_{\rm t}$, then the corresponding average SNR is expressed as $\bar{\gamma}=\frac{P_{\rm t}}{N_{\rm o}}$.

In practice, it is hard to achieve the perfect channel information due to the estimation error. For this reason, we will consider imperfect channel estimation in the performance analysis. Let \tilde{h}_{mn} represents the estimated channel coefficient, $m,n\in\{\text{s,r,d}\}$, thus the relation between the actual and estimated value can be given by [14]

$$h_{mn} = \tilde{h}_{mn} + e_{mn} \tag{5}$$

where $e_{\it mn}$ is the channel-estimation error, and it follows the complex Gaussian distribution with zero mean and variance $\sigma^2_{e_{\it mn}}$. The estimate $\tilde{h}_{\it mn}$ is independent of $e_{\it mn}$, and modeled as a zero-mean complex Gaussian distributed variable with variance $\tilde{\delta}^2_{\it mn} = \delta^2_{\it mn} - \sigma^2_{e_{\it mn}}$. According to Refs. [14] and [15], the variance of estimation error, $\sigma^2_{e_{\it mn}}$ can be set as a decreasing function of SNR.

Substituting Eq. (5) into Eqs. (3) and (4), the received signals \mathbf{y}_{sd} and \mathbf{y}_{rd} can be respectively expressed as

$$\mathbf{y}_{\mathrm{sd}} = \sqrt{P_{\mathrm{s}}} \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} + \underbrace{\sqrt{P_{\mathrm{s}}} \boldsymbol{e}_{\mathrm{sd}}^{i} + \boldsymbol{n}_{\mathrm{sd}}}_{\tilde{\boldsymbol{n}}_{\mathrm{sd}}}$$

$$\mathbf{y}_{\mathrm{rd}} = A \sqrt{P_{\mathrm{s}}} \tilde{\boldsymbol{h}}_{\mathrm{rd}} \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} +$$

$$\underbrace{A \sqrt{P_{\mathrm{s}}} \left(\tilde{\boldsymbol{h}}_{\mathrm{rd}} \boldsymbol{e}_{\mathrm{sr}}^{i} + \boldsymbol{e}_{\mathrm{rd}} \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} + \boldsymbol{e}_{\mathrm{rd}} \boldsymbol{e}_{\mathrm{sr}}^{i} \right) + A \left(\tilde{\boldsymbol{h}}_{\mathrm{rd}} + \boldsymbol{e}_{\mathrm{rd}} \right) n_{\mathrm{sr}} + \boldsymbol{n}_{\mathrm{rd}}}_{\tilde{\boldsymbol{n}}_{\mathrm{rd}}}$$

$$(7)$$

where $\tilde{\pmb{h}}_{\rm sd}^i$, $\pmb{e}_{\rm sd}^i$ are the *i*-th column of $\pmb{H}_{\rm sd}$ and the channel-estimation error matrix $\pmb{E}_{\rm sd}$, and $\tilde{h}_{\rm sr}^i$, $e_{\rm sr}^i$ are the *i*-th elements of $\pmb{h}_{\rm sr}$ and $\pmb{e}_{\rm sr}$, respectively. The variance of $\tilde{\pmb{n}}_{\rm sd}$ is $\sigma_{n_{\rm sd}}^2 = P_{\rm s}\sigma_{\rm e_{\rm sd}}^2 + N_0$. For the convenience of analysis, the variance of $\tilde{\pmb{n}}_{\rm rd}$ can be approximated as $\sigma_{n_{\rm rd}}^2 \approx A^2 \|\tilde{\pmb{h}}_{\rm rd}\|_{\rm F}^2 (P_{\rm s}\sigma_{\rm e_{\rm sr}}^2 + N_0) + P_{\rm r}\sigma_{\rm e_{\rm rd}}^2 + N_0$ [11]. Using this result, the normalized signals can be written as

$$\tilde{\mathbf{y}}_{\mathrm{sd}} = \sqrt{T}\tilde{\mathbf{h}}_{\mathrm{sd}}^{i} + \mathbf{n}_{1} \tag{8}$$

$$\tilde{\mathbf{y}}_{\rm rd} = \sqrt{G}\tilde{\mathbf{h}}_{\rm rd}\tilde{h}_{\rm sr}^i + \mathbf{n}_2 \tag{9}$$

where
$$T = \frac{P_s}{\sigma_{n_{cl}}^2}$$
, $G = \frac{A^2 P_s}{\sigma_{n_{cl}}^2}$, $n_1 = \frac{\tilde{n}_{sd}}{\sigma_{\tilde{n}_{sd}}}$ and $n_2 = \frac{\tilde{n}_{rd}}{\sigma_{\tilde{n}_{rd}}}$.

With the received signals and the channel estimates at the destination, the detection algorithm based on the ML principle is written as [13]

$$\hat{i} = \underset{j \in [1,N_{\text{t}}]}{\arg\min} \left[T \| \tilde{\boldsymbol{h}}_{\text{sd}}^{j} \|_{\text{F}}^{2} - 2\sqrt{T} \operatorname{Re} \{ \tilde{\boldsymbol{y}}_{\text{sd}}^{\text{H}} \tilde{\boldsymbol{h}}_{\text{sd}}^{j} \} + G \| \tilde{\boldsymbol{h}}_{\text{rd}}^{l} \|_{\text{F}}^{2} \| \tilde{\boldsymbol{h}}_{\text{sr}}^{i} \|_{\text{F}}^{2} - 2\sqrt{G} \operatorname{Re} \{ \tilde{\boldsymbol{y}}_{\text{rd}}^{\text{H}} \tilde{\boldsymbol{h}}_{\text{rd}} \tilde{\boldsymbol{h}}_{\text{sr}}^{j} \} \right]$$
(10)

Therefore, the estimate of active transmit antenna index \hat{i} can be attained.

2 Performance Analysis of AF-SSK System

In this section, the performance of cooperative AF-SSK system in the presence of imperfect channel information will be analyzed, and the closed-form expression of average BER will be derived. Using the detection algorithm in Eq. (10), the antenna index is detected by minimizing D_j , which is expressed as

$$D_{j} = T \|\tilde{\boldsymbol{h}}_{sd}^{j}\|_{F}^{2} - 2\sqrt{T}\operatorname{Re}\{\tilde{\boldsymbol{y}}_{sd}^{H}\tilde{\boldsymbol{h}}_{sd}^{j}\} + G\|\tilde{\boldsymbol{h}}_{rd}\|_{F}^{2} + \|\tilde{\boldsymbol{h}}_{sr}^{j}\|_{F}^{2} - 2\sqrt{G}\operatorname{Re}\{\tilde{\boldsymbol{y}}_{rd}^{H}\tilde{\boldsymbol{h}}_{rd}\tilde{\boldsymbol{h}}_{sr}^{j}\}$$
(11)

If the *i*-th antenna is activated at the source and the estimate of antenna index at the destination is j, $j \neq i$, then we can obtain

$$D_{j\neq i} = T \|\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j}\|_{\mathrm{F}}^{2} - 2\sqrt{P_{\mathrm{s}}} \operatorname{Re} \left\{ \sqrt{T} (\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i})^{\mathrm{H}} \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} + \boldsymbol{n}_{1}^{\mathrm{H}} \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \right\} + G \|\tilde{\boldsymbol{h}}_{\mathrm{rd}}\|_{\mathrm{F}}^{2} \|\tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j}\|^{2} - 2\sqrt{G} \operatorname{Re} \left\{ \sqrt{G} (\tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i})^{\mathrm{H}} \|\tilde{\boldsymbol{h}}_{\mathrm{rd}}\|_{\mathrm{F}}^{2} \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} + \boldsymbol{n}_{2}^{\mathrm{H}} \tilde{\boldsymbol{h}}_{\mathrm{rd}} \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} \right\}$$

$$(12)$$

If the detection is correct (i. e, j=i), then Eq. (12) can be simplified as

$$D_{j=i} = T \|\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i}\|_{\mathrm{F}}^{2} - 2\sqrt{P_{\mathrm{s}}}\operatorname{Re}\left\{\sqrt{T}\|\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i}\|_{\mathrm{F}}^{2} + \boldsymbol{n}_{1}^{\mathrm{H}}\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i}\right\} + G \|\tilde{\boldsymbol{h}}_{\mathrm{rd}}\|_{\mathrm{F}}^{2} \|\tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i}\|_{2}^{2} - 2\sqrt{G}\operatorname{Re}\left\{\sqrt{G}\|\tilde{\boldsymbol{h}}_{\mathrm{rd}}\|_{\mathrm{F}}^{2}\|\tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i}\|_{2}^{2} + \boldsymbol{n}_{2}^{\mathrm{H}}\tilde{\boldsymbol{h}}_{\mathrm{rd}}\tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i}\right\}$$

$$(13)$$

With Eqs. (12) and (13), the conditional PEP given by $\{\widetilde{\pmb{H}}_{sd},\widetilde{\pmb{h}}_{sr},\widetilde{\pmb{h}}_{rd}\}$ is written as

$$\operatorname{pep}(i \to j \mid \widetilde{\boldsymbol{H}}_{\operatorname{sd}}, \widetilde{\boldsymbol{h}}_{\operatorname{sr}}, \widetilde{\boldsymbol{h}}_{\operatorname{rd}}) = \operatorname{Pr}(D_{j \neq i} < D_{j = i}) = \operatorname{Pr}(T \mid \widetilde{\boldsymbol{h}}_{\operatorname{sd}}^{i} - \widetilde{\boldsymbol{h}}_{\operatorname{sd}}^{j} \mid_{\operatorname{F}}^{2} + G \mid \widetilde{\boldsymbol{h}}_{\operatorname{rd}} \mid_{\operatorname{F}}^{2} \mid \widetilde{\boldsymbol{h}}_{\operatorname{sr}}^{i} - \widetilde{\boldsymbol{h}}_{\operatorname{sr}}^{j} \mid_{\operatorname{F}}^{2} < \widetilde{\boldsymbol{n}})$$

$$(14)$$

where

 $\tilde{n} = 2\sqrt{T}\operatorname{Re}\{\boldsymbol{n}_{1}^{\mathrm{H}}(\tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i})\} + 2\sqrt{G}\operatorname{Re}\{\boldsymbol{n}_{2}^{\mathrm{H}}\boldsymbol{h}_{\mathrm{rd}}(\tilde{h}_{\mathrm{sr}}^{i} - \tilde{h}_{\mathrm{sr}}^{i})\}$, and it can be approximated as a zero-mean Gaussian random variable with variance

$$\sigma_{\tilde{n}}^{2} = 2\left(T \| \tilde{\boldsymbol{h}}_{sd}^{j} \boldsymbol{x}_{m} - \tilde{\boldsymbol{h}}_{sd}^{i} \boldsymbol{x}_{q} \|_{F}^{2} + G \| \boldsymbol{h}_{rd} \|_{F}^{2} \| \tilde{\boldsymbol{h}}_{sr}^{j} \boldsymbol{x}_{m} - \tilde{\boldsymbol{h}}_{sr}^{i} \boldsymbol{x}_{q} \|^{2} \right)$$
(15)

Utilizing the PDF of \tilde{n} , which is $f_{\tilde{n}}(t) =$

 $\frac{1}{\sqrt{2\pi\sigma_{\vec{n}}^2}} \exp\left(-\frac{t^2}{2\sigma_{\vec{n}}^2}\right)$, Eq. (14) can be rewritten

$$\begin{aligned}
\operatorname{pep}(i \to j \mid \tilde{\boldsymbol{H}}_{\mathrm{sd}}, \tilde{\boldsymbol{h}}_{\mathrm{sr}}, \tilde{\boldsymbol{h}}_{\mathrm{rd}}) &= \int_{T \parallel \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \parallel_{\mathrm{F}}^{2} + G \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} \parallel^{2}} &= \operatorname{cxp}\left(-\frac{t^{2}}{2\sigma_{n}^{2}}\right) \operatorname{d}t = \\
Q\left(\frac{T \parallel \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \parallel_{\mathrm{F}}^{2} + G \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} \parallel \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} \parallel^{2}}{\sigma_{n}}\right) = \\
Q\left(\sqrt{\frac{T \parallel \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \parallel_{\mathrm{F}}^{2} + G \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} \parallel \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} \parallel^{2}}}\right) = \\
Q\left(\sqrt{\frac{P_{\mathrm{s}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \parallel_{\mathrm{F}}^{2}}{2}} + \frac{P_{\mathrm{s}} P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}}^{i} \parallel_{\mathrm{F}}^{2} + \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} \parallel^{2}}{2}}{2}\right) = \\
Q\left(\sqrt{\frac{P_{\mathrm{s}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sd}}^{j} \parallel_{\mathrm{F}}^{2}}{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}}} \parallel_{\mathrm{F}}^{2} + \frac{P_{\mathrm{s}} P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{i} - \tilde{\boldsymbol{h}}_{\mathrm{sr}}^{j} \parallel^{2}}{2}}}{Q\left(P_{\mathrm{s}}\sigma_{e_{\mathrm{sd}}}^{2} + N_{0}\right) + (P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0}) + (P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0}) (P_{\mathrm{s}}\delta_{\mathrm{sr}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{s}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{s}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})}\right) + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{s}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})}} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{\mathrm{r}} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{0}}{2(P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})}}{2(P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{0} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{0} \parallel_{\mathrm{F}}^{2}}{2(P_{\mathrm{r}}\sigma_{e_{\mathrm{rd}}}^{2} + N_{0})} + \frac{P_{\mathrm{r}} \parallel \tilde{\boldsymbol{h}}_{\mathrm{rd}} \parallel_{\mathrm{F}}^{2} + P_{0} \parallel_{\mathrm{F}}^{2}}$$

where $C = \frac{(P_s \delta_{sr}^2 + N_0)}{(P_s \sigma_{e..}^2 + N_0)}$.

Using
$$\gamma_{sd} = \frac{P_{s} \|\tilde{\boldsymbol{h}}_{sd}^{i} - \tilde{\boldsymbol{h}}_{sd}^{j}\|_{F}^{2}}{2(P_{s}\sigma_{e_{sd}}^{2} + N_{0})}, \quad \gamma_{rd} = \frac{P_{r} \|\tilde{\boldsymbol{h}}_{rd}\|_{F}^{2}}{P_{r}\sigma_{e_{rd}}^{2} + N_{0}}, \quad \gamma_{sr} = \frac{P_{s} \|\tilde{\boldsymbol{h}}_{sr}^{i} - \tilde{\boldsymbol{h}}_{sr}^{j}\|_{F}^{2}}{2(P_{s}\sigma_{e_{sr}}^{2} + N_{0})} \text{ yields}$$

$$pep(i \rightarrow j \mid \gamma_{sd}, \gamma_{srd}) = Q\left(\sqrt{\gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{rd} + C}}\right) = Q\left(\sqrt{\gamma_{sd} + \gamma_{srd}}\right)$$

$$(17)$$

where $\gamma_{\rm srd}=\frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\gamma_{\rm rd}+C}$. Then, with Eq. (17), the average PEP is given by

$$PEP(i \rightarrow j) = \int_{0}^{\infty} \int_{0}^{\infty} pep(i \rightarrow j \mid \gamma_{sd}, \gamma_{srd}) \times f_{\gamma_{sd}}(\gamma) f_{\gamma_{srd}}(\gamma) d\gamma_{srd} d\gamma_{sd} = \int_{0}^{\infty} \int_{0}^{\infty} Q(\sqrt{\gamma_{sd} + \gamma_{srd}}) f_{\gamma_{sd}}(\gamma) f_{\gamma_{srd}}(\gamma) d\gamma_{srd} d\gamma_{sd}$$
(18)

Based on $Q(x) = \pi^{-1} \int_0^{\pi/2} \exp\left(-\frac{x}{(2\sin^2\theta)}\right) d\theta$,

we can obtain

$$PEP(i \rightarrow j) = \frac{1}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\gamma_{sd}} \left(\frac{1}{2\sin^{2}\theta} \right) M_{\gamma_{srd}} \left(\frac{1}{2\sin^{2}\theta} \right) d\theta$$
(19)

where $M_{\gamma_{\rm sd}}$ (•) and $M_{\gamma_{\rm srd}}$ (•) are MGFs of $\gamma_{\rm sd}$ and $\gamma_{\rm srd}$, respectively.

Let $t = \sin\theta$ and using the transformation of variables, Eq. (19) can be rewritten as

$$PEP(i \to j) = \frac{1}{2\pi} \int_{0}^{1} M_{\gamma_{sd}} \left(\frac{1}{2t^{2}}\right) M_{\gamma_{srd}} \left(\frac{1}{2t^{2}}\right) \frac{1}{\sqrt{1-t^{2}}} dt \cong \frac{N_{p}^{-1}}{2} \sum_{u=1}^{N_{p}} M_{\gamma_{sd}} (1/2\varphi_{u}^{2}) M_{\gamma_{srd}} (1/2\varphi_{u}^{2})$$
(20)

where $\varphi_u = \cos((2u-1)\pi/(2N_{\rm p}))$, and $N_{\rm p}$ is the order of the Chebyshev polynomial [16].

Based on the definitions in Eq. (17), the PDFs of γ_{sd} , γ_{sr} and γ_{rd} over Rayleigh channel are respectively given by

$$f_{\gamma_{\rm sd}}(\gamma) = \frac{1}{\Gamma(N_{\rm r})\overline{\gamma}_{\rm sd}} \left(\frac{\gamma}{\overline{\gamma}_{\rm sd}}\right)^{N_{\rm r}-1} \exp\left(-\frac{\gamma}{\overline{\gamma}_{\rm sd}}\right) (21)$$

$$f_{\gamma_{\rm rd}}(\gamma) = \frac{1}{\Gamma(N_{\rm r})\gamma_{\rm rd}} \left(\frac{\gamma}{\gamma_{\rm rd}}\right)^{N_{\rm r}-1} \exp\left(-\frac{\gamma}{\gamma_{\rm rd}}\right) (22)$$

$$f_{\gamma_{\rm sr}}(\gamma) = \exp(-\gamma/\bar{\gamma}_{\rm sr})/\bar{\gamma}_{\rm sr}$$
 (23)

Correspondingly, the cumulative distribution function (CDF) of γ_{sr} is obtained by

$$F_{\gamma_{\rm sr}}(\gamma) = 1 - \exp(-\gamma/\bar{\gamma}_{\rm sr}) \tag{24}$$

where
$$ar{\gamma}_{ ext{sd}} = rac{P_{ ext{s}}\delta_{ ext{sd}}^2}{(P_{ ext{s}}\sigma_{ ext{sd}}^2 + N_0)}$$
, $ar{\gamma}_{ ext{sr}} = rac{P_{ ext{s}}\delta_{ ext{sr}}^2}{(P_{ ext{s}}\sigma_{ ext{e}_{ ext{sr}}}^2 + N_0)}$, and $ar{\gamma}_{ ext{rd}} = rac{P_{ ext{r}} ilde{\delta}_{ ext{rd}}^2}{(P_{ ext{r}}\sigma_{ ext{e}_{ ext{sr}}}^2 + N_0)}$.

With the PDF of $\gamma_{\mbox{\tiny sd}}$, the corresponding MGF can be derived as

$$M_{\gamma_{\rm sd}}(s) = \mathcal{L}\{f_{\gamma_{\rm sd}}(\gamma)\} = (1 + \bar{\gamma}_{\rm sd}s)^{-N_{\rm r}}$$
 (25)

Using Eqs. (22) and (24), the CDF of $\gamma_{\mbox{\tiny srd}}$ is given by

$$F_{\gamma_{\rm srd}}(\gamma) = \Pr\left(\frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\gamma_{\rm rd} + C} < \gamma\right) =$$

$$\int_{0}^{\infty} F_{\gamma_{\rm sr}}\left(\frac{\gamma(\gamma_{\rm rd} + C)}{\gamma_{\rm rd}}\right) f_{\gamma_{\rm rd}}(\gamma_{\rm rd}) \, \mathrm{d}\gamma_{\rm rd} =$$

$$1 - \frac{2}{\Gamma(N_{\rm r})} e^{\frac{\gamma}{\gamma_{\rm sr}}} \left(\sqrt{\frac{\gamma C}{\gamma_{\rm sr}\gamma_{\rm rd}}}\right)^{N_{\rm r}} K_{N_{\rm r}}\left(2\sqrt{\frac{\gamma C}{\gamma_{\rm sr}\gamma_{\rm rd}}}\right)$$

$$(26)$$

where $K_v(\cdot)$ is the v-th order modified Bessel function of the second kind ^[17]. Then the MGF of $\gamma_{\rm srd}$ is derived as

$$\begin{split} M_{\gamma_{\rm srd}}(s) &= s \mathcal{L}\{F_{\gamma_{\rm srd}}(\gamma)\} = \\ 1 &= \frac{s\overline{\gamma}_{\rm sr}\overline{\gamma}_{\rm rd}N_{\rm r}}{C} z^{(N_{\rm r}+1)/2} \mathrm{e}^{z/2} W_{-(N_{\rm r}+1)/2,N_{\rm r}/2}(z) \end{split}$$

(27)

(31)

where $\mathcal{L}\{\bullet\}$ represents Laplace transform, $z = C/[\overline{\gamma}_{rd}(s\overline{\gamma}_{sr}+1)]$, and $W_{\lambda,\mu}(z)$ is the Whitakker function^[17].

Substituting Eqs. (25) and (27) into Eq. (20), the average PEP can be written as

$$PEP(i \to j) \cong \frac{N_{p}^{-1}}{2} \sum_{u=1}^{N_{p}} (1 + \frac{\bar{\gamma}_{sd}}{2\varphi_{u}^{2}})^{-N_{r}} \times \\
\left[1 - \frac{\bar{\gamma}_{sr}\bar{\gamma}_{rd}N_{r}}{2\varphi_{u}^{2}C} \hat{z}^{(N_{r}+1)/2} e^{\hat{z}/2} W_{-(N_{r}+1)/2,N_{r}/2} (\hat{z})\right]$$
(28)

where $\hat{z}=\frac{2C\varphi_u^2}{\left[\bar{\gamma}_{\rm rd}\left(\bar{\gamma}_{\rm sr}+2\varphi_u^2\right)\right]}$. With this result, the average BER can be obtained by a union upper bound [18]

$$P_{e} \leqslant \sum_{j=1}^{N_{t}} \sum_{i=1}^{N_{t}} \frac{N(i \rightarrow j)}{N_{t} \log_{2}(N_{t})} PEP(i \rightarrow j) = (N_{t}/2) PEP(i \rightarrow j)$$
(29)

where $N(i \rightarrow j)$ is the number of error bits between the actual active antenna index i and the es-

timate
$$j$$
 , and $\sum_{j=1}^{N_{\mathrm{t}}}\sum_{i=1}^{N_{\mathrm{t}}}N(i
ightarrow j)\!=\!rac{N_{\mathrm{t}}^2}{2}\!\log_2\!N_{\mathrm{t}}$.

Substituting Eq. (28) into Eq. (29), the closed-form BER expression of the AF—SSK system with channel estimation error can be derived as

$$P_{e} \leqslant \frac{N_{t}}{4N_{p}} \sum_{u=1}^{N_{p}} (1 + \frac{\bar{\gamma}_{sd}}{2\varphi_{u}^{2}})^{-N_{r}} \cdot \left[1 - \frac{\bar{\gamma}_{sr}\bar{\gamma}_{rd}N_{r}}{2\varphi_{v}^{2}C} \hat{z}^{\frac{N_{r}+1}{2}} e^{\frac{\hat{z}}{2}} W_{-(N_{r}+1)/2, N_{r}/2} (\hat{z})\right] (30)$$

Based on Eq. (30), the theoretical average BER can be achieved, and it is shown to have a good agreement with the corresponding simulation for both imperfect CSI and perfect CSI (i. e., $\sigma_{e_{\rm sd}}^2 = \sigma_{e_{\rm rd}}^2 = 0$).

3 Asymptotic BER Analysis and Diversity Gain

In this section, the asymptotic performance of the AF-SSK system under large SNR is analyzed, and asymptotically approximated expression of average BER is derived. With this approximation, the diversity gain of the system is attained.

Using the series representation of $K_v(\bullet)$ (which is Eq. (8. 446) in Ref. [17]),

$$K_v(2x)$$
 with small x can be approximated as $K_v(2x) \approx \frac{1}{2} \sum_{j=0}^{v-1} (-1)^j \frac{(v-j-1)!}{j!} x^{2k-v} + (-1)^{v+1} \frac{x^v}{v!} \left[\ln(x) - \frac{1}{2} \psi(1) - \frac{1}{2} \psi(v+1) \right]$

where $\psi(\bullet)$ is the psi function^[17].

Plugging Eq. (31) into Eq. (26), and considering $C \approx \bar{\gamma}_{\rm sr}$, the approximate CDF of $\gamma_{\rm srd}$ can be expressed as

$$F_{\gamma_{\rm srd}}(\gamma) \approx 1 - \frac{e^{\frac{\gamma}{\gamma_{\rm sr}}}}{\Gamma(N_{\rm r})} \left[\sum_{k=0}^{N_{\rm r}-1} \frac{\Gamma(N_{\rm r}-k)}{\Gamma(k+1)} (-\frac{\gamma}{\gamma_{\rm rd}})^k - \frac{\left(-\frac{\gamma}{\gamma_{\rm rd}}\right)^{N_{\rm r}}}{\Gamma(N_{\rm r}+1)} \left(\ln\left(\frac{\gamma}{\gamma_{\rm rd}}\right) - \psi(1) - \psi(N_{\rm r}+1) \right) \right] \approx 1 - e^{\frac{\gamma}{\gamma_{\rm sr}}} - \frac{e^{\frac{\gamma}{\gamma_{\rm sr}}}}{\Gamma(N_{\rm r})} \left[\sum_{k=1}^{N_{\rm r}-1} \frac{\Gamma(N_{\rm r}-k)}{\Gamma(k+1)} (-\frac{\gamma}{\gamma_{\rm rd}})^k - \frac{\left(-\frac{\gamma}{\gamma_{\rm rd}}\right)^{N_{\rm r}}}{\Gamma(N_{\rm r}+1)} \left(\ln\left(\frac{\gamma}{\gamma_{\rm rd}}\right) - \psi(1) - \psi(N_{\rm r}+1) \right) \right] \approx \frac{\gamma}{\gamma_{\rm sr}} - \frac{1}{\Gamma(N_{\rm r})} \left[\sum_{k=1}^{N_{\rm r}-1} \frac{\Gamma(N_{\rm r}-k)}{\Gamma(k+1)} \left(-\frac{\gamma}{\gamma_{\rm rd}} \right)^k - \frac{\left(-\frac{\gamma}{\gamma_{\rm rd}}\right)^{N_{\rm r}}}{\Gamma(N_{\rm r}+1)} \left(\ln\left(\frac{\gamma}{\gamma_{\rm rd}}\right) - \psi(1) - \psi(N_{\rm r}+1) \right) \right]$$

$$(32)$$

where the third approximate equation is obtained based on the approximation that $\exp\left(-\frac{\gamma}{\gamma_{\rm sr}}\right) \approx 1 - \frac{\gamma}{\gamma_{\rm sr}}$ and $\exp\left(-\frac{\gamma}{\gamma_{\rm sr}}\right) \approx 1$ for very large SNR $\bar{\gamma}_{\rm sr}$.

With the Laplace transform, the MGF of $\gamma_{\mbox{\tiny srd}}$ is approximated as

$$M_{\gamma_{\rm srd}}(s) \approx \frac{1}{\bar{\gamma}_{\rm sr}s} - \frac{1}{\Gamma(N_{\rm r})} \left[\sum_{k=1}^{N_{\rm r}-1} \Gamma(N_{\rm r}-k) \left(-\bar{s\gamma}_{\rm rd} \right)^{-k} + \left(-\bar{s\gamma}_{\rm rd} \right)^{-N_{\rm r}} \left(\ln(\bar{s\gamma}_{\rm rd}) + \psi(1) \right) \right]$$
(33)

Substituting Eq. (33) and the approximation $M_{\gamma_{\rm sd}}(s) \approx (s \bar{\gamma}_{\rm sd})^{-N_{\rm r}}$ into Eq. (19) yields the average PEP as

$$\begin{split} \text{PEP}(i \to j) &\approx \frac{2^{N_{\text{r}}}}{\pi \overline{\gamma}_{\text{sd}}^{N_{\text{r}}}} \int_{0}^{\pi/2} \left[\frac{2}{\gamma_{\text{sr}}} (\sin \theta)^{2N_{\text{r}}+2} - \right. \\ &\left. \sum_{k=1}^{N_{\text{r}}-1} \frac{\Gamma\left(N_{\text{r}}-k\right) \left(-2\right)^{k}}{\Gamma\left(N_{\text{r}}\right) \overline{\gamma}_{\text{rd}}^{k}} (\sin \theta)^{2N_{\text{r}}+2k} - \left(-\frac{4}{\gamma_{\text{rd}}}\right)^{N_{\text{r}}} \times \right. \end{split}$$

$$\frac{(\sin\theta)^{4N_{\rm r}}}{\Gamma(N_{\rm r})} \left(\psi(1) + \ln(\frac{\bar{\gamma}_{\rm rd}}{2}) - 2\ln\sin\theta \right) d\theta$$
(34)

With (3.621.3) and (4.387.4) in Ref. [17], we can obtain

$$\operatorname{PEP}(i \to j) \approx \left(\frac{2}{\gamma_{sd}}\right)^{N_{r}} \left[\frac{1}{\gamma_{sr}} \frac{(2N_{r} + 1)!!}{(2N_{r} + 2)!!} + \sum_{k=1}^{N_{r}-1} \frac{(-2)^{k-1} \Gamma(N_{r} - k)}{\Gamma(N_{r}) (\bar{\gamma}_{rd})^{k}} \frac{(2N_{r} + 2k - 1)!!}{(2N_{r} + 2k)!!} - \left(-\frac{2}{\bar{\gamma}_{rd}}\right)^{N_{r}} \frac{(4N_{r} - 1)!!}{2\Gamma(N_{r}) (4N_{r})!!} \left(\ln(2\bar{\gamma}_{rd}) + \psi(1) + 2\sum_{k=1}^{4N_{r}} \frac{(-1)^{k}}{l}\right)\right] \tag{35}$$

For $N_{\rm r}=1$, the average PEP expression of Eq.(35) under high SNR can be rewritten as

$$\operatorname{PEP}(i \to j) \approx \frac{3}{4\overline{\gamma}_{\mathrm{rd}}} \left[\frac{1}{\overline{\gamma}_{\mathrm{rf}}} + \frac{1}{\overline{\gamma}_{\mathrm{rd}}} \left(\ln(2\overline{\gamma}_{\mathrm{rd}}) + \psi(1) - \frac{7}{6} \right) \right] (36)$$

Thus the approximate average BER is obtained by

$$P_{e} \approx \frac{3N_{t}}{8\bar{\gamma}_{sd}} \left[\frac{1}{\bar{\gamma}_{sr}} + \frac{1}{\gamma_{rd}} \left(\ln(2\bar{\gamma}_{rd}) + \psi(1) - \frac{7}{6} \right) \right]$$
(37)

For $N_{\rm r} \geqslant 2$, the average PEP can be further simplified as

$$\left(\frac{2}{\gamma_{\rm sd}}\right)^{N_{\rm r}} \frac{(2N_{\rm r}+1)!!}{(2N_{\rm r}+2)!!} \left[\frac{1}{\gamma_{\rm sr}} + \frac{1}{(N_{\rm r}-1)\bar{\gamma}_{\rm rd}}\right]$$

$$(38)$$

Then the average BER is approximated as

$$P_{e} \approx \frac{N_{t}}{2} \left(\frac{2}{\gamma_{sd}}\right)^{N_{r}} \frac{(2N_{r}+1)!!}{(2N_{r}+2)!!} \left[\frac{1}{\gamma_{sr}} + \frac{1}{(N_{r}-1)\bar{\gamma}_{rd}}\right]$$
(39)

With Eqs. (37) and (39), the asymptotically approximated BER expressions for $N_{\rm r}=1$ and $N_{\rm r} \geqslant 2$ are attained, respectively. It is shown that they have values close to the corresponding simulations under high SNR.

In what follows, we will give the analysis of the diversity gain. Let $P_{\rm s}=r_1P_{\rm t}$, $P_{\rm r}=r_2P_{\rm t}$, where $r_1+r_2=1$ and r_1 , $r_2\in[0,1]$, we can obtain $\bar{\gamma}_{\rm sd}=\frac{r_1\bar{\gamma}\tilde{\delta}_{\rm sd}^2}{r_1\bar{\gamma}\sigma_{e_{\rm sd}}^2+1}$, $\bar{\gamma}_{\rm sr}=\frac{r_1\bar{\gamma}\tilde{\delta}_{\rm sr}^2}{r_1\bar{\gamma}\sigma_{e_{\rm sr}}^2+1}$ and $\bar{\gamma}_{\rm rd}=\frac{r_2\bar{\gamma}\tilde{\delta}_{\rm rd}^2}{r_2\bar{\gamma}\sigma_{e_{\rm sd}}^2+1}$. Considering that the estimation error

is a decreasing function of SNR, in terms of Ref.

[15], we have $\sigma_{e_{sd}}^2 = \sigma_{e_{sr}}^2 = \sigma_{e_{rd}}^2 = \frac{1}{\tau \gamma}$, where τ is the length of training sequences. Based on this, Eqs. (37) and (39) can be respectively rewritten as

$$\begin{split} P_{\rm e} &\approx \frac{3N_{\rm t}}{8\bar{\gamma}^2} \frac{r_1 + \tau}{r_1\tilde{\delta}_{\rm sd}^2\tau} \left[\frac{r_1 + \tau}{r_1\tilde{\delta}_{\rm sr}^2\tau} + \frac{r_2 + \tau}{r_2\tilde{\delta}_{\rm rd}^2\tau} \left(\ln \left(\frac{2r_2\tilde{\delta}_{\rm rd}^2\tau\bar{\gamma}}{r_2 + \tau} \right) + \psi(1) - \frac{7}{6} \right) \right] \quad (40) \\ P_{\rm e} &\approx \frac{2^{N_{\rm r}-1}N_{\rm t}}{\bar{\gamma}^{N_{\rm r}+1}} \cdot \frac{(2N_{\rm r}+1)!!}{(2N_{\rm r}+2)!!} \left(\frac{r_1 + \tau}{r_1\tilde{\delta}_{\rm sd}^2\tau} \right)^{N_{\rm r}} \cdot \frac{\left(\frac{r_1 + \tau}{r_1\tilde{\delta}_{\rm sr}^2\tau} + \frac{1}{N_{\rm r}-1} \cdot \frac{r_2 + \tau}{r_2\tilde{\delta}_{\rm rd}^2\tau} \right) \quad (41) \end{split}$$

With the results above, we can evaluate the diversity order G_d , which is an important error performance indicator. G_d is defined as the slope of average BER curve for average SNR approaching infinity, and can be derived as [19]

$$G_{\rm d} = \lim_{\bar{\gamma} \to \infty} \left[-\frac{\log(P_{\rm e})}{\log(\bar{\gamma})} \right] = N_{\rm r} + 1 \tag{42}$$

Therefore, with the estimation error modeled as the decreasing function of SNR, the AF-SSK system will obtain the diversity order of $N_r + 1$. As a result, the system performance will become better as the N_r increases.

4 Simulation Results

In this section, the BER performance of the cooperative AF-SSK system with imperfect channel estimation over Rayleigh channel is assessed by using the derived theoretical expressions and computer simulations. The number of transmit antennas at the source, $N_{\rm t}\!=\!2$ or 4, and the pathloss exponent $\alpha=3$. $d_{\rm sd}$, $d_{\rm sr}$ and $d_{\rm rd}$ are the normalized distance for source-to-destination link, source-to-relay link and relay-to-destination link, respectively, and $d_{\rm sd}:d_{\rm sr}:d_{\rm rd}=1:0.5:0.5$. The order of the polynomial $N_{\rm p}$ in Eq. (9) is 5, and $P_{\rm s}\!=\!P_{\rm r}=P_{\rm t}/2$. The variance of estimation error $\sigma_{e_{\rm sd}}^2=\sigma_{e_{\rm rd}}^2=\sigma_{e_{\rm rd}}^2=\sigma_{e_{\rm r}}^2$. The simulation results are realized by Monte Carlo method.

In Fig. 2, we plot the average BER of AF-SSK system with estimation errors for different receive antennas, where $N_{\rm r}=1$, 2, $N_{\rm t}=2$. The variance of estimation error is given by $\sigma_{\rm e}^2=\frac{1}{\tau\gamma}$ with $\tau=3$. The theoretical BERs are calculated by

Eq. (30), and they can agree well with the corresponding simulations. The asymptotical approximated BERs are calculated by Eqs. (37) and (39) for $N_{\rm r} = 1$ and 2, respectively, and they have values very close to the simulated ones for large SNR. From Fig. 2, we can obtain the diversity orders of the AF-SSK system by computing the slopes of BER curves, i. e., they are 2 and 3 for the system with $N_r = 1$ and $N_r = 2$, respectively. These results accord with the derived theoretical orders of $N_r + 1$ shown as Eq. (42). Moreover, the BER of the system with two receive antennas $(N_r=2)$ is lower than that with single receive antenna $(N_r = 1)$, which means that the BER performance becomes better with the increase of receive antennas due to higher diversity gain. The results above indicate that the theoretical analysis is effective, and the deduced expressions can evaluate the performance of AF-SSK system well.

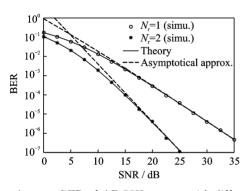


Fig. 2 Average BER of AF-SSK system with different receive antennas ($N_{\rm t}\!=\!2$)

Fig. 3 illustrates the BER performance of AF-SSK system with different receive antennas in the presence of imperfect estimations, where $N_{\rm r}=1$, 2, and $N_{\rm t}=4$. As shown in Fig. 3, the results similar to Fig. 2 can be found. Namely, the system with $N_{\rm r}=2$ has better performance than that with $N_{\rm r}=1$ because the former has larger diversity order than the latter. Moreover, the theoretical BERs can match the corresponding simulated ones, only small performance gap is observed at very low SNR. Besides, the approximated BERs are also close to the simulations at large SNR. All these results further show that the derived theoretical expressions are valid, and can provide good

performance evaluation for the AF-SSK system under imperfect channel information.

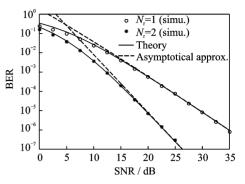


Fig. 3 Average BER of AF-SSK system with different receive antennas ($N_{\rm t} = 4$)

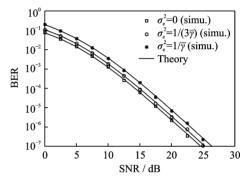


Fig. 4 Average BER of AF-SSK system with different estimation error variances

Fig. 4 shows the theoretical results and simulations of the average BER of AF-SSK system with different estimation errors. The number of transceiver antennas is $N_{\rm r}=N_{\rm t}=2$, and the variances of estimation errors are 0, $\frac{1}{\gamma}$ and $\frac{1}{3\gamma}$. In Fig. 4, the theoretical BERs are consistent with the corresponding simulations for both perfect and imperfect CSI, which testifies that the derived theoretical expressions can accurately describe the BER performance of AF-SSK system under perfect and imperfect CSI. Besides, the average BER of $\sigma_e^2 = \frac{1}{3\gamma}$ is lower than that of $\sigma_e^2 = \frac{1}{\gamma}$ because of smaller estimation error variance, and the average BER of $\sigma_e^2 = 0$ is lower than that of $\sigma_e^2 =$ $\frac{1}{3\gamma}$ due to no estimation error. These results illustrate that the BER performance of AF-SSK system degrades with the increase of estimation error

variance, as expected.

5 Conclusions

Considering imperfect estimation information in practice, we have investigated the performance of cooperative AF-SSK system over Rayleigh fading channel. In the presence of estimation errors, the PDFs and MGFs of the effective SNR are respectively derived for the performance analvsis. Based on this, closed-form average BER and the corresponding asymptotical BER are also deduced. Using the asymptotical BER, the diversity order is analyzed. As a result, the AF-SSK system can achieve the diversity order of $N_{\rm r}+1$. Simulation results illustrate that the derived theoretical BERs can match well with the corresponding simulations for imperfect estimation information, and thus the correctness and effectiveness of theoretical analysis are validated. Thereby, the system performance can be evaluated well. Considering that the SM is the extension of SSK, in the future works, we will study the performance of the cooperative SM system with AF in the presence of channel estimation error so that the corresponding system performance can be well analyzed.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Nos. 61601220, 61172077), the Foundation of Graduate Innovation Center in NUAA (No. kfjj20170410), the Fundamental Research Funds for the Central Universities, the Open Research Fund of National Mobile Communications Research Laboratory of Southeast University (No. 2017D03), and the Six Talent Peaks Project of Jiangsu Province (No. 2015-DZXX-007).

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(Production Editor: Wang Jing)