DOA Estimation Algorithm with Non-uniform Motion Synthetic Linear Array

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Abstract: Synthetic aperture using moving array is widely used for extending array aperture and improving direction of arrival (DOA) estimation performance. This paper proposes a DOA estimation algorithm with non-uniform motion synthetic linear array. The proposed method successively estimates the phase correction factors of the signals received by a moving linear array and then compensates the manifold to construct a much larger synthetic array. With the synthetic array, the DOA estimation performance is efficiently improved. Besides, due to the successive phase correction factors estimation, the proposed method can be applied regardless of the configuration and motion of the linear array and can release the request of temporal signal coherence. The simulations validate the effectiveness of the proposed method.

Key words: DOA estimation; synthetic array; moving array

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0 Introduction

Direction of arrival (DOA) estimation is an important branch of array signal processing and has vital applications in areas such as communications, radar and sonar[1]. In general, the high resolution DOA estimation of the signal requires the support of large aperture antennas[2]. However, with the tendency of miniaturization of bearing platforms, the antenna aperture is limited so that conventional array antennas cannot meet the high resolution requirements. In practice, a large aperture array can be obtained with a smaller moving array through signal processing, that is called synthetic array (SA). SAs exploit platform motion and use temporal signal coherence to achieve improved estimation performance[3-4].

Extended towed array measurement (ETAM) algorithm is a widely used SA approach in underwater acoustic detection[5], which achieves a desired aperture size using a moving towed array (MTA). The basic idea in this algorithm is a phase correction factor that is used to combine successive measurements of the MTA coherently to extend the array length. While ETAM algorithm is mainly used in underwater acoustic detection, in the electromagnetic signal, there are also many SA applications, such as sources localization using moving coprime array[6] or sources localization in near-field using moving array[7-8], and Cramer-Rao Bound (CRB) of SA is analyzed in Ref. [9].

The algorithms mentioned above basically take the ideal situation of the array uniform motion into account, and this assumption is difficult to achieve in practice. In addition, in the synthesis process, phase noise may affect the phase correction factor estimation[10], which can decrease the estimation performance greatly. Therefore, this paper presents a synthesis algorithm with

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non-uniform linear array in non-uniform motion for high resolution DOA estimation. Firstly, the phase correction factor and the initial DOA angles of the received signals are estimated by two-dimensional MUSIC spatial spectrum search successively. Then, the estimated phase correction factors are taken into the array manifold of the synthesized array, and the DOA angles are estimated by one-dimensional spatial spectrum search. As the phase correction factor is estimated successively, there is no limitation of array motion or configuration, and the requirement of temporal signal coherence is also low. Through the array expansion, the algorithm can effectively enhance the DOA estimation performance.

The rest parts of this paper are as follows. Section 1 introduces the synthetic signal model. In Section 2, we derivate the DOA estimation algorithm with synthetic linear array and analyze its performance. Section 3 shows several numerical simulations and conclusions are obtained in Section 4.

1 Signal Model

Consider a non-uniform linear array consisting of M elements moving along X-axis, as shown in Fig. 1, and the velocity is denoted by v_l $(l=1,\cdots,L)$, where L is the number of sample intervals. Assume K far-field narrowband sources impinging on the array from the bearing $\theta_k(k=1,\cdots,K)$. The sensor positions are denoted by $d_m(m=1,\cdots,M)$ and the inter-element spacing is no larger than $\lambda/2$, where λ is the smallest wavelength of the K sources. We take the first sensor on the left as a reference, so $d_1=0$. Receive the signal during L intervals and the l-th interval starts at time $T_l=lT$.

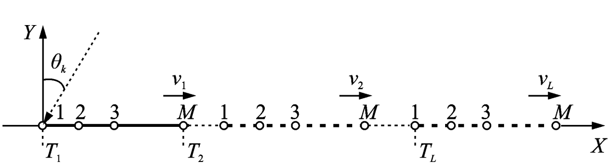

Fig. 1 Synthetic moving linear array model

During each interval, samples last for $\tau(\tau<T)$. Then, the signal received by the m-th sensor at time $t+T_l(t\in[0,\tau],l=1,\cdots,L)$ can be represented as

$$x_m(t+T_l)=\sum_{k=1}^{K}\left[e^{j2\pi f_k(v_lT_l+d_m)\sin\theta_k/c}e^{j(2\pi f_kT_l+\psi_{k,l})}s(t)\right]+\varepsilon_m(t+T_l) \quad (1)$$

where $\psi_{k,l}$ stands for the phase noise of the k-th source during interval T_l and set $\psi_{k,1}=0$; $s(t)$ is the signal received by the reference sensor at time t; $\varepsilon_m(t+T_l)$ is the additive white noise with zero mean and σ^2 variance. Notice that, for simplicity we assume the velocity v_l during each receiving interval is constant and may change at other intervals.

2 The Proposed DOA Estimation Algorithm

2.1 Phase correction factor estimation

From Eq. (1) we can see that there are two parameters to estimate: DOA angles θ_k and the phase noise $\psi_{k,l}$. So we have to estimate them jointly. Denote phase correction factor $\varphi_{k,l}=2\pi f_kT_l+\psi_{k,l}$, which stands for the uncertain phase from the l-th interval to $l+1$-th interval of the k-th source. As there are L intervals, we have $L-1$ pairs phase correction factors to estimate. Once all the factors are estimated, we can use them to properly compensate for the phase fluctuations of the received signal caused by irregularities of array motion and phase noise[5].

According to Eq. (1), the received signals during the l-th interval and the $l+1$-th interval of the k-th source have the following relationship

$$x_m(t+T_{l+1})=e^{j2\pi f_k(v_{l+1}-v_l)T\sin\theta_k/c}e^{j(2\pi f_kT+\psi_{k,l+1})}x_m(t+T_l) \quad (2)$$

and

$$\boldsymbol{X}_l=\boldsymbol{AS}_l+\boldsymbol{E}_l=[\boldsymbol{a}(\theta_1),\cdots,\boldsymbol{a}(\theta_K)]\boldsymbol{S}_l+\boldsymbol{E}_l \quad (3)$$

where $\boldsymbol{a}(\theta_k)=[1,e^{j2\pi f_kd_1\sin(\theta_k)/c},\cdots,e^{j2\pi f_kd_M\sin(\theta_k)/c}]^{\mathrm{T}}$; $k=1,\cdots,K$; $\boldsymbol{X}_l=[x_1(t+T_l),\cdots,x_M(t+T_l)]^{\mathrm{T}}$; $\boldsymbol{E}_l=[\varepsilon_1(t+T_l),\cdots,\varepsilon_M(t+T_l)]^{T}$.

From Eqs. (2)—(3), without considering the noise we can get

$$\boldsymbol{X}_{l+1}=\boldsymbol{AS}_{l+1}=[\boldsymbol{a}(\theta_1)\boldsymbol{a}_{1,l},\cdots,\boldsymbol{a}(\theta_K)\boldsymbol{a}_{K,l}]\boldsymbol{S}_l \quad (4)$$

where $a_{k,l}=e^{j2\pi f_k(v_{l+1}-v_l)T\sin\theta_k/c}e^{j(2\pi f_kT+\psi_{k,l+1})}$, $k=1,\cdots,K$.

Then, consider Eqs. (3)—(4) jointly

$$X'_l = \begin{bmatrix} X_l \\ X_{l+1} \end{bmatrix} = \begin{bmatrix} AS_l \\ AS_{l+1} \end{bmatrix} = B_l S_l = \left(\begin{bmatrix} 1 & \cdots & 1 \\ a_{1,l} & \cdots & a_{K,l} \end{bmatrix} \odot A \right) S_l \quad (5)$$

where $\odot$ stands for Khatri-Rao product; B_l can be seen as the equivalent manifold of the moving liner array from time T_l to time T_{l+1}. During this period, the data covariance matrix can be represented as

$$R_l = E[X'_l X_l^{\mathrm{H}}] = U_{sl} \Lambda_{sl} U_{sl}^{\mathrm{H}} + U_{nl} \Lambda_{nl} U_{nl}^{\mathrm{H}} \quad (6)$$

where U_{sl} and U_{nl} stand for signal subspace and noise subspace; Λ_{sl} and Λ_{nl} are diagonal matrixes which consist of K biggest eigenvalues $\lambda_1, \cdots, \lambda_K$ and the rest $2M - K$ eigenvalues. In practice, the covariance matrix R_l can be calculated as $\hat{R}_l = X'_l X_l^{\mathrm{H}} / L$.

Then, the two dimensional spatial spectrum can be described as

$$P_l(\varphi, \theta) = \frac{1}{b_l^{\mathrm{H}}(\varphi, \theta) U_{nl} U_{nl}^{\mathrm{H}} b_l(\varphi, \theta)} \quad (7)$$

where $b_l(\varphi, \theta)$ is the l-th column vector of manifold B_l; φ and θ are the phase correction factors and DOA angles to estimate during the period from time T_l to time T_{l+1}.

As here we only use two intervals' receiving data, the DOA estimation may not be precise. Once all L-1 pairs phase correction factors are estimated and L intervals' data are corrected, the accurate DOA estimation can be obtained through the whole receiving signal.

2.2 DOA estimation with SA

In the last subsection, all L-1 pairs phase correction factors $\varphi_l = [\varphi_{1,l}, \cdots \varphi_{K,l}]\ (l = 1, \cdots, L)$ are estimated ($\psi_{k,1} = 0$, so $\varphi_{k,1}$ does not need to estimate). Then, we can use them to correct the signals received by same sensors at different time as signals received by different sensors at the same time, which means the array aperture is extended.

According to Eq. (1), we can get that

$$x_{(l-1)M+m}(t) = \mathrm{e}^{-\mathrm{j}\varphi_l} x_m(t + T_l) \quad (8)$$

where $x_{(l-1)M+m}(t)$ can be seen as the signals received by the sensor whose location is $d_{(l-1)M+m}$ at time t. Actually, this sensor does not exist and is called a virtual sensor, but after phase correction from $x_m(t + T_l)$, it is equivalent to the signals that the real array receives.

Then the whole received data can be described as

$$x(t) = [x_1(t), \cdots, x_M(t), \cdots x_1(t + T_l), \cdots, x_M(t + T_l)]^{\mathrm{T}} = A_{sa} s(t) + E(t) \quad (9)$$

where $A_{sa} = B \odot A$ can be seen as the array manifold of synthetic moving linear array, $B = [b(\varphi_1, \theta_1), \cdots, b(\varphi_K, \theta_K)]$, $b(\varphi_k, \theta_k) = [\mathrm{e}^{\mathrm{j}\varphi_{k,1}} \mathrm{e}^{\mathrm{j}2\pi f_k v_1 T_1 \sin\theta_k / c}, \cdots, \mathrm{e}^{\mathrm{j}\sum_{l=1}^{L} \varphi_{k,l}} \mathrm{e}^{\mathrm{j}2\pi f_k v_L T_L \sin\theta_k / c}]^{\mathrm{T}}$, $E(t) = [\varepsilon_1(t), \cdots, \varepsilon_M(t), \cdots, \varepsilon_1(t + T_l), \cdots, \varepsilon_M(t + T_l)]^{\mathrm{T}}$.

Denote $\hat{R}_{xx} = x(t) x^{\mathrm{H}}(t) / ML$ as the covariance matrix of the whole receiving signals $x(t)$, then we can use one dimensional MUSIC algorithm to obtain the DOA estimation and the spatial spectrum can be represented as

$$P(\theta) = \frac{1}{a_{sa}^{\mathrm{H}}(\theta) U_n U_n^{\mathrm{H}} a_{sa(\theta)}} \quad (10)$$

where $a_{sa}(\theta)$ is the column vector of array manifold A_{sa}; U_n stands for the noise subspace of $\hat{R}_{xx}$.

2.3 Performance analysis

According to the analysis above, we can see that after synthetics, the number of SA's elements is extended to ML, and the aperture is $\sum_{l=1}^{L} v_l T_l + D$, where D is the original aperture of the real linear array. In the process of phase correction factors estimation, we use two intervals' data and the dimension of the manifold A_l is $2M \times K$, thus the maximum number of the sources that can be resolved is $2(M-1)$.

The main contributes of the proposed DOA method can be summarized as follows:

(1) The array aperture is largely extended, as a result the DOA estimation performance is also improved.

(2) The phase correction factor is estimated using the adjacent two intervals' data to release the request of signals' time coherence.

(3) The method can be applied in the case of non-uniform linear array and non-uniform motion, which suits the practical requests more.

And the detailed steps of the proposed DOA method are presented as follows:

Step 1 Estimate $L-1$ pairs phase correction factors $\boldsymbol{\varphi}_l=[\varphi_{1,l},\cdots,\varphi_{K,l}]$ $(l=1,\cdots,L)$ using two adjacent two intervals' data successively.

Step 2 Compute the covariance matrix $\hat{\boldsymbol{R}}_{xx}$ of the whole received signals $\boldsymbol{x}(t)$.

Step 3 Take phase correction factor into $\boldsymbol{A}_{sa}$ to eliminate the influence of phase noise.

Step 4 Obtain the DOA estimation by 1-D spatial spectrum searching by Eq. (10).

3 Simulations

In this section, we present numerical simulations to verify the effiectiveness of the proposed DOA estimation method. The parameters for simulations are denoted as follows. The velocity v_l varies from 5 00m/s to 1 000 m/s for different intervals. The interval $T=1$ ms and during each interval the snapshot $N=500$.

The root mean square error (RMSE) is defined as the performance metric

$$\mathrm{RMSE}=\sqrt{\frac{1}{CK}\sum_{c=1}^{C}\sum_{k=1}^{K}(\hat{\theta}_{c,k}-\theta_k)^2} \tag{11}$$

where C stands for the times of Monte-Carlo simulations and $\hat{\theta}_{c,k}$ is the estimation of the k-th angle for the c-th trial. In this paper, we set $C=$ 1 000.

3.1 Verification of resolution ability

In this simulation, we verify the resolution ability of the proposed method. As mentioned in Section 2, the maximum number that can be resolved with an M elements moving linear array is $2(M-1)$. So assume there 2 far-field narrowband sources with the bearing $[10°,15°]$ impinging on a two elements array. We set $L=5$ and receive the signals of 5 intervals. SNR = 10 dB. Fig. 2 shows the phase correction factor and initial DOA estimation from the 4-th interval to 5-th interval. Fig. 3 depicts the spatial spectrum of the total received signals using Eq. (10) after estimating all four pairs phase correction factors and correcting the manifold $\boldsymbol{A}_{sa}$. It is clearly that both two sources can be resolved with the two elements array, while the real two elements array can resolve only one source at most.

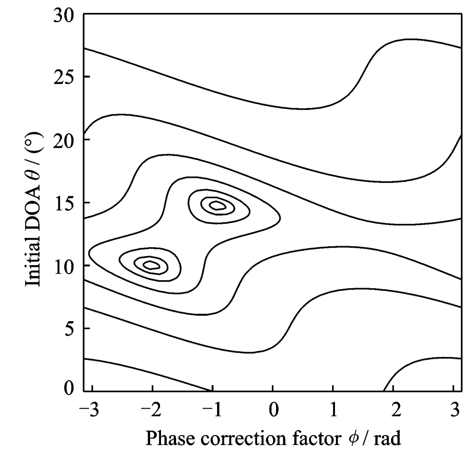

Fig. 2 Phase correction factor and initial DOA estimation

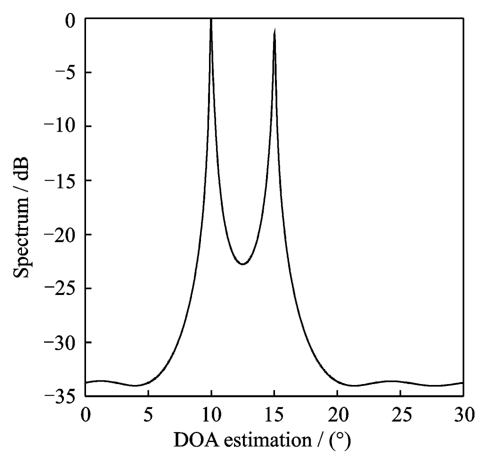

Fig. 3 Spatial spectrum of SA

3.2 Estimation performance of the proposed algorithm

We compare the estimation performance of the proposed approach with synthetic linear array and the method only using real linear array. In this simulation, the real linear array has four elements, and the moving synthetic linear array also applies this real array to sample five intervals so that the total SA has 20 elements. The other parameters are the same as that in the above simulation. The results is shown in Fig. 4 that the estimation performance is greatly improved using SA, as the SA has much more virtual sensors than the real array.

Fig. 5 shows the estimation performance of the proposed method versus different numbers of sample intervals (L). It clearly shows that the DOA estimation performance is getting better

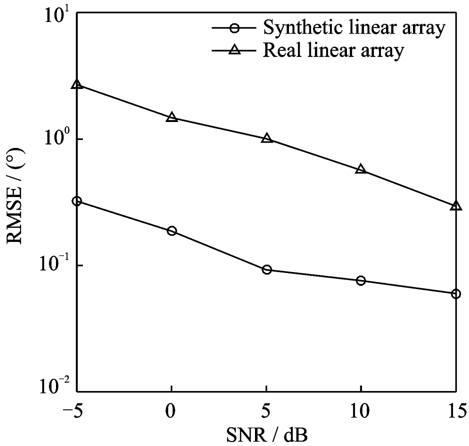

Fig. 4 Comparison of estimation performance with different arrays

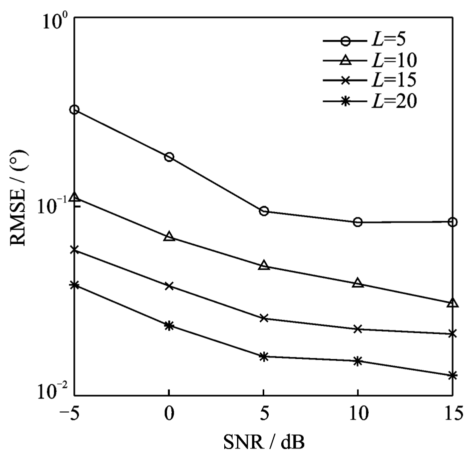

Fig. 5 DOA estimation performance with different L

with L and SNR is increasing because the larger the number of sample intervals is, the more the equivalent array elements will be and the performance improves.

4 Conclusion

Synthetic technology is a useful method to extend array aperture and improve DOA estimation performance. In this paper, the proposed DOA estimation method using a non-uniform linear synthetic linear array through non-uniform motion can meet the request of high resolution DOA estimation and the miniaturization of bearing platforms both. Besides the improvement of the DOA estimation with limited array aperture, the proposed method can also release the request of temporal signal coherence regardless of the motion and configuration of the array. The simulations show that the synthetic moving array has an improved DOA estimation performance over the real array.

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