Permutation Encoding for Pilot Coordination in Multi-user Massive MIMO

Hafiz Ahmad Khalid *

College of Electronics Science and Technology, Beijing University of Posts and Telecommunications,
Beijing 100876, P. R. China

(Received 20 March 2018; revised 20 April 2018; accepted 30 May 2018)

Abstract: Pilot plays an essential role in a duplex communication system. Several methods have been proposed for pilot assignment over specific scenarios. With the help of permutation encoding, we implemented a genetic algorithm for optimizing pilot assignment in a multi-user massive multiple input multiple output (MIMO) system. Results show improvement on existing results especially in the case of strong user estimation rates.

Key words: genetic algorithms; performance analysis; permutation encoding

CLC number: TN911 **Document code**: A **Article ID**: 1005-1120(2018)S-0059-04

0 Introduction

In a duplex communication system, pilot symbols play a vital role in a noisy environment for smooth and uninterrupted services to the users. We are considering large antenna array communication systems (Massive MIMO), in which theoretically an infinite number of antennas on the base station (BS) and single antenna mobile users (MUs) separated around pre-allocated BS[1]. Although there were several methods proposed up till now to better-optimize pilot assignment over specific scenarios, Ref. [1] describes many opportunities for better enhancements in MIMO systems. One way to improve the system is to optimize pilot assignments for MUs around a BS. Four methods used in Ref. [2] for pilot assignments are exhaustive enumeration, degradation based greedy assignment, variance-based greedy assignment and position based assignment. Among them, position based assignment outperforms above all.

Previous study[1,3-6] shows that feedback using pilot symbols is costly and impractical in frequency division duplex (FDD) system as compared with time division duplex (TDD) systems where uplink training is used for channel estimation. Therefore, the number of pilot symbols does not depend on the number of antennas on every base station at each cell.

In an uncoordinated base station type system, the performance of that system is severely degraded because of inter-cell interference in the initial phase of training[6].

The results from Refs. [1,3,5,7] made the foundation to our problem[2] which formulate the asymptotic analysis of coordination strategy problem with the help of combinatorial network utility maximization (NUM) problem. This approach improves the performance of the system up to a level that nearly resembles with an actual problem to possibly analyze the potential assistance of pilot coordination in a practical system using different pilot assignment methods.

In this paper, we studied that a genetic algorithm can be used to optimize the pilot assignment. We used genetic algorithm by using permutation encoding technique for crossover and mutation. Permutation encoding is necessary for crossover and mutation in ordered typeprob-

---

* Corresponding author, E-mail address: engineerahmadkhalid@gmail.com.

How to cite this article: Hafiz Ahmad Khalid. Permutation encoding for pilot coordination in multi-user massive MIMO[J]. Trans. Nanjing Univ. Aero. Astro., 2018,35(S):59-62.
http://dx.doi.org/10.16356/j.1005-1120.2018.S.059

lems[8]. We observed that genetic algorithm, when used with permutation encoding, gives good estimates over existing MUs position based pilot assignment model used in Ref. [2].

1 Model

In multi-cell multi-user massive MIMO system, we define the notations as there exist L base stations that contain M transmit antennas at each base station to serve K users with a single antenna. To consider the massive MIMO system, we assume that M is a very large number such that $M \gg K$ and the capability of channel reciprocity of system means the system is TDD mode of operation. Further, we assume pairwise statistically independent vector of complex channel gains in a block fading channel model. So $\boldsymbol{h}_{ijk}$ will denote the channel gains to the base station i in cell j from the user k, where each vector can be considered as zero-mean Gaussian distributed with covariance matrix $\boldsymbol{R}_{ijk}$ which shows the same relation as channel vector[2].

Therefor $\boldsymbol{H}_{ij}$ will have collection of columns as channel vector gains of the kth user to base station i in cell j. If ρ_{tr} denote the SNR for training pilots and $\boldsymbol{T}_{\mathrm{tr}}$ is available orthogonal pilot sequences, so that reception in all cells is synchronized because of training of pilot symbols is going to start at a same time for all user in the system. The received channel gain at the ith BS can be shown as

$$\boldsymbol{W}_i = \sqrt{\rho_{\mathrm{tr}}} \sum_{j=1}^{L} \boldsymbol{H}_{ij} \boldsymbol{D}_j + \boldsymbol{N}_i \tag{1}$$

where $\boldsymbol{W}_i \in \mathbf{C}^{M \times T_{\mathrm{tr}}}$; $\boldsymbol{N}_i$ is i. i. d complex Gaussian distributed noise with zero mean and unit variance. $\boldsymbol{D}_j$ is orthogonal rows pilot sequences in the jth cell for all K users. Considering the least square (LS) channel estimation if we reuse all pilot sequences in every cell and correlate the pilots with received signals, the received signal would be

$$\boldsymbol{Y}_i = \boldsymbol{W}_i \frac{1}{\sqrt{\rho_{\mathrm{tr}}}} \overline{\boldsymbol{D}}^{\mathrm{H}} = \sum_{j=1}^{L} \boldsymbol{H}_{ij} + \frac{1}{\sqrt{\rho_{\mathrm{tr}}}} \widetilde{\boldsymbol{N}}_i \tag{2}$$

where transformed noise matrix is still i. i. d complex gaussian distributed noise with zero mean and unit variance. We need to define a pool of pilot sequences in a matrix $\boldsymbol{D}$ and another matrix $\boldsymbol{P}_j$ that contains all the assignments of pilot sequence in matrix $\boldsymbol{D}$ where $\boldsymbol{P}_j \in \{0,1\}^{K \times T_{\mathrm{tr}}}$. We can define the matrix $\boldsymbol{D}$ as identity matrix to represent our practical system where the matrix represents the time-frequency block for the particular pilot symbol.

This modifies the expression as in Eq. (3) where we reuse same pilot sequences in every cell.

$$\boldsymbol{Y}_i = \boldsymbol{W}_i \frac{1}{\sqrt{\rho_{\mathrm{tr}}}} \overline{\boldsymbol{D}}^{\mathrm{H}} \boldsymbol{P}_i^{\mathrm{T}} = \boldsymbol{H}_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{L} \boldsymbol{H}_{ij} \boldsymbol{P}_j \boldsymbol{P}_i^{\mathrm{T}} + \frac{1}{\sqrt{\rho_{\mathrm{tr}}}} \widetilde{\boldsymbol{N}}_i \tag{3}$$

This estimate can be improved by using minimum mean square error (MMSE) but adds the additional complexity. For this, we need the second-order characteristics of a channel like a covariance matrix of the channel. Channel estimates are shown in Eq. (4).

$$\hat{\boldsymbol{h}}_{iik} = \boldsymbol{R}_{iik} \left(\frac{1}{\rho_{\mathrm{tr}}} \boldsymbol{I} + \sum_{(j,m) \in \kappa_{\mu(i,k)}} \boldsymbol{R}_{ijm} \right)^{-1} \boldsymbol{y}_{ik} \tag{4}$$

In Eq. (4), $\boldsymbol{y}_{ik}$ denotes the estimates of each user as in the column of $\boldsymbol{Y}_i$. The variance of the channel can be calculated as in Eq. (5).

$$\boldsymbol{\Phi}_{ik,jm} = \boldsymbol{R}_{iik} \left(\frac{1}{\rho_{tr}} \boldsymbol{I} + \sum_{(c,n) \in K_{\mu(i,k)}} \boldsymbol{R}_{icn} \right)^{-1} \boldsymbol{R}_{ijm} \tag{5}$$

In the uplink and downlink scenarios, the results of Ref. [6] are used with MMSE and matched filter as the asymptotic expression of SINRs.

Existing pilot assignment problem is not convex and a suboptimal solution with the combinatorial problem, so it is hard to solve optimally[2]. This formulates the NUM problem as combined optimization shown in Eq. (6), where orthogonality of pilot assignment matrices is a constraint.

$$\max U(\boldsymbol{r}_{ul}, \boldsymbol{r}_{dl}) \quad \text{s. t.}\ \boldsymbol{P}_i \boldsymbol{P}_i^{\mathrm{T}} = \boldsymbol{I} \ \forall i \tag{6}$$
$$\boldsymbol{P}_1, \cdots, \boldsymbol{P}_L \in \{0,\ 1\}^{K \times T_{\mathrm{tr}}}$$

Achievable rates are shown in Eq. (7) for both uplink and downlink of all users in the system represented as utility function parameters in Eq. (6).

$$\boldsymbol{r}_{ul/dl} = \log_2 (1 + \boldsymbol{\gamma}_{ik}^{ul/dl}) \tag{7}$$

We used the Network utility maximization

(NUM) problem on SINR as an asymptotic expression of Ref. [2], where author assigned equal power in downlink scenario. This NUM problem is a combinatorial optimization problem which is NP-Hard, but a suboptimal solution is available.

2 Permutation Encoding

We applied the genetic algorithm to utility function to find the optimized pilot assignment. To optimize the pilot sequences and to get the achievable data rate, we have to optimize pilot sequences. Here notable thing is that conventional genetic algorithm is not applicable because of pilot sequences cannot simply crossover.

Therefore, we need permutation encoding for optimized results of Eq. (6). We took the unitary matrix $\boldsymbol{D}$ which contains the orthogonal pilot as given below.

$$\boldsymbol{D}=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

where we can model the pilot assignment as

$$\boldsymbol{D}_j=\boldsymbol{P}_j\boldsymbol{D}$$

where $\boldsymbol{P}_j$ describes the assignment of the pilot in the jth cell but keeping the orthogonality of the $\boldsymbol{P}_j$ should be preserved as orthogonal.

$$\boldsymbol{P}_j\boldsymbol{P}_j^{\mathrm{T}}=\boldsymbol{I}$$

$$\boldsymbol{P}_j=\begin{bmatrix}0&0&0&0&1\\0&0&1&0&0\\1&0&0&0&0\\0&0&0&1&0\\0&1&0&0&0\end{bmatrix}$$

This translates the matrix $\boldsymbol{P}_j$ has the row assignment of the pilot with different positions like time-frequency block of multi-cell multi-user massive MIMO TDD system. Therefore the row assignments can also be taken as vector permutations of pilot assignments. So we have

$$\boldsymbol{P}_j=\boldsymbol{P}_j^{\mathrm{T}}$$

where $\boldsymbol{P}_j^{\mathrm{T}}=[5\ 3\ 1\ 4\ 2]^{\mathrm{T}}$, which also represents the previously defined $\boldsymbol{P}_j$. Here we need permutation encoding technique for multi-point crossover and mutation. The order is necessary when problems must have a specific arrangement such that no duplication is there because of crossover and mutation. Permutation encoding is only useful for ordered problems. Partially mapped crossover (PMX) can be used in the following case when parents make new children.

Parent 1 : 4 $\underline{2\quad 3\quad 1}$ 5
Parent 2 : 1 $\underline{3\quad 2\quad 5}$ 4
Child 1 : 4 $\underline{3\quad 2\quad 5}$ 1
Child 2 : 5 $\underline{2\quad 3\quad 1}$ 4

Similarly most commonly used mutation on permutation problems is inversion for order changing.

Chromosome 1 : 1 3′ 2 5 4
Chromosome 1^* : 1 4 2 5 3′

where Chromosome 1^* is generated after mutation process. Problems like permutation are one of them in which specific order of chromosome should not change. Hence, orthogonality of pilot sequences is preserved.

3 Simulation Results

We simulated the no-coordination strategy and compared it with position based strategy. The results from Eq. (6) are shown in Fig. 1. Position based strategy gives sub-optimal results over no coordination case[2], but results show the improvement with week user only. It is slightly pessimistic with strong users, but overall performance is acceptable and has little computational cost as compared with other methods.

The simulation results demonstrated for 200 base station antennas in every cell. We considered three cells with five users in each cell, with cell radius of 1.6 km. For the ease of simulation matrix management, we consider the same number of pilot symbols in each cell where all users are uniformly distributed in each cell.

Although position based pilot assignment shows substantial gains, a drawback is that additional pilot symbol slots cannot be used for data. However, there is some limitation of coherence interval in multi-cell multi-user massive MIMO TDD systems because the throughput of the system also depends on coherence time. Larger the

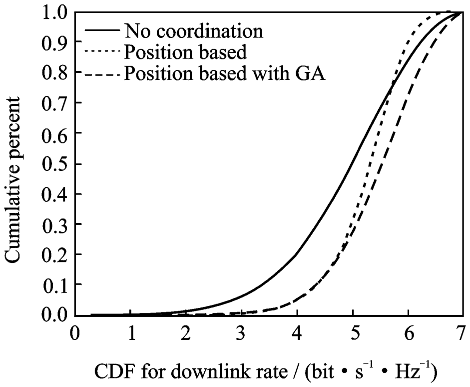

Fig. 1 Comparison of no coordination, position based and optimized position based using genetic algorithm (GA) with the help of permutation encoding

coherence time will affect the users in the system because it slows down the overall convergence time of the system.

4 Conclusions

We simulated coordination strategy problem using combinatorial network utility maximization problem with natural evolutionary optimization technique of genetic algorithm. Genetic algorithm is used with the help of permutation encoding. We found that permutation encoding can be used to implement genetic algorithm for pilot optimization. Results show that it improves the existing results especially in the case of strong user estimation rates.

References:

[1] RUSEK F, PERSSON D, LAU B K, et al. Scaling up MIMO: Opportunities and challenges with very large arrays[J]. Signal Processing Magazine IEEE, 2012, 30(1):40-60.

[2] NEUMANN D, GRUENDINGER A, JOHAM M, et al. Pilot coordination for large-scale multi-cell TDD systems[C]//International ITG Workshop on Smart Antennas. [S. l.]: IEEE, 2014:1-6.

[3] MARZETTA T L. Noncooperative cellular wireless with unlimited numbers of base station antennas[J]. IEEE Trans Wireless Commun, 2010, 9(11):3590-3600.

[4] JOSE J, ASHIKHMIN A, MARZETTA T L, et al. Pilot contamination and precoding in multi-cell TDD systems[J]. IEEE Transactions on Wireless Communications, 2009, 10(8):2640-2651.

[5] HOYDIS J, BRINK S T, DEBBAH M. Massive MIMO in the UL/DL of cellular networks: How many antennas do we need? [J]. IEEE Journal on Selected Areas in Communications, 2013, 31(2):160-171.

[6] JOSE J, ASHIKHMIN A, MARZETTA T L, et al. Pilot contamination problem in multi-cell TDD systems[C]//IEEE International Conference on Symposium on Information Theory. [S. l.]: IEEE Press, 2009:2184-2188.

[7] NEUMANN D, GRUNDINGER A, JOHAM M, et al. On the amount of training in coordinated massive MIMO systems[C]//Sensor Array and Multichannel Signal Processing Workshop. [S. l.]: IEEE, 2014: 293-296.

[8] AGGARWAL S, GOSWAMI P, GARG R. A review paper on different encoding schemes used in genetic algorithms[J]. International Journal of Advanced Research in Computer Science and Software Engineering, 2014, 4(1).

Mr. **Hafiz Ahmad Khalid** received the B. S. degree in Electronics Engineering from COMSATS Institute of Information Technology, Abbottabad, Pakistan, in 2010 and the M. S. degree in Electrical Engineering from University of Engineering and Technology, Taxila, Pakistan, in 2015, respectively. He is currently pursuing the Ph. D. degree in Electronics Science and Technology at Beijing University of Posts and Telecommunications, Beijing, China. From 2013 to 2016, he was a Lecturer in Wah Engineering College, University of Wah, Wah Cantt, Pakistan. His research interest includes the Wireless and Optical MIMO Communication.

(Production Editor: Zhang Huangqun)