Gain-Scheduled Control and Stability Analysis of Morphing Aircraft in Transition Process

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(Received 25 April 2017; revised 5 March 2018; accepted 10 March 2018)

Abstract: The large-scale morphing aircraft can change its shape dramatically to perform high flight performance. To ensure the transient stability of aircraft in the morphing process, a novel gain-scheduled control method is investigated numerically in this paper. Based on quasi-steady assumption, the linear parameter varying (LPV) model of the morphing vehicle is derived from its nonlinear equation. Afterwards, by solving a set of linear matrix inequalities along with the bound of the morphing rate via slowly varying system theory, the designed controller which considers the transition stability during the morphing process is obtained. Finally, the transition process simulations of the morphing aircraft are performed via the changes simultaneously in both span and sweep, and the results demonstrate the effectiveness of the proposed controller.

Key words: morphing aircraft; gain-scheduled; convex hull theory; slowly varying system theory; transition stability

CLC number: O313; V212 **Document code**: A **Article ID**: 1005-1120(2018)06-1080-07

0 Introduction

The conventional aircraft is a compromise that allows the flight vehicle to fly with a limited range of flight conditions, which is unable to complete different types of missions and perform extreme maneuvers with high performance. Morphing aircraft can change its shape or geometry during flight to accomplish specific mission better and provide control authority for maneuvering and the special requirements of the military[1-3]. The pattern of morphing are mainly classified into three types[4]: (1) Planform alteration involves span, sweep and chord; (2) Out-of-plane transformation involves twist, dihedral/gull and spanwise bending; (3) Airfoil adjustment involves camber and thickness.

The current morphing aircraft design has been focused on the investigations of modeling, dynamic behavior and flight control[5-8]. For instance, Seigler et al.[3] proposed the methodology for modeling the fight control of morphing aircrafts undergoing large-scale shape change. Zhao and Hu[9] developed a parameterized structural model via the substructure synthesis approach and doublet lattice method. Afterward, Zhao and Hu[10] established the model that govern the dynamic behavior of folding wing via the floating frame method and computational fluid dynamics (CFD) code, and predicted the transient responses during the morphing process. Yue et al.[11] presented the nonlinear dynamic model and the longitudinal dynamic simulation of a folding-wing morphing aircraft through the CFD method. Taking this one step further, Yue et al.[12] proposed a linear parameter varying (LPV) model and a H_∞ self-scheduled control strategy of the folding wing. In addition, He et al.[13] built a new LPV

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How to cite this article: Li Wencheng, Jin Dongping. Gain-scheduled control and stability analysis of morphing aircraft in transition process[J]. Trans. Nanjing Univ. Aero. Astro., 2018,35(6):1080-1086.
http://dx.doi.org/10.16356/j.1005-1120.2018.06.1080

model by using tensor product method and designed a controller via the parallel-distributed compensation control approach, which did not need trip map and numerical calculation. Dong et al.[14] proposed a H_∞ control for switched system via the average dwell time method and linear matrix inequalities (LMIs)[15,16], in which the controller could track the angle of attack well during the morphing process. Shi and Peng[17] presented a novel control strategy that involved virtual morphing control surface by employing the active disturbance rejection control. After considering the morphing parameters as a part of the control input variable, they proved that the energy consumption during the morphing flight will be reduced dramatically. However, the above literatures paid less attention to the transition stability of large-scale morphing aircraft. To determine the bound of morphing rate in transient stage, Shi et al.[17,18] derived the state-space equation in the term of morphing rate explicitly and obtained the minimum and maximum values of the morphing rate via the Hurwitz criterion. On the assumptions that the aerodynamic forces depend solely on the instantaneous configuration of the aircraft, Seigler and Neal[19] analyzed the transient stability of large-scale morphing aircraft by slowing varying system theory (SVST)[20,21].

By considering the morphing variables as scheduled parameters, the morphing aircraft can be modeled as a parameter-varying system, which could be analyzed through the gain-scheduled method[22] and LMIs. In this paper, the LPV model of the morphing aircraft is established after introducing the equations of motion (EOM), and a novel gain-scheduled controller is designed through the combination of convex hull theory and SVST. The approach not only proposes an effective controller with satisfactory robustness, but also gives the bounds of the rates of variables and considers the transition stability during the morphing process.

1 LPV Model of Morphing Aircraft

The flight dynamics of a kind of morphing aircraft which can change its span and sweep is considered. The body-axis coordinate system $O\text{-}X_bY_bZ_b$ and the wind-axis coordinate system $O\text{-}X_aY_aZ_a$ are showed in Fig. 1, where α is the angle of attack and β the slip angle. The aerodynamics forces are usually specified for wind axis, and the two coordinate systems can be obtained from each other via rotation matrices. In the body frame, the nonlinear equations of the morphing aircraft can be written as[11]

$$\begin{cases} \boldsymbol{p} = m\boldsymbol{V} + \dot{\boldsymbol{S}}_o + \boldsymbol{\omega} \times \boldsymbol{S}_o \\ \boldsymbol{H}_o = \boldsymbol{S}_o \times \boldsymbol{V} + \boldsymbol{J} \cdot \boldsymbol{\omega} + \sum_{i=1}^{N} \left(\frac{1}{m_i} \boldsymbol{S}_{oi} \times \frac{\mathrm{d}\boldsymbol{S}_{oi}}{\mathrm{d}t} + \boldsymbol{J}_i \cdot \boldsymbol{\omega}_i \right) \end{cases} \tag{1}$$

where $\boldsymbol{p}$ is the momentum and $\boldsymbol{H}_o$ the angular momentum. $\boldsymbol{S}_o = \int_\Omega \boldsymbol{r} \times \mathrm{d}m$ is the static moment about the origin, where the integral domain Ω is the whole aircraft. m, $\boldsymbol{V}$, $\boldsymbol{\omega}$ and $\boldsymbol{J}$ are the mass, velocity, angular velocity and inertia matrix of the morphing aircraft, respectively. The subscript i denotes the ith morphing part and N is the total number of the morphing structures.

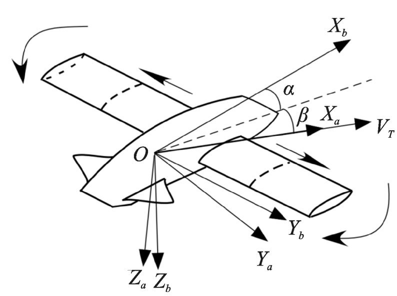

Fig. 1 Sketch of morphing aircraft and its coordinate system

The 6-DOF EOM of the morphing aircraft in the body coordinate system can be derived from Eq. (1) as follows

$$\begin{cases} \boldsymbol{F} = m(\dot{\boldsymbol{V}} + \boldsymbol{\omega} \times \boldsymbol{V}) + \dot{\boldsymbol{\omega}} \times \boldsymbol{S}_o + 2\boldsymbol{\omega} \times \dot{\boldsymbol{S}}_o + \\ \quad \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{S}_o) + \ddot{\boldsymbol{S}}_o \\ \boldsymbol{M}_o = \boldsymbol{J} \cdot \dot{\boldsymbol{\omega}} + \dot{\boldsymbol{J}} \cdot \boldsymbol{\omega} + \boldsymbol{\omega} \times (\boldsymbol{J} \cdot \boldsymbol{\omega}) + \\ \quad \boldsymbol{S}_o \times \dot{\boldsymbol{V}} + \boldsymbol{S}_o \times (\boldsymbol{\omega} \times \boldsymbol{V}) + \\ \sum_{i=1}^{N} \Big\{ \dot{\boldsymbol{J}}_i \cdot \boldsymbol{\omega}_i + \boldsymbol{J}_i \cdot \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i \times (\boldsymbol{J}_i \cdot \boldsymbol{\omega}_i) + \\ \quad \frac{1}{m_i} [\boldsymbol{S}_{oi} \times \ddot{\boldsymbol{S}}_{oi} + \boldsymbol{\omega} \times (\boldsymbol{S}_{oi} \times \dot{\boldsymbol{S}}_{oi})] \Big\} \end{cases} \tag{2}$$

It is noted that $\boldsymbol{J}$, $\boldsymbol{J}_i$, $\boldsymbol{S}_o$, $\boldsymbol{S}_{oi}$ and their derivatives change significantly with the morphing parameters during the flight, leading to a complex model. Hence, some assumptions are adopted to simplify the nonlinear EOM[19]. Firstly, the inertial forces, time derivative of inertia and static moment owing to morphing are ignored. In addition, the unsteady influence on this kind of morphing aircraft can be negligible[11] and the quasi-steady assumption is adopted. In other words, the instantaneous aerodynamic forces are completely decided by the current shape of the aircraft. Therefore, one obtains the longitudinal nonlinear EOM in a general form

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\xi}) \tag{3}$$

where $\boldsymbol{x} = [\alpha\ \theta\ q\ V_T]^{\mathrm{T}}$ is the collection of state variables, θ is the pitch angle, q is the pitch angle rate and V_T is the flight velocity. $\boldsymbol{u}=\boldsymbol{\delta}_e$ is the control input, where $\boldsymbol{\delta}_e$ is the aileron deflection. $\boldsymbol{\xi}=[\boldsymbol{\xi}_1\ \boldsymbol{\xi}_2]^{\mathrm{T}}$ is a morphing vector consisted of span and sweep.

By taking the morphing variables as scheduled parameters, Eq. (3) becomes a parameter-varying system. Several methods, such as Jacobian linearization, quasi-LPV approach, linear fractional transformation and so on, could be used to establish the LPV model of the morphing aircraft. In this work, according to Jacobian linearization technique, one has

$$\dot{z} = \boldsymbol{A}z + \boldsymbol{B}\Delta\boldsymbol{u} \tag{4}$$

where $z = \boldsymbol{x} - \boldsymbol{x}_0(\xi)$, $\Delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}_0$, $\boldsymbol{A} = \left.\dfrac{\partial \boldsymbol{f}}{\partial x}\right|_{x_0, \boldsymbol{u}_0}$, $\boldsymbol{B} = \left.\dfrac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right|_{x_0, \boldsymbol{u}_0}$; $\boldsymbol{x}_0(\xi)$ and $\boldsymbol{u}_0$ are the vector of equilibriumpoint. It should be noted that, unlike rigid aircraft, the equilibrium point and the state-space matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are the functions of the morphing variables. Furthermore, the state-space matrices depend on the aerodynamic forces, which are

$$L = \frac{1}{2}SC_L\rho V_T^2,\ D = \frac{1}{2}SC_D\rho V_T^2,$$

$$M_{\mathrm{A}} = \frac{1}{2}ScC_m\rho V_T^2 \tag{5}$$

where ρ is the density of air, S and c are the equivalent area and chord of wing; L, D and M_{A} are aerodynamic lift, drag and moment, respectively. C_L, C_D and C_m are the corresponding aerodynamic coefficients, which can be expressed as the following form at a low angle of attack

$$\begin{cases} C_L = C_{L0} + C_{L\alpha}\alpha + C_{L\delta e}\delta e \\ C_D = C_{D0} + C_{D\alpha}\alpha + C_{L\delta e}\delta e \\ C_m = C_{m0} + C_{m\alpha}\alpha + C_{mq}q + C_{m\delta e}\delta e \end{cases} \tag{6}$$

2 Gain-Scheduled Control Based on Slowly Varying System Theory

2.1 Gain-scheduled control

Rewriting an LPV plant from Eq. (4) in general form yields

$$P(\xi) := \begin{bmatrix} \boldsymbol{A}(\xi) & \boldsymbol{B}(\xi) \\ \boldsymbol{C}(\xi) & \boldsymbol{D}(\xi) \end{bmatrix} \tag{7}$$

where $\boldsymbol{A}(\xi)$, $\boldsymbol{B}(\xi)$, $\boldsymbol{C}(\xi)$, and $\boldsymbol{D}(\xi)$ depend affinely on the time-varying parameter ξ, for example

$$\boldsymbol{A}(\xi) = \boldsymbol{A}_0 + \xi_1\boldsymbol{A}_1 + \cdots + \xi_n\boldsymbol{A}_n \tag{8}$$

where $\boldsymbol{A}_0$, $\boldsymbol{A}_1$, $\cdots$, $\boldsymbol{A}_n$ are the given matrices. $\xi(t)$ varies in a polytope Θ of vertices $\omega_1, \omega_2, \cdots, \omega_k$ as

$$\xi(t) \in \Theta := Co\{\omega_1, \omega_2, \cdots, \omega_k\} = \left\{\sum_{i=1}^{k}\sigma_i(t)\omega_i : \sigma_i(t) \geqslant 0, \sum_{i=1}^{k}\sigma_i(t) = 1, k = 2^n\right\} \tag{9}$$

and the LPV plant $\boldsymbol{P}$ can be expressed as a ploytope of matrices, it reads

$$\begin{bmatrix} \boldsymbol{A}(\xi) & \boldsymbol{B}(\xi) \\ \boldsymbol{C}(\xi) & \boldsymbol{D}(\xi) \end{bmatrix} = \sum_{i=1}^{k}\sigma_i \begin{bmatrix} \boldsymbol{A}(\omega_i) & \boldsymbol{B}(\omega_i) \\ \boldsymbol{C}(\omega_i) & \boldsymbol{D}(\omega_i) \end{bmatrix} \in Co\left\{\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{B}_i \\ \boldsymbol{C}_i & \boldsymbol{D}_i \end{bmatrix}, i = 1, \cdots, k\right\} \tag{10}$$

where ξ is bounded as $\xi_i \in [\bar{\xi}_i, \bar{\xi}_i]$, $i=1,\cdots,n$. An LPV system is called "polytopic" when it satisfies Eqs. (9), (10)[23].

Lemma 1[24] Consider an LPV system as Eq. (7), for a prescribed $\gamma > 0$, if there exist matrices $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, $\boldsymbol{Y}_i$ and $\boldsymbol{X}_i > 0$ $(i = 1, \cdots, k)$, such that the following inequalities holds

$$\begin{bmatrix} \boldsymbol{W} & 0 & [\boldsymbol{X}_i\quad \boldsymbol{Y}_i^{\mathrm{T}}]\begin{bmatrix} \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{D}^{\mathrm{T}} \end{bmatrix} & \boldsymbol{L} \\ * & -\gamma^2\boldsymbol{I} & 0 & 0 \\ * & * & -\boldsymbol{I} & 0 \\ * & * & * & \boldsymbol{S}_1 + \boldsymbol{S}_1^{\mathrm{T}} \end{bmatrix} < 0 \tag{11}$$

where

$$\boldsymbol{W}=-\sum_{i=1}^{k}\dot{\sigma}_i\boldsymbol{X}_i+\boldsymbol{S}_2^{\mathrm{T}}\begin{bmatrix}\boldsymbol{A}_i^{\mathrm{T}}\\ \boldsymbol{B}_i^{\mathrm{T}}\end{bmatrix}+\begin{bmatrix}\boldsymbol{A}_i^{\mathrm{T}}\\ \boldsymbol{B}_i^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_2$$

$$\boldsymbol{L}=[\boldsymbol{X}_i \quad \boldsymbol{Y}_i^{\mathrm{T}}]-\boldsymbol{S}_2^{\mathrm{T}}-\begin{bmatrix}\boldsymbol{A}_i^{\mathrm{T}}\\ \boldsymbol{B}_i^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\boldsymbol{S}_1^{\mathrm{T}}$$

Then the gain-scheduled controller $\boldsymbol{u}(t)=\boldsymbol{K}_\sigma\boldsymbol{x}(t)$ could be obtained, where

$$\boldsymbol{K}_\sigma=\Big(\sum_{i=1}^{k}\sigma_i\boldsymbol{Y}_i\Big)\Big(\sum_{i=1}^{k}\sigma_i\boldsymbol{X}_i\Big)^{-1} \tag{12}$$

and the upper bound of the L_2-norm is γ.

Eq. (11) can be solved via LMI skills. It is noted that $\dot{\sigma}_i$ contained in Eq. (11) is able to reduce the conservatism of the controller by employing the parameter-dependent Lyapunov functions, however, the introduction of differential item $\dot{\sigma}_i$ complicates the problem. One method to solve this problem is to find the bound of $\dot{\sigma}_i$. In this paper, σ_i is the function of the morphing variables ξ, hence, the problem is converted into solving the bound of $\dot{\xi}$, which can be determined via SVST[19,21].

2.2 Slowly varying system theory[21]

Considering the linear system of Eq. (4) and a state feedback controller $u=K(\xi)z$, one has

$$\dot{\boldsymbol{z}}=[\boldsymbol{A}(\boldsymbol{\xi})+\boldsymbol{B}(\boldsymbol{\xi})\boldsymbol{K}(\boldsymbol{\xi})]\boldsymbol{z}=\bar{\boldsymbol{A}}(\boldsymbol{\xi})\boldsymbol{z} \tag{13}$$

Suppose that $\bar{\boldsymbol{A}}(\xi)$ is continuously differentiable and Hurwitz for every fixed ξ, that is

$$\lambda(\bar{\boldsymbol{A}}(\boldsymbol{\xi}))\leqslant\eta<0 \tag{14}$$

Furthermore, suppose the elements of $\bar{\boldsymbol{A}}(\xi)$ and their first partial derivatives with respect to $\boldsymbol{\xi}$ are uniform bounded, and let $\boldsymbol{Q}$ be a constant and positive-definite matrix, then there exist a positive-definite matrix that satisfies the Lyapunov equation

$$\boldsymbol{P}(\boldsymbol{\xi})\bar{\boldsymbol{A}}(\boldsymbol{\xi})+\bar{\boldsymbol{A}}^{\mathrm{T}}(\xi)\boldsymbol{P}(\xi)=-\boldsymbol{Q} \tag{15}$$

In addition, $\boldsymbol{P}(\xi)$ satisfies the following conditions for all $\boldsymbol{\xi}\in\boldsymbol{\Psi}$, where $\boldsymbol{\Psi}$ is the domain of interest.

$$c_1\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z}\leqslant\boldsymbol{z}^{\mathrm{T}}\boldsymbol{P}(\xi)\boldsymbol{z}\leqslant c_2\boldsymbol{z}^{\mathrm{T}}\boldsymbol{z} \tag{16}$$

$$\left\|\frac{\partial}{\partial\xi_i}\boldsymbol{P}(\boldsymbol{\xi})\right\|_2\leqslant\mu_i, i=1,\cdots,n \tag{17}$$

where c_1, c_2 and μ_i are positive constants independent of ξ. Moreover, suppose that $\|\partial\boldsymbol{x}_0/\partial\boldsymbol{\xi}\|_2\leqslant l$ and there exists an ε such that

$$\|\dot{\boldsymbol{\xi}}\|_2\leqslant\varepsilon<\frac{c_1}{c_2c_3}\times\frac{r}{r+2c_2l/c_3} \tag{18}$$

where r is the upper bound of $\|\boldsymbol{z}\|$, and $c_3=\sqrt{\sum_{i=1}^{k}\mu_i^2}$. Then the solutions of Eq. (13) are uniformly bounded for all $\|\boldsymbol{z}(0)\|<r\sqrt{c_1/c_2}$.

As showed in Eq. (18), the bound of $\dot{\xi}$ depends on a lot of variables. One approach to find the "best" ε is proposed in Ref. [19], which is to maximize the ratio $\lambda_{\min}(\boldsymbol{P})/\lambda_{\max}(\boldsymbol{P})$ by finding a proper $\boldsymbol{Q}$ for all $\boldsymbol{\xi}\in\boldsymbol{\Psi}$[25].

The gain-scheduled controller of Eq. (12) can be solved by combining Eq. (11) and SVST. Fig. 2 shows the flow chart:

(1) Give an initial morphing rate $\dot{\xi}_0$ firstly;

(2) The corresponding gain-scheduled controller K_σ can be obtained from Eqs. (11) and (12);

(3) The upper bound ε of the morphing rate can be determined via SVST;

(4) If the given morphing rate satisfies $\|\dot{\boldsymbol{\xi}}_0\|_2\leqslant\varepsilon$, then $\dot{\boldsymbol{\xi}}^*=\dot{\boldsymbol{\xi}}_0$ is an achieved solution and corresponding $\boldsymbol{K}_{\boldsymbol{\sigma}}$ is obtained;

(5) Otherwise, try different initial morphing rates to go through the above procedures until a proper solution is found.

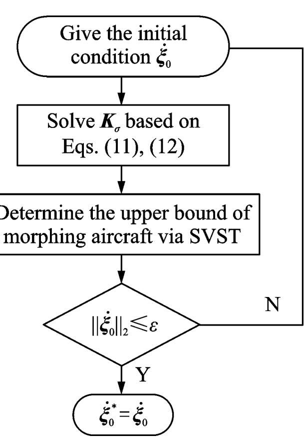

Fig. 2 Flow chart to solve the bound of morphing rate

3 Simulation

A steady andwings-level fight conditions with constant flight speed are employed, i. e.

$$\begin{cases}\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\end{bmatrix}=\begin{bmatrix}\dfrac{Z_\alpha}{V_T} & 1\\ M_\alpha & M_q\end{bmatrix}\begin{bmatrix}z_1\\ z_2\end{bmatrix}+\begin{bmatrix}\dfrac{Z_{\delta e}}{V_T}\\ M_{\delta e}\end{bmatrix}\Delta\boldsymbol{u}\\ \boldsymbol{y}=\boldsymbol{C}\boldsymbol{z}\end{cases} \tag{19}$$

where

$$Z_\alpha=-\frac{1}{m}\left(D+\frac{\partial L}{\partial \alpha}\right),\ M_\alpha=\frac{1}{J_y}\frac{\partial M_A}{\partial \alpha},$$
$$M_q=\frac{1}{J_y}\frac{\partial M_A}{\partial q},Z_{\delta e}=-\frac{1}{m}\frac{\partial L}{\partial \delta e},$$
$$M_{\delta e}=\frac{1}{J_y}\frac{\partial M_A}{\partial \delta e},\ \boldsymbol{C}=\boldsymbol{I}_2$$

These aerodynamic derivatives are the functions of morphing variables. In this paper, the vortex lattice solver Tornado[26] is used to calculate the steady aerodynamic forces for different shapes of morphing aircraft, whose span L and sweep δ varies from 5.2 m to 7.2 m and 0° to 30°, respectively.

Note that the aerodynamic derivatives can be fitted as functions of normalized morphing variables using the generalized leasts quares method, namely

$$\bar{L}=\frac{L-(L_{\max}+L_{\min})/2}{(L_{\max}-L_{\min})/2}\in[-1,1]$$
$$\bar{\delta}=\frac{\delta-(\delta_{\max}+\delta_{\min})/2}{(\delta_{\max}-\delta_{\min})/2}\in[-1,1]$$

Then Eq. (19) can be expressed as an LPV system, whose parameters are $\bar{L}$ and $\bar{\delta}$. The coefficients in Eq. (9) are set at

$$\sigma_1=\frac{1-\bar{L}}{4},\sigma_2=\frac{1-\bar{\delta}}{4},\sigma_3=\frac{1+\bar{L}}{4},\sigma_4=\frac{1+\bar{\delta}}{4} \tag{20}$$

It is noted that different convex hull models can be derived by choosing different coefficients, if the convex coordinates satisfy Eq. (9)[27]. The vertices of $\boldsymbol{A}_i$ are the values of $A(\xi)$ at four vertices of the parameter box

$$\omega_1:=(L_{\min},\delta_{\min}),\ \omega_2:=(L_{\min},\delta_{\max}),$$
$$\omega_3:=(L_{\max},\delta_{\min}),\ \omega_4:=(L_{\max},\delta_{\max})$$

According to the flow chart showed in Fig. 2, let a constant $\gamma=1.05$ and an initial morphing rate $\|\dot{\xi}_0\|_2=0.0418$, which means the morphing time is 50 s, and the corresponding $\boldsymbol{X}_i$, $\boldsymbol{Y}_i$ and $\boldsymbol{K}_\sigma$ can be determined by Eqs. (11), (12).

$$\boldsymbol{X}_1=\begin{bmatrix}0.5058 & -0.2755\\ -0.2755 & 2.3722\end{bmatrix},\ \boldsymbol{Y}_1=\begin{bmatrix}1.4077\\ 95.3112\end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{X}_2=\begin{bmatrix}0.6075 & -0.3246\\ -0.3246 & 4.9197\end{bmatrix},\ \boldsymbol{Y}_2=\begin{bmatrix}1.5936\\ 99.3573\end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{X}_3=\begin{bmatrix}0.5503 & -0.2826\\ -0.2826 & 2.252\end{bmatrix},\ \boldsymbol{Y}_3=\begin{bmatrix}1.4128\\ 78.5023\end{bmatrix}^{\mathrm{T}},$$
$$\boldsymbol{X}_4=\begin{bmatrix}0.5806 & -0.3123\\ -0.3123 & 7.597\end{bmatrix},\ \boldsymbol{Y}_4=\begin{bmatrix}1.5062\\ 71.1294\end{bmatrix}^{\mathrm{T}}$$

Afterward, the upper bound of morphing rate $\varepsilon=0.0432$ is obtained via SVST, which indicates that $\|\dot{\xi}_0\|_2<\varepsilon$ is an achieved solution. The response of the close-loop system is performed as shown in Fig. 3 using the fourth-order Ronge-Kutta method. It is found that the initial disturbance can be eliminated by using the proposed gain-scheduled controller, which can stabilize the slowly varying system.

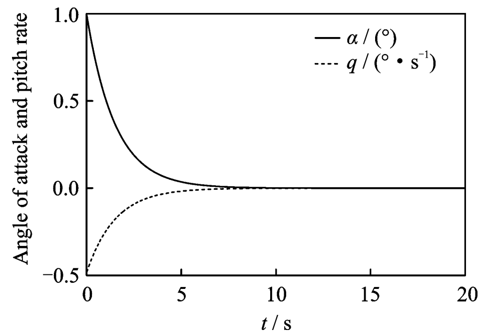

Fig. 3 Response of the close-loop system

Consider the state-space equation with an external disturbance

$$\dot{\boldsymbol{z}}=\boldsymbol{A}(\boldsymbol{\xi})\boldsymbol{z}+\boldsymbol{B}(\boldsymbol{\xi})\Delta\boldsymbol{u}+\boldsymbol{G}\omega(t)$$

where $\boldsymbol{G}=[0.1\quad 0]^{\mathrm{T}}$ and the external disturbance sine signal $\omega(t)$ is

$$\omega(t)=\begin{cases}0 & 0<t<10\\ t-10 & 10\leqslant t<13.14\\ 0 & t\geqslant 13.14\end{cases}$$

Another external disturbance square signal $\omega(t)$ is given, reads

$$\omega(t)=\begin{cases}0 & 0<t<10\\ 1 & 10\leqslant t<13.14\\ 0 & t\geqslant 13.14\end{cases}$$

The responses of the close-loop system with different external disturbances are shown in Figs. 4, 5.

In Figs. 4, 5, the effects of different disturbances can be eliminated quickly to make the close-loop system stable, which indicates a good capacity of resisting disturbance.

From Figs. 4,5, it is indicated that the designed controller makes the systemt end to zero in different cases of disturbance. The primary usefulness of this approach is that it proposes a method to solve Eq. (11) with the consideration of transition stability. Although the introduced

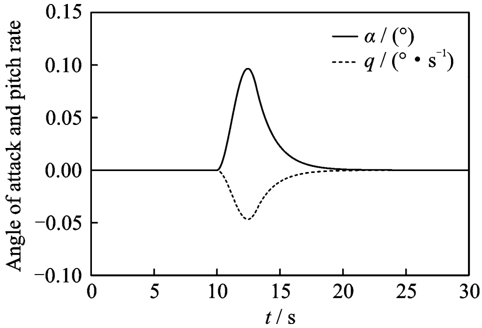

Fig. 4 Response of the close-loop system with external disturbance sine signal

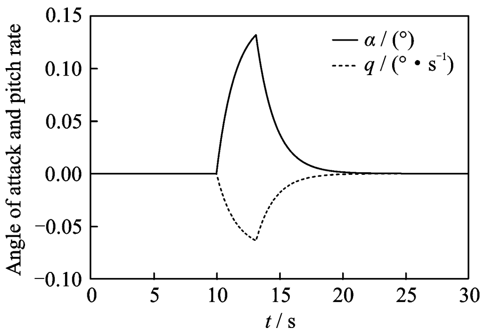

Fig. 5 Response of the close-loop system with external disturbance square signal

SVST reduces the conservatism of controller, it could provide criteria to ensure the transition stability of the slowly varying system, which is a concerned problem for morphing aircraft. In this paper, the minimum required time for large-scalemorphing is less than 1 min, which could give a reference value for flight testing.

4 Conclusions

An LPV model of a kind of morphing aircraft which can change its span and sweep is derived via convex hull theory. To reduce the conservative, the parameter-dependent Lyapunov functions are introduced along with the differential term of the variables, whose bounds are determined by slowly varying system theory. The gain-scheduled controller proposed via convex hull theory and slowly varying system theory is able to eliminate the influence of initial error and external disturbances quickly, which shows a good robustness of the close-loop system. Furthermore, the controller can guarantee the transition stability of the slowly varying system in the morphing process.

Acknowledgements

This work was supported by the Fundamental Research Funds for the Central Universities (No. 2662018QD029), the National Natural Science Foundation of China (Nos. 91016022, 61503354), and in part by the Priority Academic Program of Jiangsu Higher Education Institutions.

References:

[1] JIN D P, JI B. Topology optimization of flexible support structure for trailing edge [J]. Acta Aeronautica et Astronautica Sinica, 2015, 36(8): 2671-2677. (in Chinese)

[2] LI W C, JIN D P. Flutter suppression and stability analysis for a variable-span wing via morphing technology [J]. Journal of Sound and Vibration, 2018, 412: 410-423.

[3] SEIGLER T M, NEAL D A, BAE J S, et al. Modeling and flight control of large-scale morphing aircraft [J]. Journal of Aircraft, 2007, 44(4): 1077-1087.

[4] BARBARINO S, BILGEN O, AJAJ R M, et al. A review of morphing aircraft [J]. Journal of Intelligent Material Systems and Structures, 2011, 22(9): 823-877.

[5] HU W, YANG Z C, GU Y S. Aeroelastic study for folding wing during the morphing process [J]. Journal of Sound and Vibration, 2016, 365: 216-229.

[6] WANG X C, YANG Z C, ZHOU J, et al. Aeroelastic effect on aerothermoacoustic response of metallic panels in supersonic flow [J]. Chinese Journal of Aeronautics, 2016, 29(6): 1635-1648.

[7] YIN M, LU Y P, HE Z. LPV modeling and robust gain scheduling control of morphing aircraft [J]. Journal of Nanjing University of Aeronautics and Astronautics, 2013, 45(2): 202-208. (in Chinese)

[8] ZHANG W, SUN L, YANG X D, et al. Nonlinear dynamic behaviors of a deploying-and-retreating wing with varying velocity [J]. Journal of Sound and Vibration, 2013, 332: 6785-6797.

[9] ZHAO Y H, HU H Y. Parameterized aeroelastic modeling and flutter analysis for a folding wing [J]. Journal of Sound and Vibration, 2012, 331(2): 308-324.

[10] ZHAO Y H, HU H Y. Prediction of transient responses of a folding wing during the morphing process [J]. Aerospace Science and Technology, 2013, 24(1): 89-94.

[11] YUE T, WANG L X, AI J Q. Longitudinal linear parameter varying modeling and simulation of morphing aircraft [J]. Journal of Aircraft, 2013, 50(6): 1673-1681.

[12] YUE T, WANG L X, AI J Q. Gain self-scheduled H_∞ control for morphing aircraft in the wing transition process based on an LPV model [J]. Chinese Journal of Aeronautics, 2013, 26(4): 909-917.

[13] HE Z, YIN M, LU Y P. Tensor product model-based control of morphing aircraft in transition process [J]. Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, 2016, 230(2): 378-391.

[14] DONG C Y, LI W, WANG Q. H_∞ control of switched systems with nonidealswitchings and its application to morphing aircraft [J]. Procedia Engineering, 2013, 67: 100-109.

[15] BERARDENGO M, MANZONI S, CONTI A M. Multi-mode passive piezoelectric shunt damping by means of matrix inequalities [J]. Journal of Sound and Vibration, 2017, 405: 287-305.

[16] THEISEN L R S, NIEMANN H H, GALEAZZI R, et al. Enhancing damping of gas bearings using linear parameter-varying control [J]. Journal of Sound and Vibration, 2017, 395: 48-64.

[17] SHI R Q, PENG J. Morphing strategy design for variable-wing aircraft [C]//Proceedings of the 15th AIAA Aviation Technology, Integration, and Operations Conference. Dallas: AIAA, 2015.

[18] SHI R Q, WAN W. Analysis of flight dynamics for large-scale morphing aircraft [J]. Aircraft Engineering and Aerospace Technology, 2015, 87(1): 38-44.

[19] SEIGLER T M, NEAL D A. Analysis of transition stability for morphing aircraft [J]. Journal of Guidance Control and Dynamics, 2009, 32(6): 1947-1954.

[20] KHALIL H K, KOKOTOVIC P V. On stability of nonlinear systems with slowly varying inputs [J]. IEEE Transactions on Automatic Control, 1991, 36(2): 229.

[21] KHALIL H K. Nonlinear systems [M]. 3rd ed. New Jersey: Prentice-Hall, Upper Saddle River, 2002: 365-372.

[22] RUGH W J, SHAMMA J S. Research on gain scheduling [J]. Automatica, 2000, 36(10): 1401-1425.

[23] APKARIAN P, GAHINET P, BECKER G. Self-scheduled H_∞ control of linear parameter-varying systems: A design example [J]. Automatica, 1995, 31(9): 1251-1261.

[24] YANG G H, LUM K Y. Gain-scheduled flight control via state feedback [C]//Proceedings of the American Control Conference. Denver: IEEE, 2003: 3484-3489.

[25] KHUSAINOV D Y, YUN'KOVA E A. A method of determination of the solution of LyapuNov's equation [J]. Ukrainian Mathematical Journal, 1985, 36(4): 435-438.

[26] GHOREYSHI M, DARRAGH R, HARRISON S, et al. Canard-wing interference effects on the flight characteristics of a transonic passenger aircraft [J]. Aerospace Science and Technology, 2017, 69: 342-356.

[27] XIE W. H_2 gain scheduled state feedback for LPV system with new LMI formulation [J]. IEE Proceedings—Control Theory and Applications, 2005, 152(6): 693-697.

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(Production Editor: Zhang Tong)