Robust Adaptive Attitude Maneuvering and Vibration Reducing Control of Flexible Spacecraft Under Input Saturation

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Abstract: A robust adaptive control scheme is proposed for attitude maneuver and vibration suppression of flexible spacecraft in situations where parametric uncertainties, external disturbances, unmeasured elastic vibration and input saturation constraints exist. The controller does not need the knowledge of modal variables but the estimates of modal variables provided by appropriate dynamics of the controller. The requirements to know the system parameters and the bound of the external disturbance in advance are also eliminated by adaptive updating technique. Moreover, an auxiliary design system is constructed to analyze and compensate the effect of input saturation, and the state of the auxiliary design system is applied to the procedure of control design and stability analysis. Within the framework of the Lyapunov theory, stabilization and disturbance rejection of the overall system are ensured. Finally, simulations are conducted to study the effectiveness of the proposed control scheme, and simulation results demonstrate that the precise attitude control and vibration suppression are successfully achieved.

Key words: flexible spacecraft; attitude maneuver; modal observer; vibration suppression; input constraints

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0 Introduction

Flexible spacecraft, which often employs large flexible structures such as arrays in future space missions, will be expected to achieve high pointing and fast attitude maneuvering. However, the attitude maneuvering operation will introduce certain levels of vibration to flexible appendages, which will deteriorate its pointing performance. Worse still, the deflection of the flexible appendages makes the spacecraft dynamics highly nonlinear. Furthermore, flexible spacecraft in the practical environment is often subjected to various external disturbances, and knowledge about system parameters is usually not available, such as, the inertia matrix. Therefore, attitude maneuver and vibration control strategies robust to parametric uncertainties and external disturbance, and also suppressive to the induced vibration are in great demand in future space missions.

Recently, considerable works have been found for designing nonlinear attitude controller in the presence of the aforementioned issues. Refs.[1-2] developed. optimal and nonlinear control systems for the control of flexible spacecraft. Variable structure controllers for flexible spacecraft with large space structures are designed[3-4] because of their insensitivity to system uncertainty and external disturbance.

However, design methods in these studies require perfect knowledge of system parameters and prior information on the bounds of disturbances for the computation of control gains. Unlike these methods, nonlinear adaptive control methods, including an adaptation mechanism for tuning the controller gains, are called for. A variety of adaptive attitude controllers have been developed. A new adaptive system in Ref.[5] for rotational maneuver and vibration suppression of an orbiting spacecraft with flexi-

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ble appendages was designed. Then adaptive output regulation of the closed-loop system was accomplished in spite of large parameter uncertainties and disturbance input. Variable structure control approaches in Refs.[4, 6] have been proposed for vibration control of flexible spacecraft during attitude maneuvering, and the adaptive version of the proposed controller was achieved by releasing the limitation of knowing the bounds of the uncertainties and perturbations in advance.

Relevant drawback of these control strategies is the extra necessity to measure the modal variables and to treat the effects of the flexible dynamics affecting the rigid motion as an additional disturbance acting on a rigid structure. With regard to the latter situation, a weighted homogeneous extended state observer in Ref.[7] was designed to estimate and thus to attenuate the total disturbance in finite time, including external disturbance torque and coupling effect. As a result, the prior knowledge of the total disturbance was not required.

Unfortunately, in some cases the availability of the measured modal variables is an unrealistic hypothesis, due to the impracticability of using appropriate sensors or the economical requisite. An interesting solution for the attenuation of the flexible oscillations induced by spacecraft maneuvers is to reconstruct the unmeasured modal position and velocity by means of appropriate dynamics. It is advantageous in a case of sensor failures or the reduction of the structure and control system design costs.

A class of nonlinear controllers[8-12] has been derived for spacecraft with flexible appendages. It does not ask for measures of the modal variables, but only uses the parameters describing the attitude and the spacecraft angular velocity. The controller derived then uses estimates of the modal variables and its rate to avoid direct measurement.

However, a typical feature in all of the mentioned attitude control schemes and methods is that the control device is assumed to be able to produce a big enough control torque without considering actuator saturation. Extensive results pertaining to spacecraft attitude control systems containing actuator saturation nonlinearities were presented in Refs.[13-14], in which a robust variable structure controller has been skillfully designed to control the spacecraft attitude under input saturation. However, these control schemes lose the generality to nonlinear flexible spacecraft system. A modified adaptive backstepping attitude controller in Refs.[15-16] is developed considering external disturbance and input saturation. But the way to include the effects of the flexible dynamics in the lumped disturbance for the rigid dynamics deprives the controller of a direct compensation of the dynamic terms caused by the flexibility.

In this paper, a robust adaptive scheme is proposed at the basis of an elastic mode estimator. This very estimator only demands the angular velocity measurements, thereby avoiding the necessity of modal variables measurements in traditional methods. Also, by employing an adaptation mechanism, the requirements to know the exact system parameters and the bound of the external disturbance in advance have been eliminated. Moreover, an auxiliary system, taking the input constraints explicitly in the controller design, is constructed to rigorously enforce actuator magnitude saturation constraints. The convergence of the proposed control scheme is proved by the Lyapunov stability theorem, and the convergence domain of system attitude error can be adjusted by an explicit choice of design parameters.

1 Mathematical Model

1.1 Mathematical model of a flexible spacecraft

The mathematical model of a flexible spacecraft is briefly recalled.

The unit quaternion is adopted to describe the attitude of the spacecraft for global representation without singularities

$$\dot{\boldsymbol{q}}=\begin{bmatrix}\dot{q}_0\\ \dot{\boldsymbol{q}}_v\end{bmatrix}=\frac{1}{2}\begin{bmatrix}-\boldsymbol{q}_v^{\mathrm{T}}\\ q_0\boldsymbol{I}+\boldsymbol{q}_v^{\times}\end{bmatrix}\boldsymbol{\omega} \tag{1}$$

Under the assumption of small elastic displacements, the dynamic equations of spacecraft with flexible appendages can be found in Ref.[17] and references therein, and given by

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}+\boldsymbol{\delta}^{\mathrm{T}}\ddot{\boldsymbol{\eta}}=-\boldsymbol{\omega}^{\times}(\boldsymbol{J}\boldsymbol{\omega}+\boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\eta}})+\boldsymbol{u}+\boldsymbol{d} \tag{2}$$

$$\ddot{\boldsymbol{\eta}}+\boldsymbol{C}\dot{\boldsymbol{\eta}}+\boldsymbol{K}\boldsymbol{\eta}=-\boldsymbol{\delta}\dot{\boldsymbol{\omega}} \tag{3}$$

where $\boldsymbol{J}$ is the total inertia matrix, $\boldsymbol{\delta}$ is the coupling matrix between flexible and rigid dynamics, $\boldsymbol{\eta}$ is the modal coordinate vector, and the damping and stiffness matrices are expressed as $\boldsymbol{C}=\mathrm{diag}\left\{2\xi_i\omega_{ni}\right\}$ and $\boldsymbol{K}=\mathrm{diag}\left\{\omega_{ni}^2\right\}(i=1,\cdots,N)$ respectively, N the number of elastic modes considered, ω_{ni} the natural frequencies, ξ_i the corresponding damping, $\boldsymbol{u}$ the control input, and $\boldsymbol{d}$ the external disturbance.

Through introducing the auxiliary variable $\boldsymbol{\psi}=\dot{\boldsymbol{\eta}}+\boldsymbol{\delta}\boldsymbol{\omega}$ which represents the total angular velocity expressed in modal variables, the dynamics of the flexible spacecraft from Eqs.(2—3) can be further expressed as

$$\begin{bmatrix}\dot{\boldsymbol{\eta}}\\ \dot{\boldsymbol{\psi}}\end{bmatrix}=\begin{bmatrix}0 & \boldsymbol{I}\\ -\boldsymbol{K} & -\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{\eta}\\ \boldsymbol{\psi}\end{bmatrix}+\begin{bmatrix}-\boldsymbol{I}\\ \boldsymbol{C}\end{bmatrix}\boldsymbol{\delta}\boldsymbol{\omega} \tag{4}$$

$$\boldsymbol{J}_{mb}\dot{\boldsymbol{\omega}}=-\boldsymbol{\omega}^{\times}(\boldsymbol{J}_{mb}\boldsymbol{\omega}+\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi})+\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\psi}+\boldsymbol{K}\boldsymbol{\eta}-\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{\omega})+\boldsymbol{u}+\boldsymbol{d} \tag{5}$$

where $\boldsymbol{J}_{mb}=\boldsymbol{J}-\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$, with $\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$ as the contribution of the flexible parts to the total inertia matrix.

1.2 Preliminaries

To facilitate control system design, the following assumptions and lemmas are presented and will be used in the subsequent developments.

Assumption 1 The disturbance vector $\boldsymbol{d}$ in system Eq.(5) is assumed to be bounded by an unknown bound constant $\rho>0$, that is

$$\|\boldsymbol{d}\|\leqslant\rho \tag{6}$$

Since the bound ρ is unknown, an adaptive law will be designed to estimate ρ online, and $\hat{\rho}$ will be employed to denote the estimation.

Assumption 2 All three components of the control torque $\boldsymbol{u}$ are constrained by a saturation value, expressed by

$$|\boldsymbol{u}_i(t)|\leqslant u_{\max},\quad \forall t>0,\quad i=1,2,3 \tag{7}$$

Note that typical applications have a separate saturation limit $u_i^{\max}$ for each component u_i, but a conservative common saturation limit $u_{\max}=\min_i[u_i^{\max}]$ $(i=1,2,3)$ is adopted here to streamline the analysis, where $u_i^{\max}$ is the upper limit bound of the ith actuator.

Lemma 1 For arbitrary constant $\varepsilon>0$ and variable η, Eq.(8) holds[18-19]

$$0\leqslant|\eta|-\eta\tanh(\eta/\varepsilon)\leqslant\delta\varepsilon \tag{8}$$

with $\delta=\mathrm{e}^{-(\delta+1)}$ and $\delta=0.278\,5$.

1.3 Control problem formulation

The control objective is to achieve rest-to-rest attitude maneuver, i.e., from any initial condition to the desired quaternion $\boldsymbol{q}_d^{\mathrm{T}}=[1,0,0,0]$ and angular velocity $\boldsymbol{\omega}_d^{\mathrm{T}}=[0,0,0]$, such that

(1) The attitude orientation and angular velocity tracking errors are driven to zero, or a small set containing the origin.

(2) The vibration induced by the maneuver rotation is also suppressed in the presence of parametric uncertainties, external disturbances and input saturation constraints, i.e.

$$\lim_{t\to\infty}\boldsymbol{\eta}=0,\ \lim_{t\to\infty}\dot{\boldsymbol{\eta}}=0 \tag{9}$$

2 Robust Adaptive Constrained Backstepping Controller Design

2.1 Basic controller design

To remove the hypothesis of the measurability of the modal position and velocity, an elastic mode estimator to supply their estimates is constructed as follows[20]

$$\begin{bmatrix}\dot{\hat{\boldsymbol{\eta}}}\\ \dot{\hat{\boldsymbol{\psi}}}\end{bmatrix}=\begin{bmatrix}0 & \boldsymbol{I}\\ -\boldsymbol{K} & -\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix}+\begin{bmatrix}-\boldsymbol{I}\\ \boldsymbol{C}\end{bmatrix}\boldsymbol{\delta}\boldsymbol{\omega} \tag{10}$$

where $\hat{\boldsymbol{\eta}}$, $\hat{\boldsymbol{\psi}}$ are the estimates of modal variables, and $\boldsymbol{e}_\eta=\boldsymbol{\eta}-\hat{\boldsymbol{\eta}}$, $\boldsymbol{e}_\psi=\boldsymbol{\psi}-\hat{\boldsymbol{\psi}}$ are the estimation errors.

From Eqs.(4), (10), the response of $\boldsymbol{e}_\eta$, $\boldsymbol{e}_\psi$ can be algebraically arranged as

$$\begin{bmatrix}\dot{\boldsymbol{e}}_\eta\\ \dot{\boldsymbol{e}}_\psi\end{bmatrix}=\boldsymbol{A}\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} \tag{11}$$

Since matrix $\boldsymbol{A}$ is a Hurwitz matrix, the estimation errors $\boldsymbol{e}_\eta$, $\boldsymbol{e}_\psi$ will converge to zero asymptotically.

In what follows, a robust adaptive controller is derived in the presence of parametric uncertainties, external disturbances and unmeasured elastic vibration, where the control input limitation is not considered.

We start with Eqs.(1), (4) by considering $\boldsymbol{\omega}$ as the virtual control variable. Define the tracking error as

$$z = \omega - \alpha \tag{12}$$

where α is a virtual control input to be designed later.

The first Lyapunov candidate function is chosen as

$$\begin{aligned} V_1 &= [(1-q_0)^2 + q_v^{\mathrm{T}} q_v] + \\ &\frac{1}{2}[\hat{\eta}^{\mathrm{T}} \quad \hat{\psi}^{\mathrm{T}}] K_1 \begin{bmatrix} 2K + C^2 & C \\ C & 2I \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} + \\ &\frac{1}{2}[e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}}] K_2 \begin{bmatrix} 2K + C^2 & C \\ C & 2I \end{bmatrix} \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} \end{aligned} \tag{13}$$

where the positive definite matrices K_1 and K_2 are partitioned as

$$K_1 = \begin{bmatrix} k_{11} I & 0 \\ 0 & k_{12} I \end{bmatrix}, K_2 = \begin{bmatrix} k_{21} I & 0 \\ 0 & k_{22} I \end{bmatrix} \tag{14}$$

Using Eq.(11), the time derivative of Eq.(13) along the system trajectories is given by

$$\begin{aligned} \dot{V}_1 &= [q_v^{\mathrm{T}} + (k_{12}\hat{\psi}^{\mathrm{T}} C - 2k_{11}\hat{\eta}^{\mathrm{T}} K)\delta](z + \alpha) - \\ &[\hat{\eta}^{\mathrm{T}} \quad \hat{\psi}^{\mathrm{T}}] \begin{bmatrix} k_{11} CK & -2k_{11} K \\ 2k_{12} K & k_{12} C \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} - \\ &[e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}}] \begin{bmatrix} k_{21} CK & -2k_{21} K \\ 2k_{22} K & k_{22} C \end{bmatrix} \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} \end{aligned} \tag{15}$$

Define the stabilizing function $\alpha(q_v, \hat{\eta}, \hat{\psi})$ as

$$\alpha = -[q_v + \delta^{\mathrm{T}}(k_{12} C\hat{\psi} - 2k_{11} K\hat{\eta})] \tag{16}$$

Based on what is mentioned above, Eq.(15) becomes

$$\begin{aligned} \dot{V}_1 &= [q_v^{\mathrm{T}} + (k_{12}\hat{\psi}^{\mathrm{T}} C - 2k_{11}\hat{\eta}^{\mathrm{T}} K)\delta] z - \\ &[q_v^{\mathrm{T}} + (k_{12}\hat{\psi}^{\mathrm{T}} C - 2k_{11}\hat{\eta}^{\mathrm{T}} K)\delta] \cdot \\ &[q_v + \delta^{\mathrm{T}}(k_{12} C\hat{\psi} - 2k_{11} K\hat{\eta})] - \\ &[\hat{\eta}^{\mathrm{T}} \quad \hat{\psi}^{\mathrm{T}}] \begin{bmatrix} k_{11} CK & -2k_{11} K \\ 2k_{12} K & k_{12} C \end{bmatrix} \begin{bmatrix} \hat{\eta} \\ \hat{\psi} \end{bmatrix} - \\ &[e_\eta^{\mathrm{T}} \quad e_\psi^{\mathrm{T}}] \begin{bmatrix} k_{21} CK & -2k_{21} K \\ 2k_{22} K & k_{22} C \end{bmatrix} \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix} \end{aligned} \tag{17}$$

Take the derivate of z left-multiplied by inertia matrix J_{mb} with respect to Eq.(5), then we have

$$\begin{aligned} J_{mb}\dot{z} = &-\omega^\times (J_{mb}\omega + \delta^{\mathrm{T}}\psi) + u + d - J_{mb}\dot{\alpha} + \\ &\delta^{\mathrm{T}}(C\psi + K\eta - C\delta\omega) \end{aligned} \tag{18}$$

Although the inertia matrix J_{mb} is unknown for the system design, it can be observed that the inertia parameters $J_{mb,ij}$ $(i,j = 1,2,3)$ appear linearly in Eq.(18).

To isolate these parameters, a linear operator $L(\cdot): \mathrm{R}^3 \to \mathrm{R}^{3\times 6}$ acting on $\xi = [\xi_1 \quad \xi_2 \quad \xi_3]^{\mathrm{T}}$ is introduced as follows

$$L(\xi) = \begin{bmatrix} \xi_1 & 0 & 0 & \xi_2 & \xi_3 & 0 \\ 0 & \xi_2 & 0 & \xi_1 & 0 & \xi_3 \\ 0 & 0 & \xi_3 & 0 & \xi_1 & \xi_2 \end{bmatrix} \tag{19}$$

Let $\theta_{mb}^{\mathrm{T}} = [J_{mb,11}, J_{mb,22}, J_{mb,33}, J_{mb,12}, J_{mb,13}, J_{mb,23}]$, it follows that $J_{mb}\xi = L(\xi)\theta_{mb}$ and then Eq.(18) can be rewritten as

$$\begin{aligned} J_{mb}\dot{z} = &F\theta_{mb} - \omega^\times \delta^{\mathrm{T}}\psi + u + d + \\ &\delta^{\mathrm{T}}(C\psi + K\eta - C\delta\omega) \end{aligned} \tag{20}$$

with $F = -\omega^\times L(\omega) - L(\dot{\alpha})$.

The design procedure can be summarized in the following theorem.

Theorem 1 Considering the flexible spacecraft system governed by Eqs.(1), (4) and (5) with the Assumptions 1—2. If the control law is designed by

$$\begin{aligned} u = &-[q_v + \delta^{\mathrm{T}}(k_{12} C\hat{\psi} - 2k_{11} K\hat{\eta})] + \\ &\delta^{\mathrm{T}} C\delta\omega + [\omega^\times \delta^{\mathrm{T}}\hat{\psi} - \delta^{\mathrm{T}}(C\hat{\psi} + K\hat{\eta})] - \\ &\frac{1}{2}(\delta\omega^\times)^{\mathrm{T}}(\delta\omega^\times z) - \frac{1}{2}(C\delta)^{\mathrm{T}}(C\delta z) - \\ &\frac{1}{2}(K\delta)^{\mathrm{T}}(K\delta z) - F\hat{\theta}_{mb} - \\ &K_3 z - \frac{b\hat{\rho} z}{\|z\| + \varepsilon} \end{aligned} \tag{21}$$

and the adaptive control is selected as

$$\begin{cases} \dot{\hat{\theta}}_{mb} = \mathrm{Proj}_{\hat{\theta}_{mb}}(\Gamma F^{\mathrm{T}} z) \\ \dot{\hat{\rho}} = \dfrac{ab\|z\|^2}{\|z\| + \varepsilon} \end{cases} \tag{22}$$

where the projection operator to avoid the parameter drift problem is defined in Ref.[21], then the attitude orientation and angular velocity tracking errors are uniformly ultimately bounded.

Proof Consider the composite Lyapunov function V_2

$$V_2 = V_1 + \frac{1}{2} z^{\mathrm{T}} J_{mb} z + \frac{1}{2}\tilde{\theta}_{mb}^{\mathrm{T}} \Gamma^{-1} \tilde{\theta}_{mb} + \frac{1}{2a}\tilde{\rho}^2 \tag{23}$$

In view of control law Eq.(21), taking the derivative of the above Lyapunov function along the system in Eqs.(1), (4), (5), it follows that

$$\begin{aligned}\dot{V}_2 = &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot\\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} -\\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\begin{bmatrix} k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K}\\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix} -\\ &[\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\begin{bmatrix} k_{21}\boldsymbol{CK} & -2k_{21}\boldsymbol{K}\\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} +\\ &\left(\tilde{\boldsymbol{\theta}}_{mb}^{\mathrm{T}}\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}_{mb} - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{F}\tilde{\boldsymbol{\theta}}_{mb}\right) +\\ &\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{d} - \frac{b\hat{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon} + \frac{1}{a}\tilde{\rho}\dot{\hat{\rho}}\right) -\\ &\frac{1}{2}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z}) - \frac{1}{2}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z}) -\\ &\frac{1}{2}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z}) - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{e}_\psi +\\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{e}_\psi + \boldsymbol{K}\boldsymbol{e}_\eta)\end{aligned} \tag{24}$$

The last terms in Eq.(24) can be expanded as

$$\begin{cases} -\boldsymbol{z}^{\mathrm{T}}\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{e}_\psi \leqslant \frac{1}{2}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\psi^{\mathrm{T}}\boldsymbol{e}_\psi\\ \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{e}_\psi \leqslant \frac{1}{2}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\psi^{\mathrm{T}}\boldsymbol{e}_\psi\\ \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{e}_\eta \leqslant \frac{1}{2}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\eta^{\mathrm{T}}\boldsymbol{e}_\eta \end{cases} \tag{25}$$

Substituting Eq. (25) and the updating law Eq. (22) into Eq.(24), one obtains

$$\begin{aligned}\dot{V}_2 \leqslant &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot\\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} -\\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} +\\ &\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{d} - \frac{b\hat{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon} + \frac{b\tilde{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon}\right)\end{aligned} \tag{26}$$

where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are given by

$$\boldsymbol{P}_1 = \begin{bmatrix} k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K}\\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}$$

$$\boldsymbol{P}_2 = \begin{bmatrix} k_{21}\boldsymbol{CK} - \frac{1}{2}\boldsymbol{I} & -2k_{21}\boldsymbol{K}\\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C} - \boldsymbol{I}\end{bmatrix}$$

If $\|\boldsymbol{z}\| \geqslant \varepsilon/(b-1)$, and $k_{1i}, k_{2i}\ (i=1,2)$ is chosen such that $\boldsymbol{P}_1 > 0$, $\boldsymbol{P}_2 > 0$, the following inequality holds

$$\begin{aligned}\dot{V}_2 \leqslant &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot\\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} -\\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} \leqslant 0\end{aligned} \tag{27}$$

It is clear that the condition of the negative definitiveness of $\dot{V}_2$ is guaranteed on the boundary of the following compact set

$$R = \left\{\boldsymbol{z}\,\middle|\,\|\boldsymbol{z}\| \geqslant \frac{\varepsilon}{b-1}\right\} \tag{28}$$

which can be made arbitrarily small by increasing b to sufficiency and decreasing the value of ε. From the definitions of Eqs.(12), (16), it is concluded that the attitude orientation and angular velocity tracking errors are uniformly ultimately bounded.

This completes the proof of Theorem 1.

2.2 Robust adaptive constrained controller design

In Subsection 2.1, we have shown how to design a robust adaptive controller for the flexible system with parametric uncertainties, external disturbances and unmeasured elastic vibration. However, the actuator saturation limits are not explicitly considered, which can lead to serious discrepancies between commanded input signals and actual control effort. To overcome this problem, a modified robust adaptive controller is proposed hereinafter.

Considering the actuator saturation in Eq.(7), the control input $\boldsymbol{u}^{\mathrm{T}} = [u_1, u_2, u_3]$ in Eq.(20) can be further defined as

$$u_i = \mathrm{sat}(\boldsymbol{u}_{ci}) = \begin{cases} u_{\max} & u_{ci} > u_{\max}\\ u_{ci} & -u_{\max} \leqslant u_{ci} \leqslant u_{\max}\\ -u_{\max} & u_{ci} < -u_{\max}\end{cases} \tag{29}$$

where $\boldsymbol{u}_{ci}$ is the ith element of the designed control law. To handle the input saturation, the following auxiliary system[22-23] is constructed as follows to compensate the effect of the input saturation

$$\dot{\boldsymbol{e}}_u = \begin{cases} -\boldsymbol{K}_u\boldsymbol{e}_u - \dfrac{f(\Delta\boldsymbol{u}, \boldsymbol{z})\boldsymbol{e}_u}{\|\boldsymbol{e}_u\|^2} - \Delta\boldsymbol{u} & \|\boldsymbol{e}_u\| \geqslant \vartheta_1\\ 0 & \|\boldsymbol{e}_u\| < \vartheta_1\end{cases} \tag{30}$$

where $f(\Delta\boldsymbol{u}, \boldsymbol{z}) = |\boldsymbol{z}^{\mathrm{T}}\Delta\boldsymbol{u}| + (1/2)\Delta\boldsymbol{u}^{\mathrm{T}}\Delta\boldsymbol{u}$, $\Delta\boldsymbol{u} = \boldsymbol{u} - \boldsymbol{u}_{\mathrm{c}}$ is the difference between the applied control and the designed control input, $\boldsymbol{K}_u = \boldsymbol{K}_u^{\mathrm{T}} > 0$, and $\boldsymbol{e}_u$ is the state of the auxiliary design system. The design parameter ϑ_1 is a positive constant which should be chosen as an appropriate value in accordance with the requirement of the tracking performance.

Using the elastic mode estimator Eq.(10) and the above designed auxiliary system, the constrained robust strategy is proposed as follows

$$\begin{aligned}\boldsymbol{u}_c = &-[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] + \\ &\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{\omega} + [\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\hat{\boldsymbol{\psi}} - \boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\hat{\boldsymbol{\psi}} + \boldsymbol{K}\hat{\boldsymbol{\eta}})] - \\ &\frac{1}{2}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times})^{\mathrm{T}}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z}) - \frac{1}{2}(\boldsymbol{C}\boldsymbol{\delta})^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z}) - \\ &\frac{1}{2}(\boldsymbol{K}\boldsymbol{\delta})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z}) - \boldsymbol{F}\hat{\boldsymbol{\theta}}_{mb} - \\ &\boldsymbol{K}_3(\boldsymbol{z} - \boldsymbol{e}_u) - \frac{b\hat{\rho}\boldsymbol{z}}{\|\boldsymbol{z}\| + \varepsilon} - \frac{\boldsymbol{z}g(\boldsymbol{z})}{\varsigma^2 + \|\boldsymbol{z}\|^2}\end{aligned} \tag{31}$$

where ς is the state of the following design system

$$\dot{\varsigma} = \begin{cases} -\dfrac{g(\boldsymbol{z})\varsigma}{\varsigma^2 + \|\boldsymbol{z}\|^2} - k_4\varsigma & \|\boldsymbol{z}\| \geqslant \vartheta_2 \\ 0 & \|\boldsymbol{z}\| < \vartheta_2 \end{cases} \tag{32}$$

$$g(\boldsymbol{z}) = \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z}$$

The design parameter ϑ_2 is a positive constant which should be chosen as a small value.

The stability of the closed-loop control system with the proposed control strategy is stated by the following theorem.

Theorem 2 Considering the system Eqs.(1), (4), (5), with the Assumptions 1—2. If the robust control law is designed in Eq.(31) subject to saturation as shown in Eq.(29) and the adaptive law is selected as in Eq.(22), the attitude orientation and angular velocity tracking errors are uniformly ultimately bounded.

Proof This result is proved by considering the following augmented Lyapunov function candidate

$$V_3 = V_2 + \frac{1}{2}\boldsymbol{e}_u^{\mathrm{T}}\boldsymbol{e}_u + \frac{1}{2}\varsigma^2 \tag{33}$$

When the control law Eq.(31) is used instead of Eq.(21), the derivative of Eq.(23) can be algebraically rearranged in steps identical to those employed in deriving Eq.(26), namely

$$\begin{aligned}\dot{V}_2 \leqslant &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} + \\ &\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{d} - \frac{b\hat{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon} + \frac{b\bar{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon}\right) + \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{e}_u - \frac{\boldsymbol{z}g(\boldsymbol{z})}{\varsigma^2 + \|\boldsymbol{z}\|^2} + \boldsymbol{z}^{\mathrm{T}}\Delta\boldsymbol{u}\end{aligned} \tag{34}$$

Taking the derivative of Eq.(33) along with Eqs.(30) and (32), the following inequality holds

$$\begin{aligned}\dot{V}_3 \leqslant &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - k_4\varsigma^2 - \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} + \\ &\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{d} - \frac{b\hat{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon} + \frac{b\bar{\rho}\|\boldsymbol{z}\|^2}{\|\boldsymbol{z}\| + \varepsilon}\right) - \\ &\boldsymbol{e}_u^{\mathrm{T}}\boldsymbol{K}_u\boldsymbol{e}_u + (\boldsymbol{z}^{\mathrm{T}}\Delta\boldsymbol{u} - |\boldsymbol{z}^{\mathrm{T}}\Delta\boldsymbol{u}|) + \\ &\left(\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{e}_u - \boldsymbol{e}_u^{\mathrm{T}}\Delta\boldsymbol{u} - \frac{1}{2}\Delta\boldsymbol{u}^{\mathrm{T}}\Delta\boldsymbol{u}\right)\end{aligned} \tag{35}$$

Considering the following fact

$$\begin{aligned}&\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{e}_u - \boldsymbol{e}_u^{\mathrm{T}}\Delta\boldsymbol{u} - \frac{1}{2}\Delta\boldsymbol{u}^{\mathrm{T}}\Delta\boldsymbol{u} \leqslant \\ &\frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} + \frac{1}{2}\boldsymbol{e}_u^{\mathrm{T}}\boldsymbol{e}_u + \\ &\frac{1}{2}\boldsymbol{e}_u^{\mathrm{T}}\boldsymbol{e}_u + \frac{1}{2}\Delta\boldsymbol{u}^{\mathrm{T}}\Delta\boldsymbol{u} - \frac{1}{2}\Delta\boldsymbol{u}^{\mathrm{T}}\Delta\boldsymbol{u} = \\ &\frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} + \boldsymbol{e}_u^{\mathrm{T}}\boldsymbol{e}_u\end{aligned} \tag{36}$$

Then if $\|\boldsymbol{z}\| \geqslant \varepsilon/(b-1)$, and $k_{1i}, k_{2i}(i=1,2)$ is chosen such that $\boldsymbol{P}_1 > 0$, $\boldsymbol{P}_2 > 0$, the following inequality holds

$$\begin{aligned}\dot{V}_3 \leqslant &-[\boldsymbol{q}_v^{\mathrm{T}} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &[\boldsymbol{q}_v + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})] - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} - \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \boldsymbol{e}_u^{\mathrm{T}}(\boldsymbol{K}_u - \boldsymbol{I})\boldsymbol{e}_u - k_4\varsigma^2 \leqslant 0\end{aligned} \tag{37}$$

Similar to Section 2.1, it can be concluded that the attitude orientation and angular velocity tracking errors are uniformly ultimately bounded.

Remark 1 It is worth to point out that input constraint is taken into account directly during the design stage to avoid the occurrence of saturation. As shown in the following numerical simulations, control inputs are in their valid operating ranges.

3 Simulation

To verify the effectiveness and performance of the proposed control scheme including modal observer in Eq.(10), parameter updating law in Eq.(22) and control law in Eq.(31), numerical simulations have been conducted in this section using the flexible spacecraft system governed by Eqs.

(1), (4), (5). The spacecraft is characterized by a nominal main body inertia matrix

$$\boldsymbol{J}=\begin{bmatrix}350 & 3 & 4\\ 3 & 280 & 10\\ 4 & 10 & 190\end{bmatrix}\text{kg}\cdot\text{m}^2$$

and by the coupling matrix

$$\boldsymbol{\delta}=\begin{bmatrix}6.456\,37 & 1.278\,14 & 2.156\,29\\ -1.256\,19 & 0.917\,56 & -1.672\,64\\ 1.116\,87 & 2.489\,01 & -0.836\,74\\ 1.236\,37 & -2.658\,1 & -1.125\,03\end{bmatrix}\text{kg}^{1/2}\cdot\text{m}\cdot\text{s}^{-2}$$

respectively. Then the matrix $\boldsymbol{J}_{mb}=\boldsymbol{J}-\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$ is given by

$$\boldsymbol{J}=\begin{bmatrix}303.961\,3 & -3.593\,0 & -9.697\,5\\ -3.593\,0 & 264.263\,8 & 7.870\,9\\ -9.697\,5 & 7.870\,9 & 180.586\,9\end{bmatrix}\text{kg}\cdot\text{m}^2$$

The first four elastic modes have been considered for the implemented spacecraft model resulting from the modal analysis of the structure, with natural frequency and damping presented in Table 1.

Table 1 Parameters of the flexible dynamics

Mode	Natural frequency/(rad·s^{-1})	Damping
Mode 1	1.097 3	0.05
Mode 2	1.276 1	0.06
Mode 3	1.653 8	0.08
Mode 4	2.289 3	0.025

In the following simulations, the rest-to-rest slew maneuver is considered to bring a flexible spacecraft with any initial nonzero attitude to zero and then to keep it resting at zero attitude. The initial attitude and angular velocity are chosen to be $\boldsymbol{q}^{\mathrm{T}}(0)=[0.173\,648,\ 0.837\,087,\ -0.443\,163,\ 0.269\,701]$ and $\boldsymbol{\omega}^{\mathrm{T}}(0)=[0,0,0]$. In addition, the initial modal variables and its time derivative are given by $\boldsymbol{\eta}(0)=0$ and $\boldsymbol{\psi}(0)=\dot{\boldsymbol{\eta}}(0)+\boldsymbol{\delta\omega}(0)=0$.

To examine the robustness to external disturbance, simulation is done corresponding to the following periodic disturbance torque

$$\boldsymbol{d}(t)=\begin{bmatrix}0.03\cos(0.01t)+0.1\\ 0.015\sin(0.02t)+0.03\cos(0.025t)\\ 0.03\sin(0.01t)+0.01\end{bmatrix}\text{N}\cdot\text{m}$$

For the purpose of comparison, two different sets of simulation are conducted to demonstrate the effectiveness of the proposed approach as follows:

(1) Attitude control using the constrained control law Eq.(31) and its combination with the elastic mode estimator Eq.(10) integrated with auxiliary system Eq.(30) to handle the input saturation, labeled as "Ⅰ" in figure legend.

(2) Attitude control using the control law Eq.(21) and its combination with the elastic mode estimator Eq.(10), labeled as "Ⅱ" in figure legend.

In the following simulations, the control and adaptation gains are selected by trial-and-error until a good performance is obtained for above cases, which are tabulated in Table 2. The maximum value of the control torque of actuators is assumed to be 30 N·m.

Table 2 Design parameters for the different controllers

Control scheme	Parameter and value
Proposed controller in Eq.(21)	$k_{11}=k_{12}=1,\ k_{21}=k_{22}=10,\ K_3=I_3$ $\Gamma=0.01I_6,\ a=0.000\,1,\ b=101,\ \varepsilon=0.000\,1$
Proposed controller in Eq.(31)	$k_{11}=k_{12}=1,\ k_{21}=k_{22}=10,\ K_3=I_3$ $\Gamma=0.01I_6,\ a=0.000\,1,\ b=101,\ \varepsilon=0.000\,1$ $K_u=2I_3,\ k_4=1,\ \vartheta_1=\vartheta_2=0.01$

3.1 Constrained robust adaptive control design

The results of the proposed constrained robust adaptive controller are summarized from Figs.1—11 in the presence of parametric uncertainties, external disturbances, unmeasured elastic vibration and input saturation constraints. From Figs.1, 2, it can be seen that attitude quaternion almost converge to its equilibrium point around the time of 100 s, and the steady error of each quaternion component is less than 0.001 4. Fig.3 shows the response of angular velocity of the flexible spacecraft and the steady errors are no more than 1.32e−5 rad/s.

The behavior of the modal displacements and their estimates are given in Fig.5. It is noted that all the elastic vibrations and their rates approach zero at time 80 s. It can be observed that not only the vibrations induced by attitude maneuver are effectively suppressed but also the modal displacements can be well estimated by the modal observer, whose performance is explicitly demonstrated in Fig. 6. The steady observation errors of modal observer in

Eq.(10) are tabulated in Table 3.

Table 3 Steady observation errors of modal observer

Parameter		Steady observation errors
Mode 1	$\lvert \eta_1 - \hat{\eta}_1 \rvert$	7.381e−6
Mode 2	$\lvert \eta_2 - \hat{\eta}_2 \rvert$	1.61e−7
Mode 3	$\lvert \eta_3 - \hat{\eta}_3 \rvert$	0
Mode 4	$\lvert \eta_4 - \hat{\eta}_4 \rvert$	3.92e−7

The responses of estimated inertial parameters corresponding to update law of Eq.(22) are illustrated in Figs.7, 8. It is clear that the convergence of these estimated parameters can be achieved, but not to the true values. That is because sufficient frequency components in the tracking error states are not guaranteed. In other words, the persistent excitation (PE) condition is not satisfied.

Furthermore, time responses of the demand control torque are depicted from Figs. 9—11. Thanks to the auxiliary system introduced to compensate the saturation effect, we can see that the restrictions on the actuator output torque magnitude are satisfied.

Specifically, from the curves of the control torques, the control input derived from the proposed strategy is saturated in the initial transient phase. After this period of time, the control saturation no longer takes effect, and the attitude quaternion and angular velocity converge to the equilibrium with a good performance.

3.2 Robust adaptive control design

For comparison, the system is also controlled by using the robust adaptive control law in Eq.(21). The same simulation is repeated under the same initial conditions, and the results are shown in Figs.1—11 (dotted line). As demonstrated in Fig.4 (dotted line), it can be observed that a significant amount of oscillations of the angular velocity occur in steady state regimes. The steady errors of quaternion and angular velocity are 0.001 6 and 1.4e−4 rad/s, respectively, which are larger than those of the proposed constrained robust adaptive control method mentioned in Section 3.1.

In addition, in Figs.9—11, the constraints on the control magnitude are violated and excessive control chattering exists since the actuator saturation is not explicitly considered in the control law Eq.(21).

From the comparison, it is shown that the proposed constrained robust control approach can accomplish quick attitude rotational maneuver, and

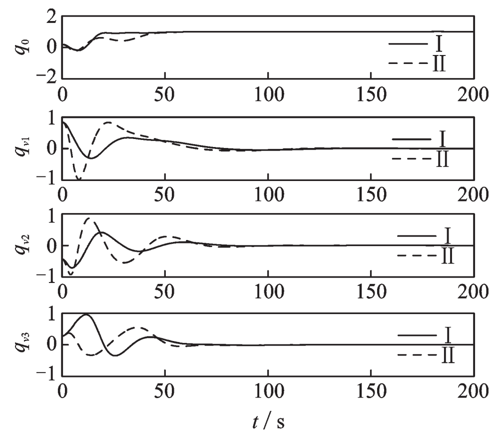


Fig. 1 Time response of quaternion

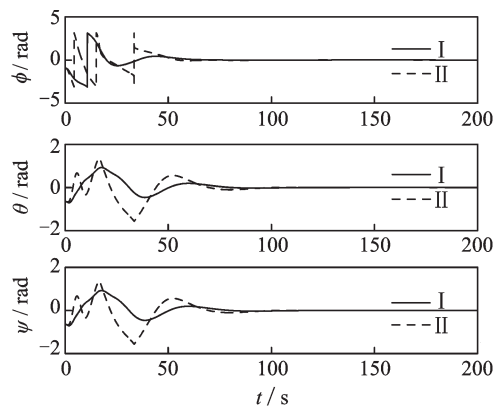


Fig. 2 Time response of Euler angles

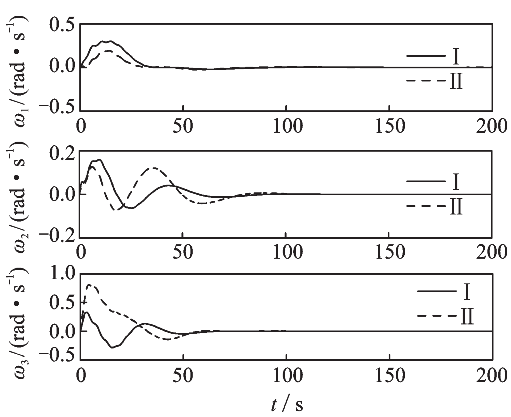


Fig. 3 Time response of angular velocity

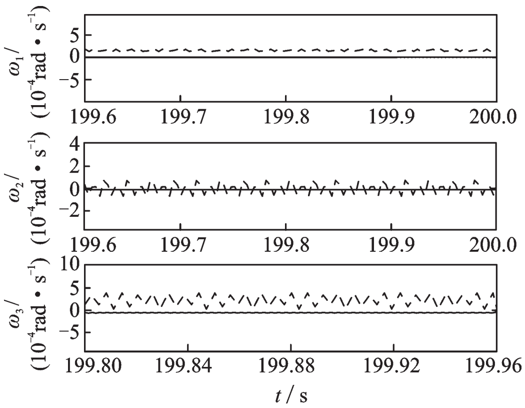


Fig. 4 Time response of the enlarged angular velocity

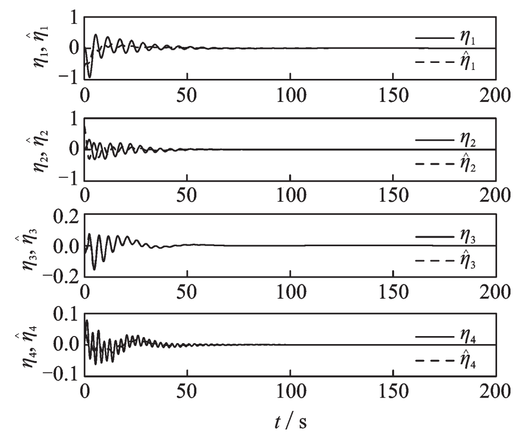


Fig. 5 Time response of vibration displacements and their estimates

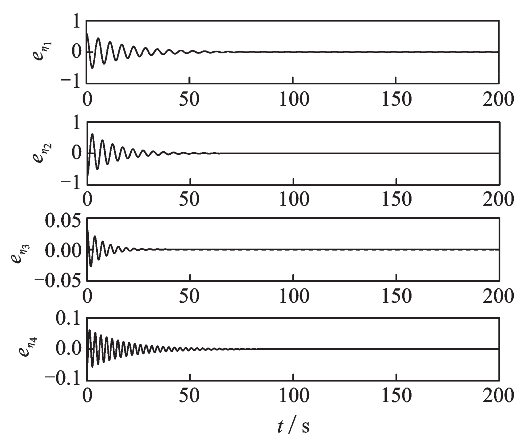


Fig. 6 Time response of vibration estimate errors

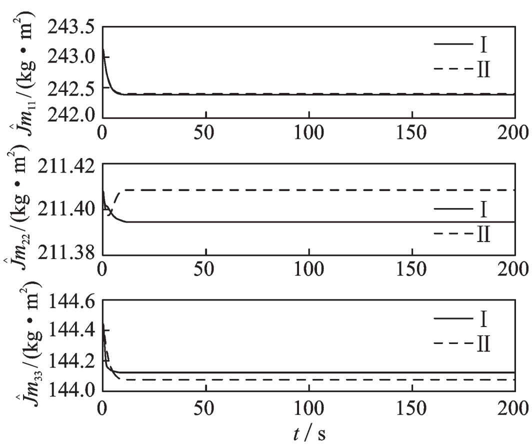


Fig. 7 Time response of the estimated parameters of inertia

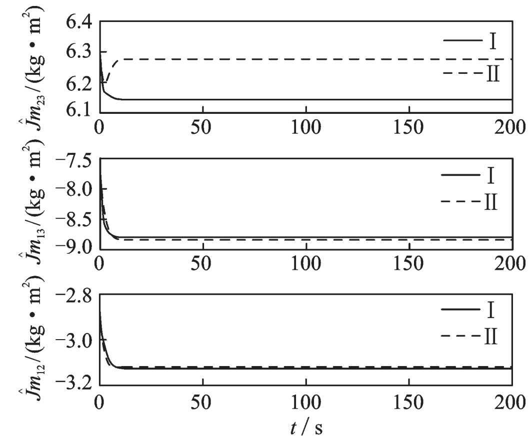
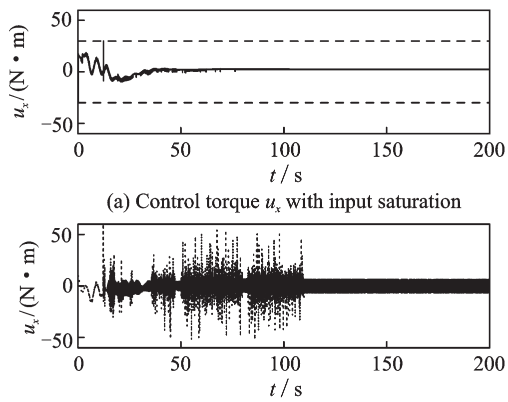


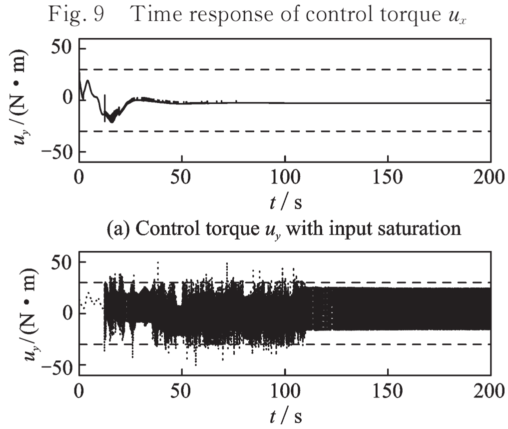
Fig. 8 Time response of the estimated parameters of the product of inertia



(a) Control torque u_x with input saturation

(b) Control torque u_x without input saturation

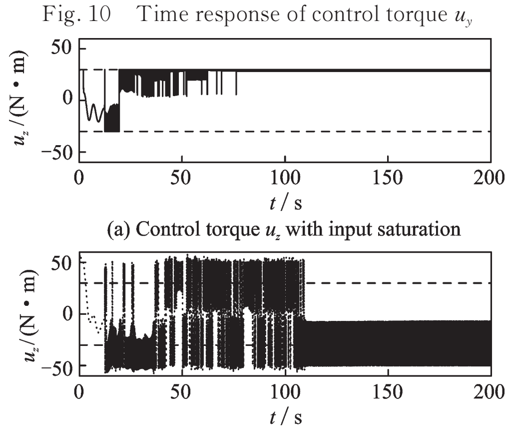
Fig. 9 Time response of control torque u_x



(a) Control torque u_y with input saturation

(b) Control torque u_y without input saturation

Fig. 10 Time response of control torque u_y



(a) Control torque u_z with input saturation

(b) Control torque u_z without input saturation

Fig. 11 Time response of control torque u_z

suppress the undesired vibrations of the appendages even though the uncertainties, external disturbances and control input saturation are explicitly considered. Furthermore, the information of upper bound of the perturbations and uncertainty is not required beforehand when the adaptive law of the developed control is adopted.

4 Conclusions

The problem of attitude maneuver and vibration suppression of flexible spacecraft has been investigated taking parametric uncertainties, external disturbances, unmeasured elastic vibration and input saturation constraints into account simultaneously. Firstly, a modal observer to supply elastic modal estimates is constructed by utilizing the inherent physical properties of flexible appendages. Then, an adaptive law is derived to remove the requirements to know the upper bound of parametric uncertainties and external disturbance. Furthermore, an auxiliary design system is introduced to analyze and compensate the input saturation effect. For the closed-loop system, the stability is rigorously proved by using the Lyapunov method and the convergence of all closed-loop signals is guaranteed. Finally, numerical simulations are conducted to verify and demonstrate the effectiveness of the proposed control methodology.

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Author contributions Dr. **TAO Jiawei** designed the study, complied the models, conducted the analysis, interpreted the results and wrote the manuscript. Prof. **ZHANG Tao** contributed to the discussion and background the study. All authors commented on the manuscript draft and approved the final manuscript.

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