New Progress in Helicopter Rotor Wake Research

*XU Guohua**, *SHI Yongjie*, *ZHAO Qijun*

National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, P. R. China

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Abstract: Rotor wake analysis, a fundamental research of helicopter technology, has been widely applied for rotor aerodynamic analysis. This paper summarizes the research of different rotor wake models at home and abroad and reviews the development process of rotor wake methods as well as the research achievement obtained in each stage. Then, the new progress of helicopter rotor wake methods is described in detail. It includes constant circulation contours modeling method of rotor wake, pseudo-implicit relaxation iteration and time-accurate solution method, research on aerodynamic interaction characteristics of helicopter rotor/fuselage by wake method, research on the rotor blade-vortex interaction noise and interaction of coaxial rigid rotor aerodynamics by viscous vortex particle method, and application of free wake method to helicopter flight dynamics modeling. In the end, some prospects for the research of helicopter rotor wake method are put forward, which clarifies the ideas for the future development of rotor wake method.

Key words: helicopter; rotor; free wake; viscous vortex particles; aerodynamic characteristics; flight dynamics; wake progress

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0 Introduction

Rotor is the key component of helicopter which provides the lift and control force. One of the remarkable characteristics of typical rotor flow field is the existence of strong rotor tip vortices in the flow field[1]. As shown in Fig. 1, in hover and low-speed forward flight, the trailed vortices pulled out from the rotor tip region roll up in space and form a helical tip vortex[2-4]. The tip vortex plays a dominant role in the whole flow field of the rotor and changes the inflow distribution on the rotor disc, which ultimately has an important impact on the performance, vibration, noise and fatigue characteristics of a helicopter[5-6]. Therefore, the accurate calculation of the rotor wake and aerodynamic characteristics is the basis of the research on aerodynamics and dynamics of the rotorcraft, which is also one of the most important and difficult research subjects in the field of helicopter technology.

Fig.1 Helicopter rotor tip vortex

Up to now, there are mainly two methods used to analyze the aerodynamic characteristics of helicopter rotors: the wake analysis method and the computational fluid dynamics (CFD) method. In recent years, although great progress has been made in CFD method for calculating the rotor flow field, the computation speed and storage requirements are still big challenges for CFD method. When the collective pitch of the rotor is increased sharply, the rotor

*Corresponding author, E-mail: ghxu@nuaa.edu.cn.

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will tend to perform angular or tilting motion, causing the rotor's dynamic and highly unsteady aerodynamic characteristics. The CFD method with higher computational capability has been proposed to deal with these cases[7-10]. Compared with the CFD method, the rotor wake analysis method is directly based on the helical wake to study the aerodynamic characteristics of the rotor. Hence, it has higher computational efficiency and requires much less computational resources, which is very important for the complex rotor aerodynamic load analysis. After years of development, great progress has been made in the rotor wake analysis method for successfully predicting the aerodynamic loads of the rotor in hover and forward flight[11].

The key aspect of rotor wake analysis lies in the wake modelling. Generally, rotor wake models can be divided into three categories: the rigid wake model, the prescribed wake model and the free wake model[12].

The rigid wake model[5] (Fig. 2) is analytical and can be understood easily[1]. It has been widely used for the rotor aerodynamic analysis in the 1960s and 1970s. However, the contraction of wake and the distortion of vortices are not taken into account in this wake model, which makes the wake shape of this model quite different from the real wake shape.

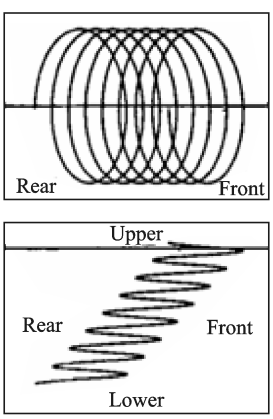


Fig.2 Rotor rigid wake model[5]

Fig.3 is the sketch of hovering rotor wake configuration[2,13]. The prescribed wake model determines the geometrical shape of the wake based on the semi-empirical formulas for the tip vortices and inner vortex sheets summarized from the flow visualization experiments. Hence, the actual contraction of the vortices can be taken into account and the axial displacement of the vortices can be calculated more accurately[5].

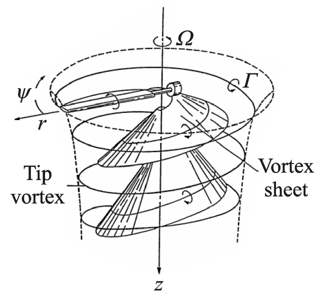


Fig.3 Hovering rotor wake configuration[13]

The free wake (or distorted wake) model[5] (Fig.4) allows the vortices to move freely with the local airflow velocity and the geometrical shape of the wake is solved by the force-free condition of free vortex sheets. Compared with the rigid wake and the prescribed wake model, the free wake model attempts to simulate the change of the actual wake shape by allowing the vortices to move freely, especially by considering the effect of wake on itself and the interaction between blade and wake. Therefore, it became the main method of rotor wake analysis in the 1980s.

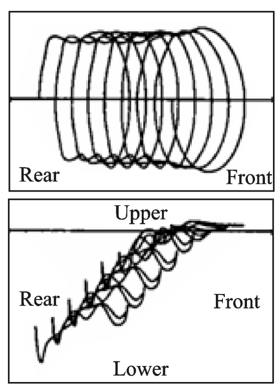


Fig.4 Rotor free wake model[5]

1 A Brief Review of Rotor Wake Research

Researchers began to focus on the rotor wake analysis in the early 1960s[5]. The work in this period included theoretical analysis and experimental measurement. The theoretical analysis was mainly to establish a model for wake calculation, so as to determine the shape and position of the tip vortex, and then to calculate the induced velocity field. At that time, the developed wake analysis method was basically a fixed wake model. Fixed wake is also known as rigid wake. Its main feature is that the wake position can be expressed as an analytical formula, which is easy to be understood. Therefore, in the 1960s and 1970s, it was widely used in the rotor aerodynamic analysis[5,14-16]. However, this type of model does not take into account the contraction and distortion of the wake, so its wake shape is quite different from the real wake shape, which makes it unsuitable for the aerodynamic analysis of rotor blade-vortex interaction[17].

The prescribed wake was gradually developed in the 1970s. In Ref.[14], Landgrebe first presented a practical prescribed wake model. The wake geometry is determined based on the semi-empirical formulas for the tip vortices and inner vortex sheets summarized from the flow visualization experiments. The geometry of the tip vortex and the inner vortex sheet is expressed as a function of rotor parameters and thrust. Later, Kocureck and Tangler et al performed a striated flow visualization experiment of the rotor wake, improved the displacement formula of the tip vortex, and carefully considered the influence of blade torsion[15]. Based on the experimental data, the prescribed wake model can obtain more accurate results within a certain range, and became an effective means for hovering rotor wake and aerodynamic analysis in the 1980s[5,18]. Compared with the fixed wake model, the prescribed wake model can account for the actual contraction of the vortex and calculate the axial displacement of the vortex more accurately. However, its deficiency is that it cannot fully describe the characteristics of the real wake under various flight conditions, and there are limitations to the consideration of wake deformation as the test conditions are finite. In addition, the existing prescribed wake model is only suitable for rotors in hover. Although some researchers have attempted to study the prescribed wake model for forward flight conditions based on the flow visualization experimental measurement, the developed model is difficult for practical application. The wake shape is very complicated in forward flight, which makes it hard to summarize the relationships between the tip vortices and the rotor parameters from the experimental data[5]. In 1985, in Ref.[19], Beddoes developed a modified wake model for forward flight conditions using a different approach. He corrected the wake deformation based on the fixed wake, and calculated the rotor blade aerodynamic loads with the modified wake model. In fact, the Beddoes model is still a prescribed wake model. Its main feature is that analytical formulas are used, and can account for the distortion and contraction of the vortex are considered as the free wake model. Therefore, the Beddoes model can be used for analyzing the rotor blade-vortex interaction.

Compared with fixed wake and prescribed wake model, free (distorted) wake model uses numerical method to solve the wake governing equation. In this way, the transformation of actual wake shape can be better simulated and the helicopter rotor velocity field under complex motion state and instantaneous state can be solved. Hence, the free wake model is a more effective and accurate theoretical model.

Landgrebe was one of the first researchers to analyze rotor free wake. In his free wake model[17,20], the lift line model was used to represent the blade, and the flow field was divided into the far field and the near field. The induced velocity distribution of the far field was calculated only once, and each update step only targeted at the near field. In addition, the update of wake position introduces explicit time stepping algorithm, and the criterion of convergence is based on the period wake shape. The number of time steps calculated by Lagrangian wake is greatly affected by the initial wake shape and flight conditions, and the convergence is not satisfactory. Later, it was found that a very important problem for free wake solution was the stability of the algorithm for wake update[21]. In Ref.[22], Sadler used an explicit forward time integration algorithm to calculate the rotor wake in forward flight. The wake model developed by Sadler near the blade is composed of trailing vortex and bound vortex, and the solid rotating vortex core model is used. It belongs to the "starting process" method, that is, free wake is generated by the starting process similar to rotor blade. The wake method has been used to calculate the aerodynamic load of rotor. Its main shortcoming is that it is not easy to obtain the convergent wake solution, and the calculation cost is high.

Compared with the fixed and prescribed wake, the free (distortion) wake uses a numerical solution to solve the wake governing equation, which can better simulate the change of the real wake shape, and solve the helicopter rotor velocity field in complex motion state and transient state. It is a more efficient and accurate calculation model.

Miller established a simplified hovering free wake model[23]. All vortex lines were considered in the near wake, while the far wake was represented by a semi-infinite cylinder, with three vortex lines at the tip, root and middle as the transition wake. Egolf simulated wake trailing vortexes and bound vortexes into quadrilateral vortex cell[24]. The radius of vortex core was determined according to the initial vortex cell. The criterion for wake convergence is the time-averaged drag coefficient, not the shape of the wake. In 1987, Bliss used curved vortex elements to analyze the free wake of the forward flying rotor[25]. The wake solution uesd a prediction-correction algorithm. However, this method is only effective for high speed forward flight, because the numerical stability is still a problem at low speed. It is worth mentioning that Johnson used free wake in his famous computing softwares CAMRAD and CAMRAD II[26], and He et al introduced free wake into their well-known FLIGHLAB flight dynamics analysis software[27]. After years of efforts, great progress had been made in the research of rotor free wake, which has gradually developed into two main calculation methods of free wake: time marching method (time stepping method) and relaxation iteration method[11-12,17].

In recent years, the free wake method has been further developed. In 1993, Crouse and Leishman introduced the relaxation method, which used the central difference of space to strengthen stability in the second-order prediction-correction method with periodic boundary conditions[28]. In Refs.[29-31], Bagai and Leishman established a pseudo-implicit prediction-correction (PIPC) method. In Refs.[32-34], Bhagwat and Leishman developed a "time-accurate" free wake method by using the central difference method and the three-step backward difference method. The method takes the convergent wake shape calculated by the relaxed iterative free wake method as the initial wake and time stepping calculation for wake vortex lines. In Refs.[35-37], Quackenbush, Lam, and Bliss et al, established a new vortex / surface interaction methods for the prediction of wake-induced airframe Loads based on their curved vortex element and constant circulation contour description method of wake.

In recent years, the rotor free wake method has been studied in China[38-47]. For example, Wang Shicun and Xu Guohua[38-39] established a hovering and forward-flying free wake model with the time-stepping method, which was used to solve the aerodynamic load of the blade. Cao Yihua[40] started from the generalized wake and established a calculation method for the vortex wake and downwash flow field of the rotor. Zhao Jinggen and Gao Zheng[42-44] applied five-point central difference method to establish a relaxed iterative free wake model, which was applied to the research on the characteristics of rotor/fuselage interaction flow field. Li Chunhua[11] presented a new two-step second-order prediction-correction difference algorithm, which established a time-accurate rotor free wake model, and was applied to the study of unsteady aerodynamic characteristics of rotor.

Due to the dominant characteristics of rotor tip vortices in the rotor flow field, as well as the efficiency and effectiveness of wake analysis, up to now, many researchers have further improved and developed rotor wake analysis[42-53]. Especially, in 2009, Zhao and He[46-47] proposed a new method for vortex particle analysis of rotor wake, On the basis of their work, Refs.[48-53] further studied the vortex particle method and its application to the interactional aerodynamics, BVI noise computations and

Flight dynamics modelling.

2 Recent Research Progress of Helicopter Rotor Wake

The following shows the research progress of helicopter rotor wake analysis, and it includes constant circulation contours modeling method of rotor wake, pseudo-implicit relaxation iteration and time-accurate solution method, research on aerodynamic interaction characteristics of helicopter rotor / fuselage by wake method, research on the rotor blade-vortex interaction noise and interaction of coaxial rigid rotor aerodynamics by viscous vortex particle method, and application of free wake method in helicopter flight dynamics modeling.

2.1 Constant circulation contours of rotor wake modeling method

The analysis of the rotor wake involves two aspects: the description of the wake structure and the solution of the wake geometry. However, in the past, the main emphasis was on the second aspect, and the research on the first aspect was rarely studied. The vortex grid method is usually used in the wake analysis to describe the wake of the rotor, that is, the vorticity is decomposed into the trailing vorticity and the shed vorticity. This conventional description does not actually provide structural information of the vortex system, such as the concentration of the vorticity distribution, the winding of the vortex system, and the like. In 1987, Bliss et al. proposed an innovative constant circulation contour description method[25]. In this way, the vortex lines are placed on equal-strength lines, and the vorticity of each vortex line is constant (Fig. 5). Bliss uesd contours of constant sheet strength to describe the wake vorticity field as a function of blade load[25]. The vorticity contained in any two contours is the same, the vorticity along the contour is constant, and the local vortex vectors are always tangent to the contour. The constant circulation contour defined in this way is not equivalent to the true vorticity line of the wake vortex. The constant circulation contour is actually a vortex line that coincides with a certain vorticity line, and has the same vortex intensity, or it represents a scroll around the vorticity line. The contours of constant sheet strength are a set of vorticity lines. Because of its same intensity, the vorticity field of the wake vortex can be understood from the degree of density and the trend of the contour. Compared to the vortex grid, it is not only intuitive and reasonable, but also reduces the number of vortex elements by half. The degree of density and orientation of such a set of vortex lines on the wake vortex surface provides a visual image of the vortex field distribution of the wake vortex surface. The use of constant circulation contours to describe the wake structure, because it is directly based on the vortex field, which is more natural and reasonable than the vortex grid description[54-55]. Moreover, after the wake is discretized, the number of vortices can be reduced by half because constant circulation contour automatically is counted to the amount of the bound and trailing vortex[56-57]. The update of the wake description method has brought some new understandings on the rotor wake structure. For example, when in high-speed forward flight, the tip of advancing blade is found to have a negative vorticity area.

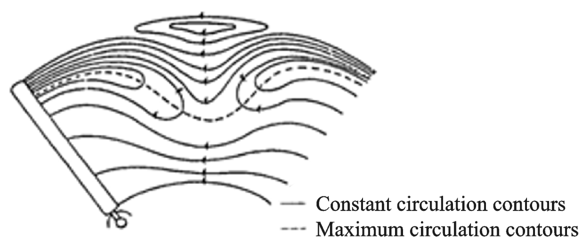


Fig.5 Constant circulation contours to describe wake structure[25]

An important problem often encountered in free wake solving is the numerical instability of the wake solution in hovering and low-speed flight states. In order to get convergent solutions, artificial numerical damping is often introduced in some previous wake analysis. In 1990, Miller & Bliss proposed a "periodic inverse solution" approach[58]. This method uses the non-Lagrangian method to describe the wake geometry. By imposing a periodic solution in the wake solution, the numerical instability is greatly improved, but the amount of calculation of the "periodic inverse solution" method is too large[5]. It is not suitable for actual wake calculation. In 1993, Crouse & Leishman first introduced the predictor-correction method in the wake solution and improved the numerical stability of the wake solution[28].

2.2 Pseudo-implicit relaxation iteration method and time-accurate solution method for rotor wake analysis

In Refs.[29-31], Bagai and Leishman developed a pseudo-implicit predictor-corrector (PIPC) relaxation algorithm with five-point central difference in space for the numerical solution of the nonlinear partial differential equations that govern the helicopter rotor free wake. The scheme is used to predict the locations of the vortices trailed by the rotor blades in hover and forward flight, and to predict the rotor induced velocity field. Their work is on the development of a robust and versatile free-wake scheme that can be used on a routine basis and incorporated into comprehensive rotor analyses.

In their work, numerical convergence, stability, and accuracy characteristics of the methodology are also discussed, and the pseudo-implicit numerical scheme is compared with conventional explicit algorithms to show better stability and convergence trends. Their method is also extended to investigate the induced velocity fields of tandem, side-by-side and coaxial rotor configurations[31].

The method is applied to examine rotor wake geometries for a wide variety of geometric rotor configurations and flight conditions, including variations in advance ratio, rotor thrust, number of blades and rotor shaft angles[31]. Comparisons between predictions and experiment for wake geometries, wake boundaries, rotor induced inflow distributions and flow field velocities, help validate the predicted capabilities of the method. The good agreement between the predictions and experiment are demonstrated.

In their free wake method, blade Weissinger-L model is used and the strengths of the blade bound vorticity depend upon the aerodynamic lift produced by each segment of the blade, as shown in Fig. 6. The lift is obtained from the angle of attack at each control point, which in turn depends upon the induced velocities. The contribution of the far-wake consists of the concentrated tip vortices.

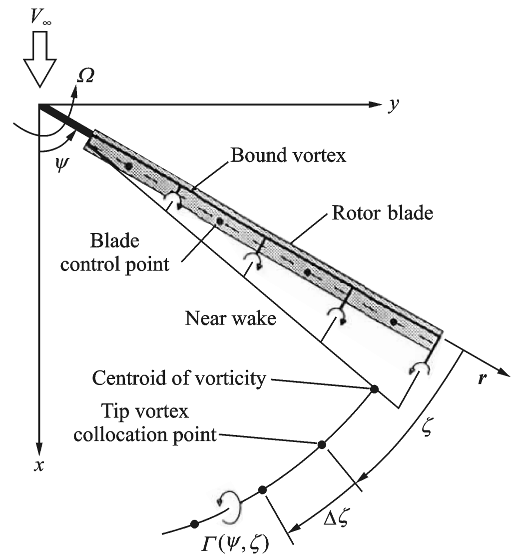


Fig. 6 Blade Weissinger-L model as well as near and far wake[29]

Bagai and Leishman gave the differential equation which governs the motion of a vortex filament in the rotor flow field by the vorticity transport theorem as follows[30]

$$\frac{\mathrm{d}\boldsymbol{r}(\psi,\zeta)}{\mathrm{d}t}=\boldsymbol{V}\big(\boldsymbol{r}(\psi,\zeta)\big) \tag{1}$$

where $\boldsymbol{r}(\psi,\zeta)$ defines the position vector of an element, p, lying on a vortex filament which is trailed from a rotor blade located at an azimuth ψ at time t (see Fig. 7).

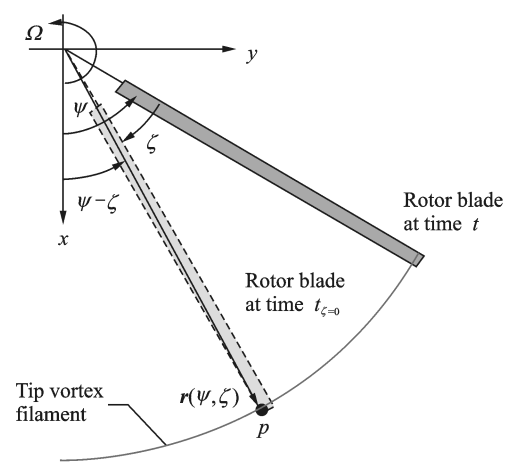


Fig.7 Definition of $\boldsymbol{r}(\psi,\zeta)$[30]

Refs.[29-31] give a PDE governing the geometry of the rotor tip vortices and it is introduced as

$$\frac{\partial r(\psi,\zeta)}{\partial\psi}+\frac{\partial r(\psi,\zeta)}{\partial\zeta}=\frac{1}{\Omega}V\left(r(\psi,\zeta)\right) \tag{2}$$

where the ψ coordinate represents the rotor azimuthal position and is, therefore, a time variable. The variable ζ represents the angular location of an element on a tip vortex fllament trailed from the blade, and essentially represents a spatial coordinate.

Then, a numerical integration scheme is developed to transform the PDE's into finite difference equations (FDE's). The formulation of the finite difference equations is described by transforming the discretized physical domain into a computational domain. As shown in Fig.8, the abscissa of the computational domain represents the ψ direction and the vertical coordinates the ζ direction.

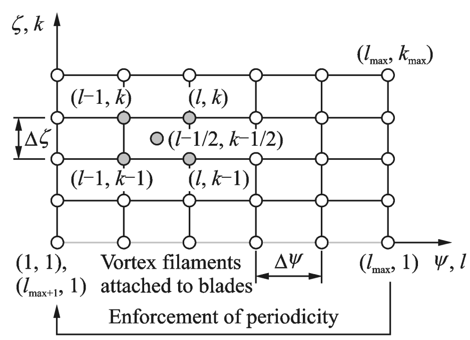


Fig.8 Wake updating performed by pseudo-implicit predictor-corrector (PIPC) scheme as Ref.[31]

Predictor

$$\begin{aligned}\boldsymbol{r}_{l,k}^{n}=&\boldsymbol{r}_{l-1,k-1}^{n}+\left(\boldsymbol{r}_{l,k-1}^{n}-\boldsymbol{r}_{l-1,k}^{n}\right)\left(\frac{\Delta\psi-\Delta\zeta}{\Delta\psi+\Delta\zeta}\right)+\\&\frac{2}{\Omega}\left(\frac{\Delta\psi\Delta\zeta}{\Delta\psi+\Delta\zeta}\right)\cdot\\&\left(\boldsymbol{V}_{\infty}+\frac{1}{4}\left(\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l-1,k-1}^{n-1}\right)+\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l-1,k}^{n-1}\right)+\right.\right.\\&\left.\left.\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l,k-1}^{n-1}\right)+\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l,k}^{n-1}\right)\right)\right)\end{aligned} \tag{3}$$

Corrector

$$\begin{aligned}\boldsymbol{r}_{l,k}^{n}=&\boldsymbol{r}_{l-1,k-1}^{n}+\left(\boldsymbol{r}_{l,k-1}^{n}-\boldsymbol{r}_{l-1,k}^{n}\right)\left(\frac{\Delta\psi-\Delta\zeta}{\Delta\psi+\Delta\zeta}\right)+\\&\frac{2}{\Omega}\left(\frac{\Delta\psi\Delta\zeta}{\Delta\psi+\Delta\zeta}\right)\cdot\\&\left(\boldsymbol{V}_{\infty}+\frac{1}{4}\left(\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l-1,k-1}\right)+\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l-1,k}\right)+\right.\right.\\&\left.\left.\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l,k-1}\right)+\boldsymbol{V}_{\text{ind}}\left(\boldsymbol{r}_{l,k}\right)\right)\right)\end{aligned} \tag{4}$$

where $\boldsymbol{V}_{\text{ind}}$ is the induced velocity by all wake vortex filament.

Fig.9 is the predicted wake geometries comparison for the single-rotor configurations by the PIPC free-wake methodology using the pseudo-implicit and explicit methods.

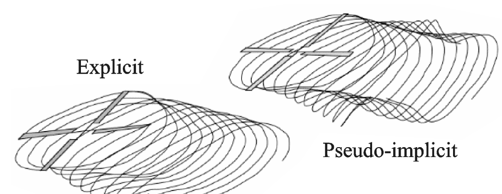


Fig. 9 Isometric views of the predicted wake structure, pseudo-implicit vs. explicit schemes, three turns of free wake[31]

Different from Bagai and Leishman's pseudo-implicit relaxation iteration method, Bhagwat and Leishman present a time-accurate solution method for rotor wake analysis in Refs.[32-34]. In Bhagwat and Leishman's wake solution method, a second-order accurate predictor-corrector type algorithm is developed in order to obtain a time-accurate solution of the vortex wake generated by a helicopter rotor. The rotor blade flapping solution is fully integrated with the wake geometry solution using the same time-marching algorithm. Their method can be used to predict the locations of wake vortex filaments at transient flight conditions, where the rotor wake may not be periodic at the rotational frequency. As numerical examples, the method is used for the prediction of the rotor induced velocity field and blade air loads during helicopter transient flights and maneuvers. A technique for increasing the order of accuracy is also added.

Bhagwat and Leishman[34] gave two newly developed time marching algorithms. The first time-marching algorithm, which is referred to as the PCC scheme, uses a five-point central differencing scheme, and the second algorithm, referred to as the PC2B scheme, uses a central difference scheme for the spatial (ζ) derivatives, but a second-order backward difference scheme for the temporal (ψ) derivatives. Figs. 10, 11 respectively show the node schematic diagram of these two formats, for both the time-marching algorithms, the governing equations are evaluated up to a second-order accuracy.

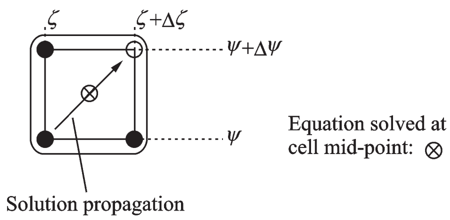


Fig.10 PCC time-marching algorithm[34]

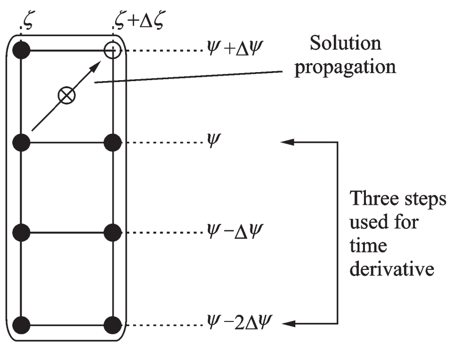


Fig.11 PC2B time-marching algorithm[34]

In Ref.[34], Bhagwat & Leishman applied the PC2B algorithm to free-vortex wake geometry solution in forward flight. The numerical case, a forward flight case at an advance ratio $\mu = 0.1$ to verify wake solution convergence. These calculations are performed for a four-bladed rotor. A forward shaft tilt angle of 3° is used with a rotor thrust coefficient of CT = 0.008. The top view of the wake geometry is shown in Fig.12 for different levels of discretization ($\Delta\psi$). It is also interesting to note that the strong distortions in the wake structures are well-preserved.

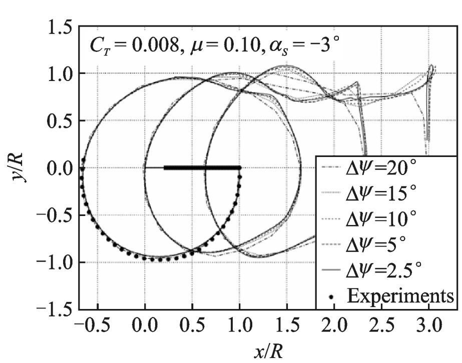


Fig.12 PC2B algorithm applied to free-vortex wake geometry solution in forward flight[34]

In Fig. 13[34], Bhagwat & Leishman further used their time-marching free-vortex wake method to evaluate the results for a ramp increase in rotor collective pitch angle at a rate of 200 °/s, the experimental data came from the measurements of Carpenter & Friedovich[59]. Fig.13(a) is the input of blade pitch angle. Note that the collective input in the

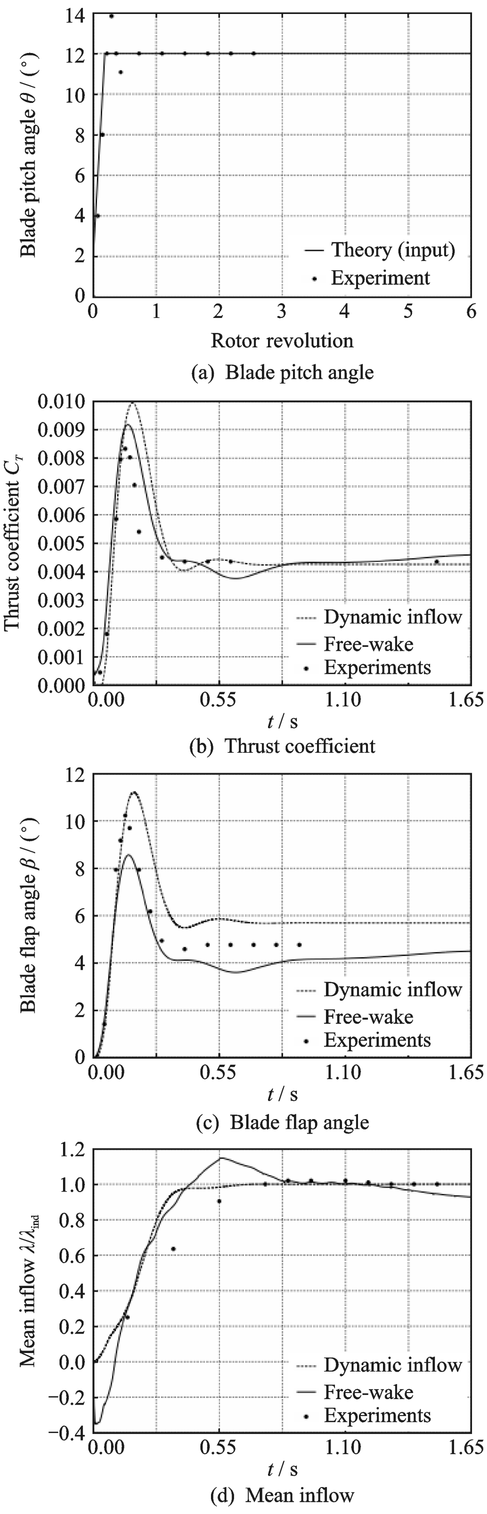


Fig.13 Ramp increase in rotor collective pitch angle at rate of 200 °/s (Experimental measurements in Ref.[59])

experiments shows some oscillatory behavior, because of blade torsion. As a result, the transient overshoot in flapping is slightly under-predicted. However, the predicted rotor thrust response and build-up in the inflow show good agreement with the experimental measurements. The slow build-up of inflow is found to be a result of the transient evolution of the trailed rotor wake, i.e., a circulatory effect. The dynamic inflow response approximately resembles the first-order linear system, the inflow reaches the steady-state value in about three time periods, i.e., after six rotor revolutions or 0.81 s. This effect is predicted very well by the time-accurate wake algorithm. In summary, a good correlation with measured rotor thrust, inflow and blade flapping angles are obtained for a ramp rate changes in collective pitch.

2.3 Viscous vortex particle method for rotor wake analysis

Refs.[46-47] study the viscous vortex particle model with higher wake capture accuracy and no physical dissipation. For the unsteady flow field analysis of the rotor, the wake vortex is spiral and the vorticity is mainly concentrated on the tip of the larger Mach number, and the external flow field is regarded as incompressible. Therefore, this incompressible non-viscous flow can be described by Navier-Stokes equation[47]

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}=-\frac{\nabla p}{\rho}+\nu\nabla^2\boldsymbol{u} \tag{5}$$

where $\boldsymbol{u}$ is the flow rate, p is the pressure, ρ is the density, v is the kinematic viscosity coefficient, and ∇ is the differential operator. The traditional rotor CFD method uses the finite volume method or the finite difference method. The numerical dissipation problem in the discrete format will cause the non-physical attenuation of the wake vortex of the spatial flow field, thus reduces the accuracy of the flow field simulation. In addition, the grid quality and time step requirements are high during the calculation process, and the calculation time becomes another important factor limiting the grid-based rotor CFD method.

In the viscous vortex particle method, the vorticity dynamics equation in the form of vorticity-velocity is more targeted for the simulation of the helicopter flow field dominated by the wake vortex. For the incompressible non-viscous flow, the vorticity transport equation can be obtained by using the formula described by the Navier-Stokes equation[47-48] to calculate the rotation of the left and right sides.

This equation, together with the vortex kinematics equation forms the governing equation for the viscous vortex particle method. The vorticity transport equation is then solved. In order to ensure the numerical stability of the governing equation solution, integral solution is performed using a prediction correction format applied by a second-order Adams-Bashforth (A-B) explicit prediction format combined with the Adams-Moulton (A-M) implicit format[46-47].

In the process of simulating the rotor wake using the viscous vortex particle method, the induced velocity of all vortex particles need to be calculated by integrating each time step. When the number of vortex elements is large, in order to improve the computational efficiency, an acceleration algorithm is added to reduce the number of iterations to speed up the calculation process, and the viscous vortex particle method is better applied to rotor design. The acceleration algorithm used in the reference is the fast multipole method (FMM)[47-48]. The fast multipole method has high computational efficiency. When applied to the calculation process of self-induced velocity and velocity gradient, compared with the direct calculation, the computational magnitude $O(N^2)$ can reduce the computational complexity to $O(N)$.

Fig.14 shows the calculation for the state where the pitch angle is 8.94°. The Mach number of the tip is 0.617. Firstly, the transient distribution of the vortex is given after the calculated convergently. It can be seen that the rotor wake forms a typical contracted geometric distortion shape, which indicates that the method can effectively simulate the wake transport process of the rotor.

Furthermore, combining the viscous vortex particle method with the CFD method, a VVM/CFD hybrid method is developed in Ref.[48]. The spatial vorticity distribution of the UH-60 helicopter rotor flying state is shown in Fig.15. The vorticity of the CFD calculation domain is represented by the vortex cloud image on the outer surface of the blade grid, and the vorticity of the viscous vortex calculation domain is represented by the vorticity iso-sur-

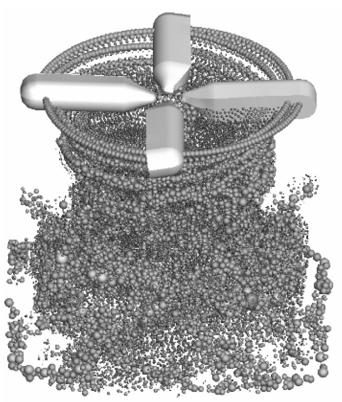


Fig.14 Obtained transient distribution diagram of vortex element in rotor wake ($\phi_{0.7} = 8.94°$)[48]

face. It can be seen from the figure that the VVM/CFD hybrid method built in Ref.[48] can still capture the distortion of the rotor wake and the interaction between the tip vortex in forward flight, although no test data of wake in the flight test of UH-60 can be used to verify the wake prediction. It is obvious that the hybrid method of this paper still maintains the advantage of the Viscous Vortex method in wake prediction, which can accurately predict the rotor wake in the forward flight state. At the same time, it can be seen that, the vorticity concentration regions of the two computational domains are completely consistent, which is similar to the results of the hovering state and indicates that the inter-domain information exchange strategy adopted in this method is also effective in forward flight.

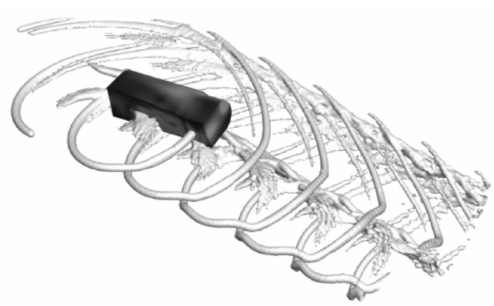


Fig. 15 Vorticity diagram calculated by hybrid method of VVM/CFD

In Ref.[48], as a verification of the rotor load prediction capability, a comparison of the normal force coefficient distributions at different sections of the blade is further given in Fig.16. It can be seen from the figure that although the assumption of rigid

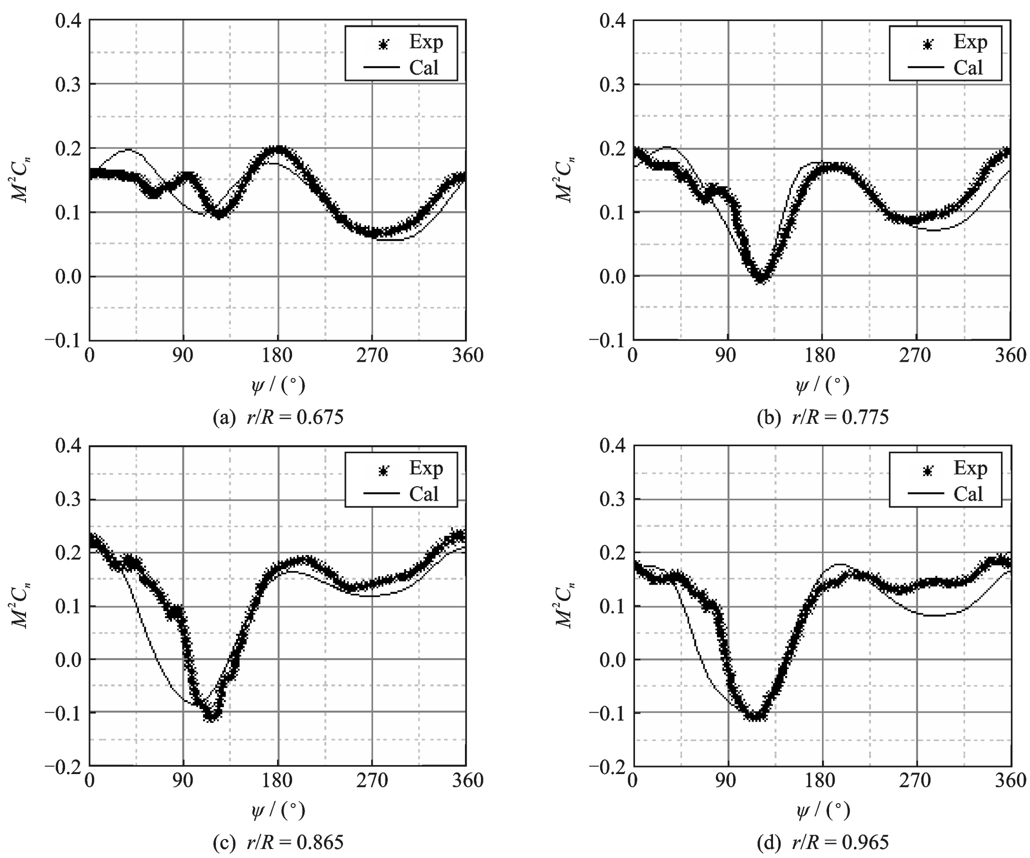


Fig.16 Normal force coefficient distributions at different sections of blade[48]

blade is adopted in the calculation model and the flapping dynamic model only considers the first-order influence, as a whole, the calculated values in this paper are still in good agreement with the flight test results, both in magnitude and in azimuth, despite it is well known that accurate calculation of the rotor aerodynamic load distribution is challenging[48].

The above calculation and analysis on UH-60A rotor in forward flight shows that the calculated rotor loads have higher accuracy as a whole.

2.4 Aerodynamic interaction of helicopter rotor/fuselage based on wake method

In Ref.[44], a full-span free wake model and a tip vortex free wake model are established respectively. In order to verify the correctness of the rotor vortex system model, the flow field of the independent rotor state in the Georgia Tech rotor/fuselage aerodynamic interaction experiment[60] is calculated, and the calculation results of the two wake models and experimental data are compared and analyzed. The coordinates and calculation points are shown in Fig.17.

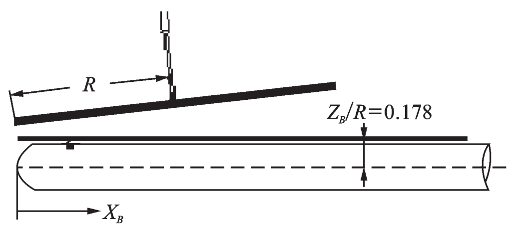


Fig.17 Calculation node position coordinate diagram

Fig.18 shows the calculation results and experimental data comparison of the two wake models. The solid line in the figure is the result of full-span free wake model, while the dotted line is the result of rolled up tip vortex model. It can be seen from Fig.18(a) that the horizontal velocity prediction results of the full-span free wake model and the tip vortex model are basically consistent. However, near the nose, the results of the tip vortex model are obviously larger, while the results of the full-span free wake are closer to the experimental values. It can be seen from the comparative analysis of the vertical flow field velocity in Fig.18 (b) that the results of the full-span free wake model and the tip vortex model are relatively consistent with the experimental data, but the results of the tip vortex model are

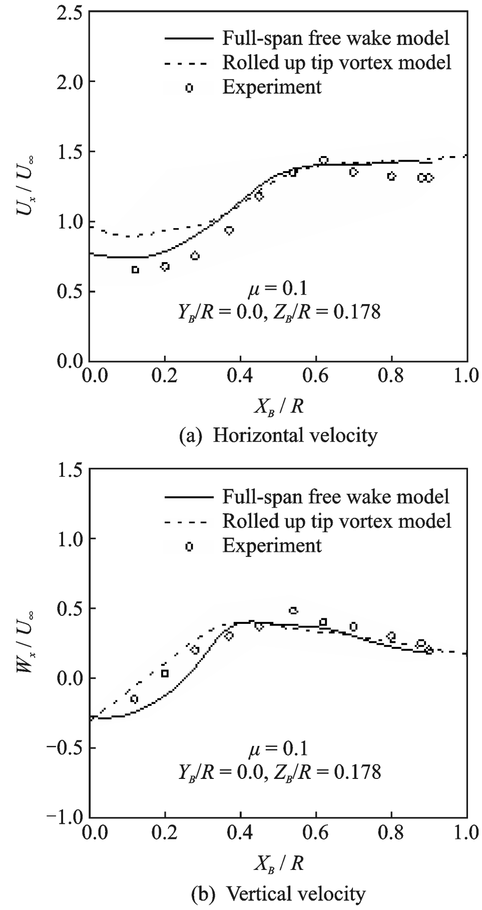
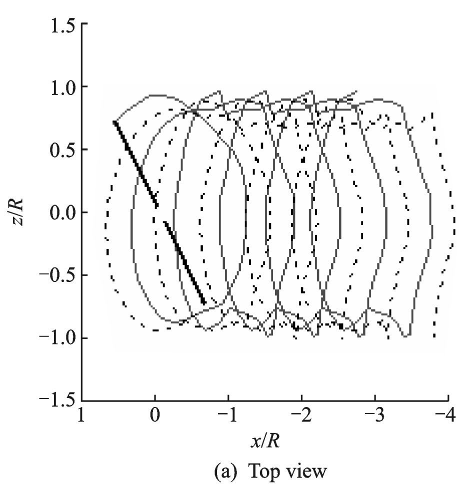


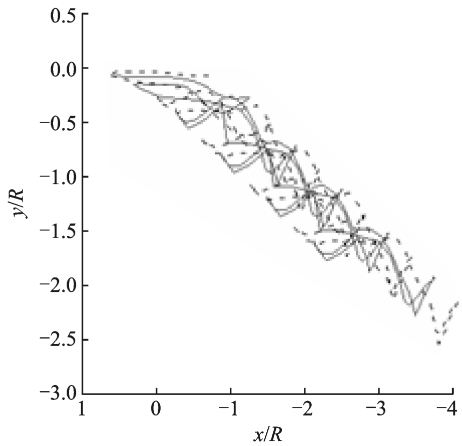
Fig. 18 Comparison of calculated results on average induced velocity along a straight line under the disc[44]

better.

The wake of two kinds of rotors flying forward at a typical low speed forward flight was calculated using the tip vortex model. The wake shapes after convergence are shown in Fig.19.

Ref.[44] also further developed a rotor/fuselage aerodynamic interaction analysis method based on the established free wake model. In the aerody-





(b) Lateral view

Fig.19 Calculation result of rotor of Georgia Tech test model (μ=0.075)[44]

namic interaction analysis of his rotor/fuselage, referring to Refs.[35-37], a simple and efficient analytical model is established by using analytical and numerical matching method (ANM) to analyze and calculate the vortex/surface interaction close to it.

Fig.20 is the comparison between the calculated value and experimental value of the average velocity field of Georgia Tech rotor / fuselage model[44]. The calculated point in the figure is 12.7 mm away from the top line of the fuselage, the horizontal and vertical velocity distributions are compared. As can be seen from the figure, in this case, the predicted values of most points are in good match with the experimental values. In comparison, the coincidence degree along the vertical direction is better. Only the maximum speed at the fuselage head was predicted to be low.

Fig.21 is the comparison between the calculation results and experimental results of the unsteady velocity field of Georgia Tech combination model in Ref.[60]. In Fig.21, the calculation point is located near the head of the fuselage. As this point falls outside the range swept by the rotor wake, the calculated results are consistent with the experimental results on the whole. Among them, the horizontal direction of the coincidence is better, the amplitude of the prediction is more accurate, but there is some deviation in the phase.

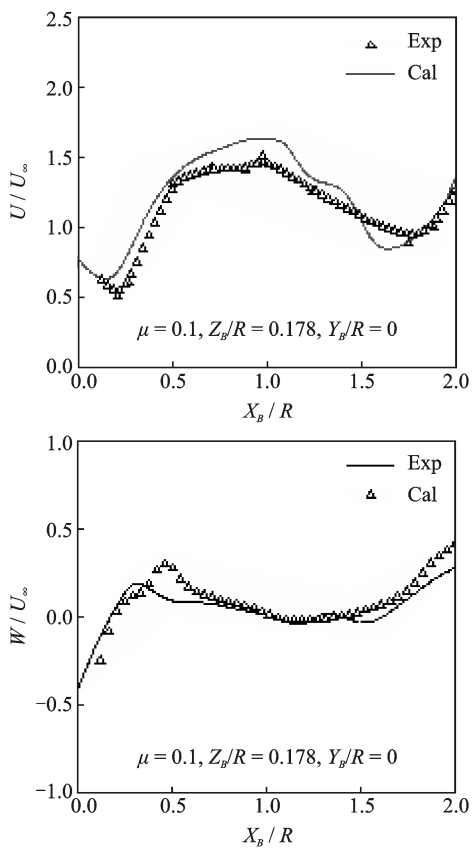


Fig. 20 Comparison between the calculated value and the experimental value of the average velocity field of Georgia Tech rotor/fuselage model[44]

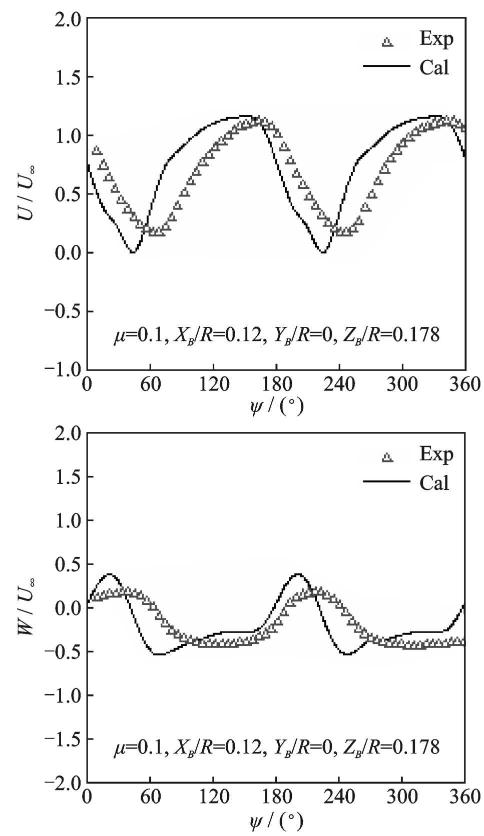


Fig. 21 Comparison between the calculation results and experimental results of the unsteady velocity field of Georgia Tech combination model[44]

2.5 Rotor BVI noise research based on viscous vortex particle method

In Ref.[51], the vortex particle method is further applied to the research of aerodynamic noise of rotor[46].

Based on the CFD/ viscous vortex particle hybrid method developed by Ref.[51] and further combined with the FW-H equation, a method which is suitable for the prediction of aerodynamic noise of rotor unsteady BVI are constructed. In order to fully verify the reliability and accuracy of the method, Ref.[51] calculated and simulated the sound pressure time history of AH-1 /OLS rotor at different observation points, and compared it with the experimental value. The results show that the noise hybrid calculation model developed in Ref.[51] is effective and can accurately predict the time history of noise pressure under rotor blade vortex interaction.

In Ref.[51], Figs.22,23 respectively give the calculation results of the time history of sound pressure at different observing points of the AH-1/OLS rotor under two blade-vortex interaction conditions.

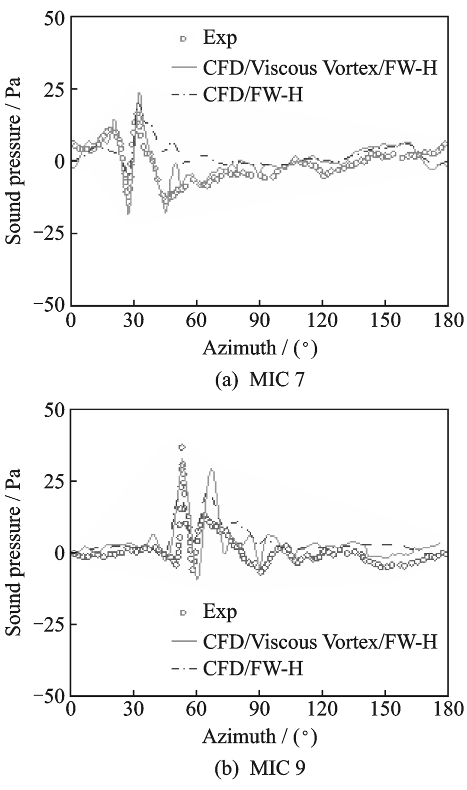


Fig. 22 Calculation and comparison of noise pressure of AH-1/OLS rotor under the condition of typical blade-vortex interaction "10014"

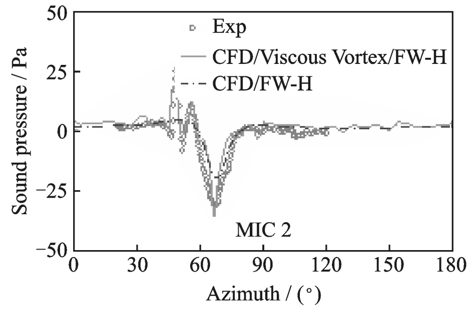


Fig.23 Calculation and comparison of noise pressure of AH-1/OLS rotor under the condition of typical blade-vortex interaction "10017"

In order to make a better comparison analysis, the results obtained by the CFD / viscous vortex particle/FW-H hybrid method and the CFD/FW-H method are presented, the experimental values are expressed in small circles. It can be seen from Fig.22 that the noise characteristics of each observation point can be distinguished by the method of Ref. [51], that is to say, the positive peak sound pressure of AH-1/OLS rotor at the 45° azimuth of the forward side can be calculated accurately in both phase and amplitude, which is the typical feature of blade-vortex interaction. The blade-vortex interaction noise dominates at MIC 7 and MIC 9, and the sound pressure signals at each observation position are composed of positive and negative pulses, which is caused by the interaction on the forward side and the backward side of rotor disk, respectively. In addition, there is an obvious negative pulse sound pressure near the 60° azimuth of MIC 2 observation position, which is mainly caused by the blade thickness noise. Comparing with the time history of noise pressure calculated by the CFD/FW-H equation, it can be seen that the better noise peak prediction in phase and amplitude can be captured by the method of CFD/viscous vortex particle/FW-H hybrid method. This is due to the fact that the viscous vortex particle mixing method in Ref.[46] can accurately predict the rotor wake and blade surface loads in the presence of blade-vortex interaction. This also demonstrates the ability of the method in Ref.[51] to accurately predict the aerodynamic and noise characteristics of rotor blade-vortex interaction.

2.6 Application of free wake method to helicopter flight dynamics modeling

In Refs.[49-50], Li Pan and Chen Renliang analyzed the shortcomings of existing rotor wake and aerodynamic models in flight dynamics modeling and application, and on this basis, a rotor unsteady free wake model for helicopter maneuvering flight is developed. Specifically, the unsteady free wake model of the rotor and the unsteady dynamic stall aerodynamic model of the airfoil are combined. At the same time, the experiments of collective pitch, cyclic pitch and the sudden increase of rotor angular velocity are carried out on a whirling-arm testing apparatus to simulate the maneuvering flight environment and measure the response of the rotor tension and moment. The experimental results are used to validate the unsteady aerodynamic model of the rotor and the validity of the free wake model under the condition of maneuvering flight. The relationship between the dynamic distortion of rotor wake and the inflow of rotor disc and the transient characteristics of rotor load are studied.

The position of the collective pitch rod and the required power of the rotor are given with the change of flight speed, the results include two cases: free wake model and dynamic inflow model. It can be seen from Figs.24,25 that the balance values of the total pitch of the free wake model and the required power of the rotor agree well with the measured values in the whole flight speed range. When using dynamic inflow model, the total rotor pitch and required power are underestimated in hover and low-speed forward flight. The results show that the

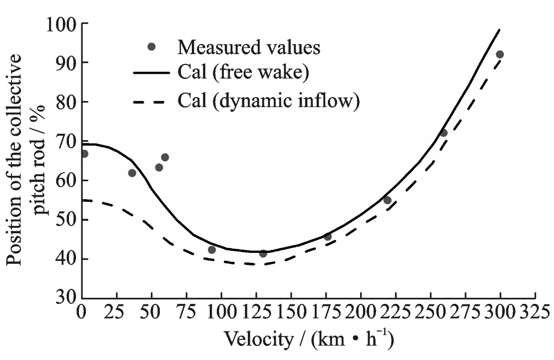


Fig.24 Influence of wake model on balance of rotor collective pitch[50]

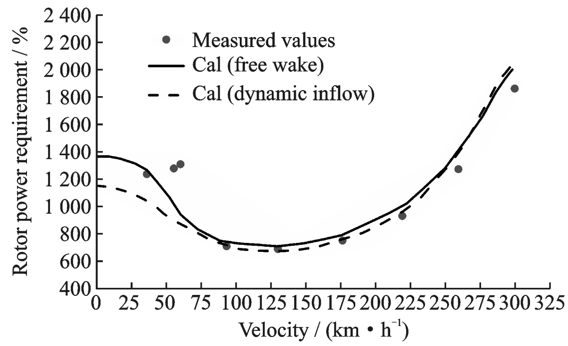


Fig.25 Influence of wake model on rotor power requirement[50]

free wake model can significantly improve the prediction accuracy of total rotor pitch and required power in hovering and low-speed flight.

In order to verify the validity of the dynamic response calculation method and the accuracy of flight dynamics model, the maneuvering flight of helicopter hover and forward flight with pushing steering rod to the right is simulated, then compared with flight test results, and the effects of free wake model, blade elastic deformation and aerodynamic interaction on the accuracy of helicopter dynamic response prediction are investigated, emphasis is laid on the accuracy and influencing factors of prediction of off-axis response.

The comparison between the measured and calculated results of the roll angular velocity response (on-axis response) is given. The results include the effects of wake model and elastic deformation. As shown in Fig.26, the influence of wake bending parameters on the calculation results is very small when the dynamic inflow model is adopted, which indicates that the introduction of wake bending distortion effect has little effect on the on-axis response. From Fig.26, it can be seen that the elastic deformation effect of the blade improves the calculation accuracy of the on-axis response to a certain extent, and the rigid blade model will produce greater overshoot.

The wake bending effect in the dynamic inflow model is completely determined by the wake bending parameters, because its direction is always opposite to the direction of the moving inflow gradient. Therefore, the dynamic inflow model considering

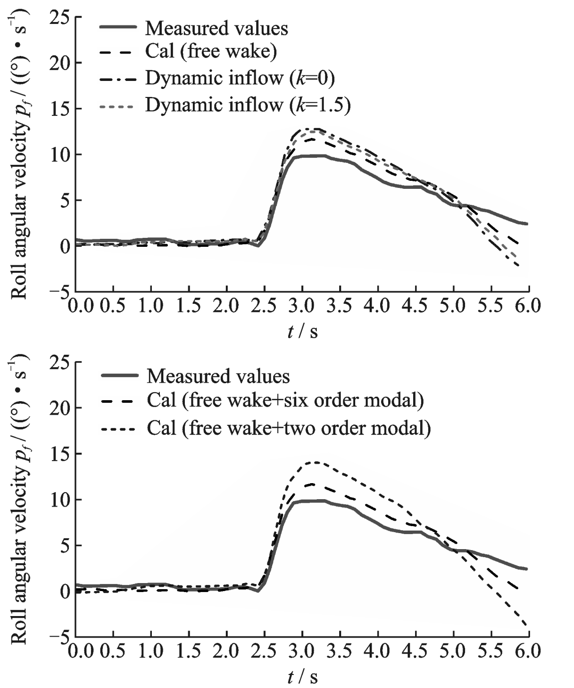


Fig.26 Influence of wake model and blade dynamics model on roll angular velocity response[50]

the dynamic wake distortion effect is difficult to be applied to any flight state. On the contrary, the free-wake model does not make too many artificial assumptions about the effect of wake distortion, and it has a wider scope of application and versatility.

2.7 Application of viscous vortex particle wake method to the study of helicopter interaction aerodynamics

Refs.[52-53] have made a thorough study on the calculation of the aerodynamic interaction of helicopter rotor/fuselage/tail/tail rotor by the vortex particle method. In their work, the viscous vortex particle method based on Lagrangian system is used to describe the rotor wake. The unsteady panel method is constructed by adding the unsteady term of the rotor wake to calculate the aerodynamic loads of the blade, fuselage and horizontal tail. then based on vorticity equivalence and Neumann boundary conditions, an unsteady panel/viscous vortex particle hybrid method for unsteady aerodynamic interaction analysis of rotor/fuselage/tail/tail rotor is established in their work.

With this method, Tan Jianfeng et al calculated and studied the unsteady pressure of the fuselage and rotor thrust characteristics under rotor/fuselage aerodynamic interaction. Then the effects of rotor parameters such as the distance between rotor and fuselage, number of blades, forward inclination angle of rotor axis, blade tip shape on rotor/fuselage aerodynamic interaction were also analyzed.

Ref.[53] studies the influence of rotor/horizontal tail aerodynamic interaction on horizontal tail aerodynamic load, and analyzed the influence of horizontal tail configuration on horizontal tail aerodynamic load and helicopter control characteristics, according to the equivalent angle of attack of the horizontal tail, the design method of the installation angle of the movable horizontal tail to eliminate the "head-up" phenomenon is proposed.

In Fig. 27, the downward load of the front, middle and rear horizontal tails along with the forward flight velocity is calculated by Ref.[53].

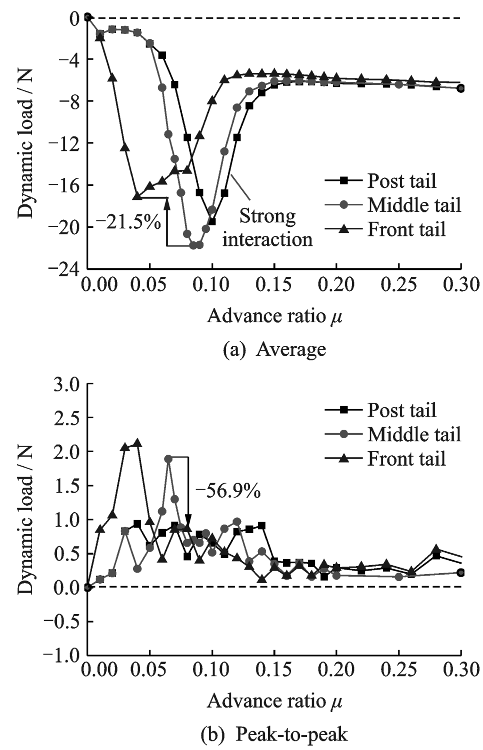
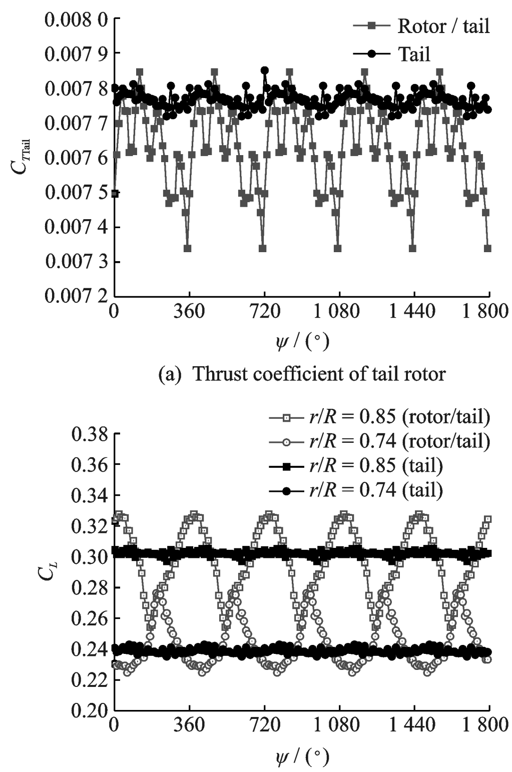


Fig.27 Mean and peak-to-peak value of dynamic load of tail varies with forward flying velocity[53]

Meanwhile, based on the developed method, Tan Jianfeng et al. studies the influence of rotor/tail rotor aerodynamic interaction on the tail rotor aerodynamic characteristics in hover, various wind directions and right sideslip states (Fig.28). Afterwards, the effects of the rotor rotation direction, height and longitudinal position on the performance of the tail rotor were analyzed, which revealed that the performance degradation characteristics of the tail rotor in the 60° right-sided sliding state were mainly determined by the change and position of the rotor tip vortex.



(a) Thrust coefficient of tail rotor

(b) Sectional thrust coefficient of tail rotor

Fig. 28 Tail rotor tension and section load in hover state[53]

2.8 Application of wake analysis method to unsteady aerodynamic interference of ABC rotor helicopter

Compared with the traditional single-rotor helicopter with tail rotor, the propeller-augmented compound advancing blade concept (ABC) rotor helicopter greatly improves flight speed, but its unique configuration also brings more serious vortex-wake interaction problems.

In 2008, Kim and Brown et al.[61-63] firstly constructed an aerodynamic model for aerodynamic interaction analysis of the propeller-augmented compound ABC rotor helicopter. The vorticity transport model (VTM)[64-65] is used to solve and calculate in the Cartesian three-dimensional mesh around the helicopter, and this method is still essentially a viscous vortex method. Kim calculated in detail the aerodynamic interaction characteristics of the main rotor, and the horizontal tail and the propulsion rotor at different flight speeds. Subsequently, in 2009, Kim further analyzed the influence of fuselage on the above aerodynamic interaction characteristics[62]. In 2013, based on the developed free wake analysis method, Lv and Xu investigated the aerodynamic characteristics of the propeller-augmented compound ABC rotor helicopter[66-68].

In the calculation of aerodynamic interaction of helicopters, especially the propeller-augmented compound ABC rotor helicopter, the correct control must be input, which can only be obtained by proper trimming calculation. For this reason, Lyu Weiliang and Xu Guohua have established a new trimming model of the propeller-augmented compound ABC rotor helicopter. On this basis, a new method for calculating the aerodynamic interaction of the propeller-augmented compound ABC rotor helicopter is established by combining the free wake model and the helicopter trimming model, in order to comprehensively analyze the interaction effects of upper and lower main rotors, propulsion rotors, horizontal tails and other aerodynamic components. With this method, the aerodynamic interaction between the upper and lower main rotors, the aerodynamic interaction of the main rotor wake on the propeller and the horizontal tail, and the aerodynamic forces of the main rotor and the propeller were calculated and studied at different forward speeds. At the same time, the influence of different vertical spacing of main rotor on aerodynamic interaction, the influence of periodic pitch control on the aerodynamic characteristics of main rotor and the influence of installation position of propelling rotor on its aerodynamic characteristics were analyzed.

Fig.29[68] gives the side view of the tip vortices of upper and lower main rotor blades with different advance ratios, in which the lower rotor wakes are represented by solid lines and the upper rotor wakes are represented by dotted lines, in addition, several major aerodynamic components, including upper and lower main rotor, horizontal tails and propulsion rotor, have been marked on the diagram.

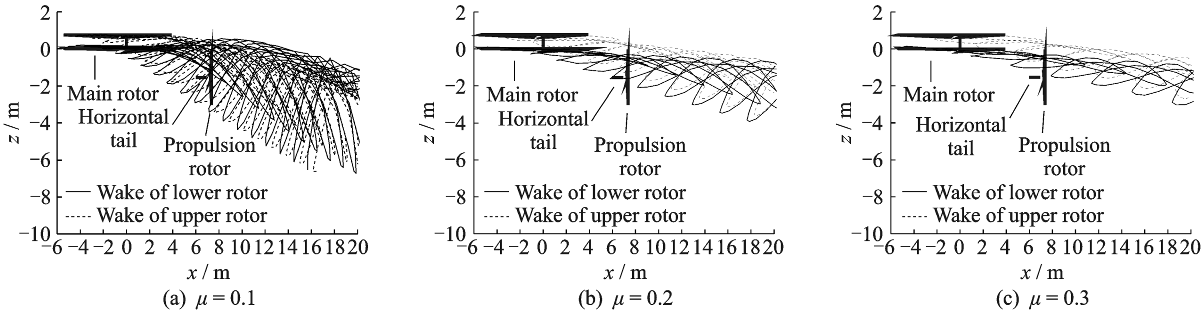


Fig.29 Lateral view of rotor wake at different forward ratio[68]

In Fig.29 (a), the wake side view of upper and lower main rotor with advance ratio of 0.1 is given, at this time, the wake of the upper and lower rotor affects most of the area below and behind the rotor disk in a more severe distortion form. It is noteworthy that the wake of the upper and lower main rotor completely covers the horizontal tail and propulsion rotor of the rear side of the helicopter, which can exert severe aerodynamic interaction on them, however, when the speed is slightly higher, the influence area of the main rotor wake on the tail aerodynamic components (horizontal tail and propulsion rotor) is smaller.

In Ref.[68], the variation of the power coefficient of the main rotor during its rotation cycle is calculated in Fig.30 with different advance ratios.

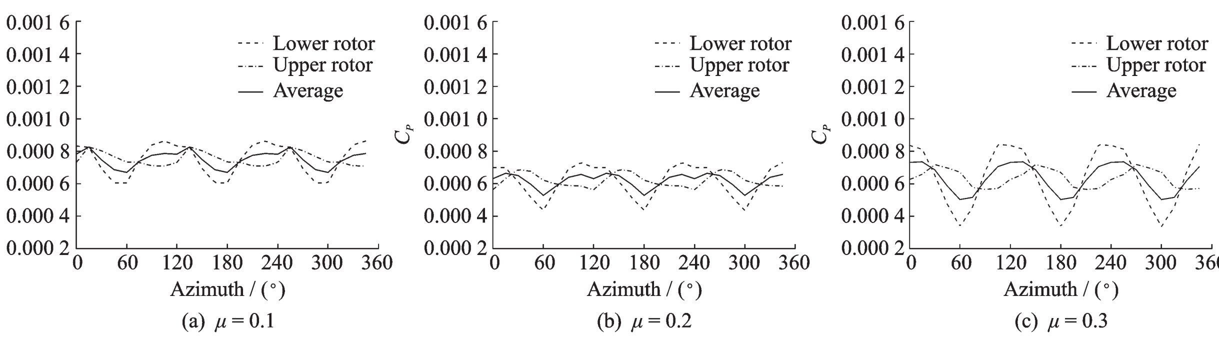


Fig. 30 Power coefficient variation of the main rotor in a rotation cycle for different advance ratios[68]

Ref.[68] further gave the side view of wake geometry where the coaxial rotors have different vertical spacing in Fig. 31. As seen from Fig. 31 (a), when the vertical spacing of the coaxial rotors is close, the wakes of the upper and lower rotors almost coincide with each other, and the wake of the upper rotor sweeps most of the lower rotor. When the vertical spacing of the main rotors increases, the wake of the upper rotor has almost no direct impact on the horizontal tail, and only a small portion of the upper end of the rotor disc is disturbed by the impact.

3 Prospect of Helicopter Rotor Wake Research

In the field of rotor wake analysis, despite great progress has been made and the wake characteristics have been well understood, there are still a number of fundamental problems that have not been thoroughly revealed, and some existing analysis methods still need to be further developed. The helicopter rotor wake analysis, and helicopter aerodynamics, dynamics and flight dynamics all require advanced rotor wake modeling technique. Therefore, rotor wake analysis will continue to be a long-term and in-depth research topic. The following trends are listed for reference by other researchers.

(1) Although CFD technology has great potential in capturing the details of rotor flow field, the rotor wake analysis method still has considerable advantages in the field of blade aerodynamic loads calculation and will continue to be the main method for

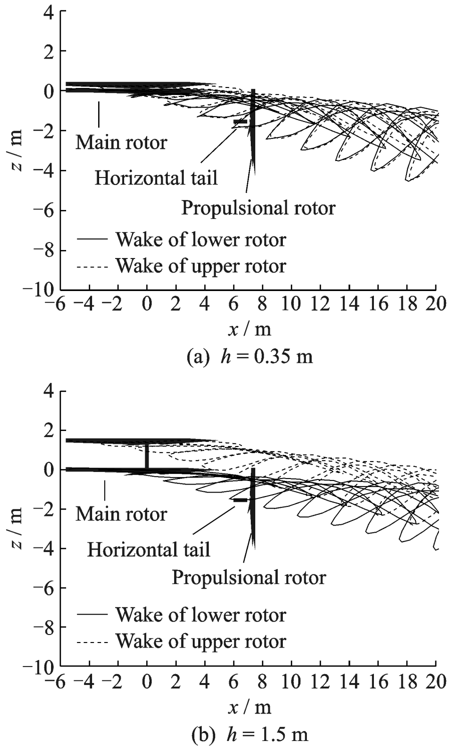


Fig. 31 Lateral view of main rotor wake with different vertical spacing[68]

calculating blade aerodynamic loads in the future.

(2) The rotor wake analysis method may be a more suitable method for the cases where the induced velocity distribution in the rotor flow field needs to be calculated. One of the typical cases is studying the influence of helicopter rotor downwash on missile launching.

(3) In order to improve the calculation of blade aerodynamic loads, in addition to the development of rotor wake analysis methods, more sophisticated aerodynamic models of airfoils need to be developed to accurately simulate the unsteady aerodynamic loads of blades.

(4) New wake analysis methods need to be developed to accurately consider the effects of air compressibility for the conditions with higher Mach number at the blade tip, especially for those with significant compressibility effects.

(5) Current wake analysis methods are mostly used in hover, vertical flight and forward flight. It's rare to conduct research on rotor loads and control responses in maneuvering flight using wake analysis method. Moreover, helicopter wake characteristics in complex unsteady flight states such as maneuvering flight have not been thoroughly studied. Hence, time-accurate wake calculation methods need to be further developed.

(6) Up to now, there is not enough research on applying free wake model to the aerodynamic characteristics analysis of the rotors with new-type tips. It is a direction of the future development to improve the method of rotor wake analysis, so that the method can be well applied to the research of new-type tips.

(7) Although many studies have been carried out on rotor wake analysis methods, most of them are confined to pure aerodynamic researches, such as rotor wake modeling, induced velocity calculation and aerodynamic loads prediction. In recent years, an important trend in this field is applying the rotor wake analysis method to rotor aeroelastic analysis. Therefore, taking into account the blade elasticity and aeroelastic coupling in the rotor wake analysis is an important development direction in the future.

(8) The analysis of rotor wake can be divided into the analysis of rotor blade vortices and the analysis of rotor wake. The former can be called internal problem, while the latter can be called external problem. Combining the wake analysis method with CFD method is a long-term development direction to simulate the internal and external problems of the rotor wake flow field more effectively.

(9) Because of taking into account the viscous effect, the vortex particle method has better calculation accuracy than the previous vortex wake analysis method and will be an important research direction of the rotor wake analysis method.

References

[1] JOHNSON W. Helicopter theory, Princeton University Press[M]. New Jersey: [s.n.], 1980.

[2] CONLISK A T. Modern helicopter rotor aerodynamics [J]. Progress in Aerospace Sciences, 2001, 37 (5): 419-476.

[3] LEISHMAN J G, BAGAI A. Challenges in understanding the vortex dynamics of helicopter rotor wakes [J]. AIAA journal, 1998, 36(7): 1130-1140.

[4] EGOLF T A, LANDGREBE A J. Generalized wake geometry for a helicopter in forward flight and effect of

wake deformation on airloads[R]. NASA CR 3726, 1983.

[5] WANG S C, XU G H. Progress of helicopter rotor aerodynamics [J]. Journal of Nanjing University of Aeronautics & Astronautics, 2001, 33(3): 203-211. (in Chinese)

[6] BHAGWAT M J, LEISHMAN J G. Correlation of helicopter rotor tip vortex measurements [J]. AIAA Journal, 2000, 38(2): 301-308.

[7] YANG A M, QIAO Z D. Navier-Stokes computation for a helicopter rotor in forward flight based on moving overset grids[J]. Acta Aeronautica et Astronautica Sinica, 2001, 22(5): 434-436.

[8] ZHAO Q J, XU G H, ZHAO J G. Numerical simulations of the unsteady flowfield of helicopter rotors on moving embedded grids [J]. Aerospace Science and Technology, 2005, 9(2): 117-124.

[9] HUBERT P, SIEGFRIED W, WAGNER. Navier-Stokes analysis of helicopter rotor aerodynamics in hover and forward flight [J]. Journal of Aircraft, 2012, 39(5): 813-821.

[10] BANGALORE A, SANKAR L N. Forward-flight analysis of slatted rotors using Navier-Stokes methods [J]. Journal of Aircraft, 2015, 34(1): 80-86.

[11] LI C H. Modeling on time-accurate free wake method and investigation on aerodynamic characteristics of rotor and tiltrotor[D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2007.

[12] XU Guohua. Research on aerodynamic characteristics of helicopter new-tip rotors by free wake analysis[D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 1996. (in Chinese)

[13] GRAY R B. An aerodynamic analysis of a single bladed rotor in hovering and low speed forward flight as determined from smoke studies of the vorticity distribution in the wake[M]. Princeton, New Jersey, USA: Princeton University, 1956

[14] LANDGREBE A J. The wake geometry of a hovering rotor and its influence on rotor performance[J]. Journal of the American Helicopter Society, 1972, 17(4): 3-15.

[15] KOCUREK J D, TANGLER J L. A prescribed wake lifting surface hover performance analysis[J]. Journal of the American Helicopter Society, 1977, 22(1): 24-35;12.

[16] WANG S C. The induced velocity distribution of the lift propeller on the disc[J]. Acta Mechanica Sinica, 1964, 7(3): 43-53. (in Chinese)

[17] SONG C Y. Computation of rotational noise and research of blade-vortex interaction noise for rotors based upon wake analysis[D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2010.

[18] SHENOY K R, GRAY R B. Iterative lifting surface method for thick bladed hovering helicopter rotors[J]. Journal of Aircraft, 1981, 18(6): 417-424.

[19] BEDDOES T S. A wake model for high resolution airloads[C]// International Conference on Rotorcraft Basic Research. North Carolina, USA:[s.n.],1985.

[20] LANDGREBE A J, EGOLF T A. Prediction of helicopter induced flow velocities using the rotorcraft wake analysis[C]// Proceedings of the 32nd Annual National Forum of American Helicopter Society. [S. l.]:AHS, 1976.

[21] QUACKENBUSH T R, LAM C G, WACHPRESS D A, et al. Analysis of high resolution unsteady airloads for helicopter rotor blades[C]//Proceedings of the AHS 50th Annual Forum. Washington, D.C. :[s. n.],1994.

[22] SADLER S G. Development and application of a method for predicting rotor free wake positions and resulting rotor blade air loads[R]. NASA-CR-1911, 1971.

[23] MILLER R H. A simplified approach to the free wake analysis of a hovering rotor[C]// Proceedings of the 7th European Rotorcraft Forum. [S.l.]:[s.n.],1981.

[24] SPARKS S S, Egolf T A. A full potential rotor analysis with wake influence using an inner-outer domain technique[C]//Proceedings of the 42nd Annual AHS Forum. [S.l.]:AHS, 1986.

[25] BLIS S, DONAL D, DADON E, et al. Rotor wake modeling for high speed applications [C]//Proceedings of the American Helicopter Society 43rd Annual Forum. [S.l.]: AHS, 1987.

[26] JOHNSON W. A comprehensive analytical model of rotorcraft aerodynamics and dynamics[R]. NASA TM -81182, 1980.

[27] HE C, LEE C S, CHEN W B. Rotorcraft simulation model enhancement to support design[J]. Testing and Operational Analysis, 1999, 45(4): 284-292.

[28] CROUSE G, LEISHMAN J G. A new method for improved rotor free-wake convergence, Aerospace Sciences Meeting[C]//Aerospace Sciences Meeting. [S. l.]:[s.n.],1993.

[29] LEISHMAN J G, BAGAI A. Rotor free-wake modeling using a relaxation technique-including comparisons with experimental data [J]. Journal of the American Helicopter Society, 1995, 40(3): 29-41.

[30] BAGAI A, LEISHMAN J G. Rotor free-wake model-

ing using a pseudo-implicit technique-including comparisons with experimental data [J]. Journal of the American Helicopter Society, 1995, 40(13): 29-41.

[31] BAGAI A. Contributions to the mathematical modeling of rotor flow fields using a pseudo-implicit free wake analysis[D]. City of College Park: University of Maryland, 1995.

[32] BHAGWAT M J, LEISHMAN J G. Rotor aerodynamics during maneuvering flight using a time-accurate free-vortex wake[J]. Journal of the American Helicopter Society, 2003, 48(3): 143.

[33] BHAGWAT M J, LEISHMAN J G. Stability, consistency and convergence of time-marching free-vortex rotor wake algorithms[J]. Journal of the American Helicopter Society, 2001, 46(1): 59-71.

[34] BHAGWAT M J. Mathematical modeling of the transient dynamics of helicopter rotor wakes using a time-accurate free-vortex method [D]. City of College Park : University of Maryland, 2001.

[35] QUACKENBUSH T R, LAM C-M G, BLISS D B. Vortex methods for the computational analysis of rotor/body interaction[J]. Journal of the American Helicopter Society, 1994, 39(4): 14-24.

[36] QUACKENBUSH T R, BLISS D B. Free wake calculation of rotor flow fields for interactional aerodynamics[C]// the 44th Annual Forum of the American Helicopter Society. Washington D.C.: [s.n.], 1988.

[37] QUACKENBUSH T R, LAM C-M G, BLISS D B. New vortex/surface interaction methods for the prediction of wake-induced airframe loads[C]//Presented at the 46th Annual Forum of the American Helicopter Society, Washington D. C. :[s.n.], 1990.

[38] XU G H, WANG S C. Calculation of free wake of hovering rotor [J]. Journal of Nanjing University of Aeronautics & Astronautics, 1998, 30(2): 126-131. (in Chinese)

[39] XU G H, WANG S C. Calculation of free wake of helicopter rotor in forward flight[J]. Journal of Nanjing University of Aeronautics & Astronautics, 1997, 29 (6): 648-653. (in Chinese)

[40] CAO Y H. Calculation method of vortex wake and downwash flow field of rotor [J]. Journal of Beijing University of Aeronautics and Astronautics, 2000, 26 (2):174-177. (in Chinese)

[41] YUE H L, XIA P Q. The unsteady dynamic inflow model of rotor wake bending of tilt-rotor aircraft during flight transition[J]. Science in China, 2009, 39(12): 1992-2000. (in Chinese)

[42] ZHAO J G, GAO Z, XU G H. Calculation method of helicopter rotor/ fuselage aerodynamic interaction [J]. Journal of Nanjing University of Aeronautics & Astronautics, 2000, 32(4):369-374. (in Chinese)

[43] XU G H, ZHAO Q J, GAO Z, et al. Prediction of aerodynamic interactions of helicopter rotor on its fuselage [J]. Chinese Journal of Aeronautics, 2002, 15 (1):12-17.

[44] ZHAO J G. Research on aerodynamic interference of helicopter rotor / fuselage [D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2000. (in Chinese)

[45] JOHNSON W. A general free wake geometry calculation for wings and rotors[C]// the American Helicopter Society 51th Annual Forum. Fort Worth, Texas: [s.n.],1995.

[46] ZHAO J G, HE C J. A viscous vortex particle model for rotor wake and interaction analysis [J]. Journal of the American Helicopter Society, 2010, 55 (1) : 12007-12014.

[47] HE C J, ZHAO J G. Modeling rotor wake dynamics with viscous vortex particle method [J]. AIAA Journal, 2009, 47(4):902-913.

[48] WEI P. Research on viscous vortex numerical algorithm and its hybrid method for rotor unsteady flowfield [D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2012.

[49] LI P, CHEN R. Aerodynamics analysis of a rotor undergoing maneuvers using an efficient time marching free vortex method[C]// Proceedings of the 3rd International Basic Research Conference on Rotorcraft Technology. [S.l.]:[s.n.], 2009.

[50] LI P. Rotor unsteady free-vortex wake model and investigation on high-fidelity modeling of helicopter flight dynamic [D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2010.

[51] ZHAO Y Y. Study on rotor impeller vortex interference noise of CFD / viscous vortex particle mixing method[D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2017.

[52] TAN J F, WANG H W, WU C, et al. Unsteady aerodynamic disturbances of rotor/horizontal tail based on unsteady surface element / viscous vortex particle hybrid method [J]. Acta Aeronautica et Astronautica Sinica 2014, 35(3): 643-656. (in Chinese)

[53] TAN J F. Research on helicopter aerodynamic interaction with unsteady panel/viscous vortex particle hybrid method[D]. Beijing: Tsinghua University, 2013.

[54] LOU W J, WANG S C. Research progress of helicopter rotor wake [J]. Acta Aeronautica et Astronautica

Sinica, 1990, 11(3): A113-A119. (in Chinese)

[55] WANG F X, WU J, LIAN Z Z. A comprehensive method for the generation of constant vorticity contours wake[J]. Acta Aerodynamica Sinica, 2006, 24(1): 67-72. (in Chinese)

[56] LOU W J, WANG S C. A new efficient method of constant vorticity contours wake modeling for a helicopter rotor[J]. Journal of Nanjing University of Aeronautics and Astronautics, 1997, 29(1): 102-105. (in Chinese)

[57] LOU W J, WANG S C. A new wake modeling technology using circulation contours and its application to rotor rigid wake[J]. Journal of Nanjing University of Aeronautics and Astronautics, 1989, 21(4): 38-45. (in Chinese)

[58] MILLER W O, BLISS D B. Direct periodic solutions of rotor free wake calculations [J]. Journal of the American Helicopter Society, 1993, 38(2): 53-60.

[59] CARPENTER P J, FRIDOVICH B. Effect of a rapid blade-pitch increase on the thrust and induced-velocity response of a full-scale helicopter rotor[R]. NACA TN 3044, 1953.

[60] KOMERATH N M, MAVRIS D M, LIOU S G. Prediction of unsteady pressure and velocity over a rotorcraft in forward flight [J]. Journal of Aircraft, 1991, 28(8):509-516.

[61] KIM H W, KENYON A R, DURAISAMY K, et al. Interactional aerodynamics and acoustics of a propeller-augmented compound coaxial helicopter [C]// The American Helicopter Society Specialists' Conference on Aeromechanics. San Francisco, CA:[s.n.], 2008.

[62] KIM H W, KENYON A R, DURAISAMY K, et al. Interactional aerodynamics and acoustics of a hingeless coaxial helicopter with an auxiliary propeller in forward flight[J]. Aeronautical Journal, 2009, 113(1140): 65-78.

[63] KIM H W, KENYON A R, DURAISAMY K, et al. Interactional aerodynamics and acoustics of a propeller-augmented compound coaxial helicopter [C]//The American Helicopter Society Specialists' Conference on Aeromechanics. San Francisco, CA: American Helicopter Society, 2008.

[64] BROWN R E. Rotor wake modeling for flight dynamic simulation of helicopters[J]. AIAA Journal, 2000, 38(1): 57-63.

[65] BROWN R E, LINE A J. Efficient high-resolution wake modeling using the vorticity transport equation [J]. AIAA Journal, 2005, 43(7): 1434-1443.

[66] LYU W L, XU G H. New-trim-method-based investigation on the cyclic-pitch-effected advancing-blade-concept helicopter aerodynamics [J]. Journal of Aircraft, 2015, 52(4):1-7.

[67] LYU W L, XU G H. Interactional effect of propulsive propeller location on counter-rotating coaxial main rotor[J]. Journal of Aircraft, 2018, 55(6):2538-2545.

[68] LV W L. Research on the stability, numerical method of rotor wake systems and the aerodynamic characteristics of ABC helicopters[D]. Nanjing: Nanjing University of Aeronautics and Astronautics, 2013. (in Chinese)

Authors Dr. **XU Guohua** is professor and doctoral superviser at National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics. His research focuses on rotorcraft aerodynamics, aeroacoustics, and CFD.

Dr. **SHI Yongjie** is associate professor and master superviser at National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics. His research focuses on rotorcraft aerodynamics, aeroacoustics, and CFD.

Dr. **ZHAO Qijun** is professor and doctoral superviser at National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, Nanjing University of Aeronautics and Astronautics. He is Vice Dean of College of Aerospace Engineering, NUAA. His research focuses on rotorcraft aerodynamics, aeroacoustics, and CFD.

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