Sliding Mode Fault Tolerant Attitude Control Scheme for Spacecraft with Actuator Faults

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Abstract: An active fault tolerant control scheme is investigated for the attitude control systems of spacecraft with external disturbance and actuator faults by using the sliding mode technique. Firstly, the dynamic equations and kinematic equations of spacecraft are given. For the dynamic mode of spacecraft in faulty case, a fault diagnosis component is used for fault detection and estimation by using a nonlinear observer. According to the fault estimation information obtained during the fault diagnosis, the fault tolerant control scheme is developed by adopting the backstepping sliding mode control technique. Meanwhile, the Lyapunov theory is used to analyze the stability of the closed-loop attitude systems. Finally, simulation results for the attitude dynamics models show the feasibility of the proposed fault tolerant scheme.

Key words: fault tolerant control; fault estimation; actuator faults; sliding mode control

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0 Introduction

Due to the diversity and complexity of the spacecraft mission, and its harsh working conditions (such as vacuum, weightlessness, high temperature and strong radiation), the spacecraft components are faced with the question of aging and fault. Component failure can not only reduce the performance of the attitude system, but also lead to the instability of attitude control system and even the failure of the entire space mission. Therefore, the spacecraft has put forward higher requirements on the safety and reliability of the attitude control system, especially in the condition of actuator fault. To improve the performance of spacecraft, the fault tolerant control (FTC) scheme must be considered in the design process of attitude control systems[1].

In the past ten years, some research results have been reported about fault tolerant control of spacecraft. Gao et al.[2] proposed an integrated fault diagnosis and fault tolerant control scheme for nonlinear satellite attitude control systems. With the design of a bank of unscented Kalman filters and state augmentation, the unknown fault parameters and system states can be jointly estimated. Han et al.[3] developed an adaptive fault-tolerant control method for spacecraft experiencing two kinds of actuator failures. Shen et al.[4] designed a fault-tolerant control scheme for spacecraft attitude stabilization with external disturbances. The approach is based on integral-type sliding mode control strategy to compensate for actuator faults without controller reconfiguration. Zhang et al.[5] investigated the problem of robust fault-tolerant control for flexible spacecraft with disturbances and actuator failures. The linear matrix inequality technique based conditions are formulated for the existence of the admissible controller, which ensures that the faulty closed-loop system is asymptotically stable with a disturbance attenuation level and partial loss of actuator effectiveness. Ma et al.[6] provided a fault-tolerant adaptive control approach

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for attitude tracking of flexible spacecraft with external disturbance, unknown inertia parameters and actuator faults. Using an uncertainty decomposition, the uncertainties of flexibility and dynamics are parameterized, and the control gain matrix uncertainty is handled. Additionally, an observer-based fault diagnosis scheme has been incorporated into a control allocation framework to solve the attitude control problem of spacecraft in the presence of actuator faults, external disturbances and actuator saturation[7]. To best of our knowledge, the important issue of fault tolerant tracking control for spacecraft in actuator fault case has not been fully investigated yet, which remains challenging and motivates us to do this study.

Herein, a dynamical model of spacecraft with external disturbances and an actuator fault is firstly presented. The main results are given later, which includes a nonlinear fault detection observer and a nonlinear fault estimation observer. Then, a sliding model control based fault tolerant control scheme is proposed for the faulty dynamic system of spacecraft. Simulation results show the effectiveness of the proposed technique, and finally some conclusions are drawn.

1 Attitude Control Systems Description

A rigid spacecraft system can be described by the following attitude kinematics and dynamics equations[8]

$$\begin{bmatrix}\dot{\varphi}\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix}=\frac{1}{\cos\theta}\begin{bmatrix}\cos\theta & \sin\varphi\sin\theta & \cos\varphi\sin\theta\\ 0 & \cos\varphi\cos\theta & -\sin\varphi\cos\theta\\ 0 & \sin\varphi & \cos\varphi\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}+\frac{\omega_0}{\cos\theta}\begin{bmatrix}\sin\psi\\ \cos\theta\cos\psi\\ \sin\theta\sin\psi\end{bmatrix} \tag{1}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}=-\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega}+3\omega_0^2\boldsymbol{\zeta}^{\times}\boldsymbol{J}\boldsymbol{\zeta}+\boldsymbol{u}+\boldsymbol{d}(t) \tag{2}$$

where φ, θ and ψ are the pitch angle, yaw angle, and roll angle, respectively. $\boldsymbol{\omega}=[\omega_x,\omega_y,\omega_z]^{\mathrm{T}}$ are the pitch rate, yaw rate, and roll rate, respectively. The nonlinear element ζ is defined as $\boldsymbol{\zeta}=[-\sin\theta,\sin\varphi\cos\theta,\cos\varphi\cos\theta]^{\mathrm{T}}$. $(\cdot)^{\times}$ represents the skew-symmetric matrix. ω_0 is the constant orbital rate. $\boldsymbol{u}=[u_x,u_y,u_z]$ denotes the combined control torque produced by the actuator and can be written as follows [9]

$$\boldsymbol{u}=\boldsymbol{D}\boldsymbol{\tau}(t) \tag{3}$$

where $\boldsymbol{D}=[\boldsymbol{D}_1,\boldsymbol{D}_2,\boldsymbol{D}_3,\boldsymbol{D}_4]\in\mathbf{R}^{3\times4}$ is configuration matrix of the reaction wheel, representing the influence of each wheel on the angular acceleration of the spacecraft, and $\boldsymbol{\tau}=[\tau_1,\tau_2,\tau_3,\tau_4]^{\mathrm{T}}$ denotes the torques produced by the four reaction fly-wheels. We define $\boldsymbol{\sigma}=[\varphi,\theta,\psi]^{\mathrm{T}}$. For small attitude angles, the attitude dynamic model can be described as

$$\begin{cases}\dot{\boldsymbol{\sigma}}=\boldsymbol{\omega}+\boldsymbol{F}\boldsymbol{\sigma}\\ \dot{\boldsymbol{\omega}}=\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega}+\boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{\tau}+\boldsymbol{J}^{-1}\boldsymbol{d}(t)\\ \boldsymbol{y}=\boldsymbol{\sigma}\end{cases} \tag{4}$$

where $\boldsymbol{J}\in\mathbf{R}^{3\times3}$ is the total inertia matrix of the spacecraft, and $\boldsymbol{d}(t)=[d_1,d_2,d_3]^{\mathrm{T}}\in\mathbf{R}^3$ denotes the external disturbance torque. $\boldsymbol{F}_{\omega}=-\boldsymbol{\omega}^{\times}\boldsymbol{J}\boldsymbol{\omega}+3\omega_0^2\boldsymbol{\zeta}\times\boldsymbol{J}\boldsymbol{\zeta}$, and

$$\boldsymbol{F}=\begin{bmatrix}0 & 0 & \omega_0\\ 0 & 0 & 0\\ -\omega_0 & 0 & 0\end{bmatrix}$$

In particular, the situation in which the actuator loses complete or partial control power is considered. $\tau_i^F(t)$ is used to describe the control sent from the ith the four reaction wheels as follows

$$\tau_i^F=e_i\tau_i(t)\quad e_i\in(0,1]$$

where e_i is an unknown constant representing the effectiveness factor of the ith control wheel. τ_i denotes the desired control signal of the ith actuator generated by the controller($i=1,2,3,4$).The case $e_i(t)=1$ implies that the ith actuator is healthy, and $0<e_i(t)<1$ corresponds to the case in which the ith actuator partially loses its actuating power but still works all the time. We define $\boldsymbol{E}=\mathrm{diag}\{\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3,\boldsymbol{e}_4\}$. Then the general nonlinear spacecraft attitude dynamics models with actuator fault can be rewritten as

$$\dot{\boldsymbol{\omega}}=\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega}+\boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{E}\boldsymbol{\tau}+\boldsymbol{J}^{-1}\boldsymbol{d}(t) \tag{5}$$

The objective of this study is to design a fault tolerant controller for the rigid spacecraft attitude systems Eq.(4) with external disturbance and actuator faults by using the sliding mode control technique such that the signals of the closed-loop system are bounded and the attitude angle $\boldsymbol{\sigma}$ can asymptotical track the desired attitude command $\boldsymbol{\sigma}_{\mathrm{d}}$. Fig. 1 shows the structure of fault tolerant attitude control

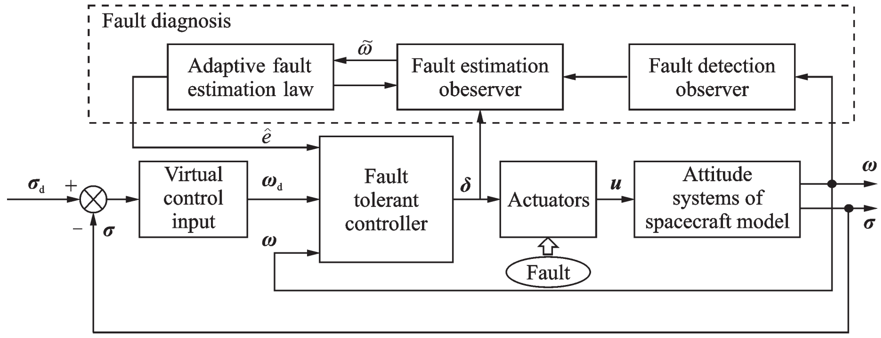

Fig. 1 The structure of fault tolerant attitude control system

system.

Assumption 1 The time-varying external disturbance $\boldsymbol{d}(t)$ is unknown bounded continuous function, namely, there exist a known positive constant scalar $\bar{d}>0$, such that $\|\boldsymbol{d}\|<\bar{d}$.

Assumption 2 The nonlinear function $\boldsymbol{\phi}(\boldsymbol{\omega},t)=\boldsymbol{\omega}\times \boldsymbol{J\omega}$ is locally Lipschitz nonlinear, i.e., there exists a constant β satisfying $\|\boldsymbol{\phi}(\boldsymbol{\omega},t)-\boldsymbol{\phi}(\hat{\boldsymbol{\omega}},t)\|\leqslant\beta\|\boldsymbol{\omega}-\hat{\boldsymbol{\omega}}\|$.

Lemma 1[10] (Young's inequality) For any vectors $\boldsymbol{x},\boldsymbol{y}\in \mathrm{R}^n$, the following inequality must exist

$$\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y}\leqslant\frac{a^p}{p}\|\boldsymbol{x}\|^p+\frac{1}{qa^q}\|\boldsymbol{y}\|^q$$

where $a>0,p>1$, and $(p-1)(q-1)=1$.

Remark 1 For a sufficiently small time, assumption 2 is reasonable because the system state $\boldsymbol{\omega}$ is first-order differentiable and its derivative is bounded [11].

2 Faults Detection and Estimation Design

In this section, we consider the rigid spacecraft in fault case. In order to quickly determine the time of actuator fault occurred, a nonlinear fault detection observer is designed for the dynamics equation of spacecraft in actuator healthy case, that is

$$\dot{\bar{\boldsymbol{\omega}}}=-\boldsymbol{\Lambda}(\bar{\boldsymbol{\omega}}-\boldsymbol{\omega})+\boldsymbol{J}^{-1}\hat{\boldsymbol{F}}_{\omega}+\boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{\tau}+\boldsymbol{J}^{-1}\bar{d}\boldsymbol{\varrho} \tag{6}$$

where $\bar{\boldsymbol{\omega}}$ is the estimated angular rate vector, $\boldsymbol{\Lambda}=\mathrm{diag}\{\lambda_1,\lambda_2,\lambda_3\}$, $\lambda_i>0$ are the eigenvalues of fault detection observer, which are determined in advance. $\boldsymbol{\varrho}=[1,1,1]^{\mathrm{T}}$.

Let $\boldsymbol{e}_{\omega}=\boldsymbol{\omega}-\bar{\boldsymbol{\omega}}_{\mathrm{d}}$ and $\boldsymbol{\kappa}=\boldsymbol{d}-\bar{d}\boldsymbol{\varrho}$, and by subtracting Eq. (6) from Eq. (5), the error dynamic equation of fault detection observer can be obtained as

$$\begin{aligned}\dot{\boldsymbol{e}}_{\omega}=&-\boldsymbol{\Lambda}\boldsymbol{e}_{\omega}-\boldsymbol{J}^{-1}(\boldsymbol{\phi}(\boldsymbol{\omega},t)-\boldsymbol{\phi}(\hat{\boldsymbol{\omega}},t))\boldsymbol{J}^{-1}\boldsymbol{\kappa}-\\&\boldsymbol{J}^{-1}\boldsymbol{D}(\boldsymbol{I}-\boldsymbol{E})\boldsymbol{\tau}\\&\boldsymbol{r}=\boldsymbol{e}_{\omega}\end{aligned} \tag{7}$$

The remaining important task is the evaluation of the generated residual $\boldsymbol{r}$. One of the widely adopted approaches is to choose a so-called threshold J_{th}, and based on this, the following logical relationship for fault detection can be used.

$$\|\boldsymbol{r}\|_{2,T}>J_{\mathrm{th}}\ \Rightarrow\ \text{A fault occurs}\Rightarrow\ \text{Alarm}$$
$$\|\boldsymbol{r}\|_{2,T}\leqslant J_{\mathrm{th}}\ \Rightarrow\ \text{No fault}$$

where the so-called residual evaluation function $\|\boldsymbol{r}\|_{2,T}$ is determined by

$$\|\boldsymbol{r}\|_{2,T}=\int_0^T\boldsymbol{r}^{\mathrm{T}}(t)\boldsymbol{r}(t)\mathrm{d}t \tag{8}$$

where $t\in(0,T]$ is the finite-time window. Note that the length of time window is finite. Since an evaluation of residual signal over the whole time range is impractical, it is desired that the faults will be detected as early as possible. When an actuator fault is timely detected by using fault detection observer in Eq. (6), the next task is to estimate the fault so as to utilize effectively the estimated fault information for fault tolerant control design. Here, the disturbance observer idea is used and the adaptive nonlinear observer for reconstructed actuator the loss of effectiveness failure information is designed. As $\boldsymbol{E}(t)$ is a diagonal, the term $\boldsymbol{E}(t)\boldsymbol{\tau}(t)$ in Eq. (5) can be rearranged as

$$\boldsymbol{E}(t)\boldsymbol{\tau}(t)=\boldsymbol{U}(t)\boldsymbol{e}(t) \tag{9}$$

where $\boldsymbol{U}=\mathrm{diag}\{\tau_1,\tau_2,\tau_3,\tau_4\}$ and $\boldsymbol{e}=[e_1,e_2,e_3,e_4]^{\mathrm{T}}$. Using Eq. (9), the faulty dynamic equation can be written as

$$\dot{\boldsymbol{\omega}}=\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega}+\boldsymbol{J}^{-1}\boldsymbol{DUe}+\boldsymbol{J}^{-1}\boldsymbol{d}(t) \tag{10}$$

Let $\tilde{\boldsymbol{\omega}}=\boldsymbol{\omega}-\hat{\boldsymbol{\omega}}$, the sliding mode fault estimation observer for the information of the effectiveness of failure and disturbances by the angular velocity loop can be designed as

$$\dot{\hat{\boldsymbol{\omega}}}=-\boldsymbol{L}(\hat{\boldsymbol{\omega}}-\boldsymbol{\omega})+\boldsymbol{J}^{-1}\hat{\boldsymbol{F}}_{\omega}+\boldsymbol{J}^{-1}\boldsymbol{DU}\hat{\boldsymbol{e}}+\boldsymbol{J}^{-1}\bar{d}\,\mathrm{sign}(\tilde{\boldsymbol{\omega}}) \tag{11}$$

where $\hat{\boldsymbol{\omega}}$ is the estimation value of $\boldsymbol{\omega}$. $\hat{\boldsymbol{e}}=[\hat{e}_1,\hat{e}_2,\hat{e}_3,\hat{e}_4]^{\mathrm{T}}$ denotes the estimation of the loss of effectiveness factor, which is obtained as

$$\dot{\hat{\boldsymbol{e}}}=-\lambda\hat{\boldsymbol{e}}+\gamma\boldsymbol{U}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\boldsymbol{J}^{-1}\tilde{\boldsymbol{\omega}},\ \dot{\lambda}=-v\lambda,\ \lambda(0)>0 \tag{12}$$

where $\gamma>0$ and $v>0$ are two positive scalars. Let $\tilde{\boldsymbol{e}}=\boldsymbol{e}-\hat{\boldsymbol{e}}$ and $\boldsymbol{\kappa}=\boldsymbol{d}-\bar{d}\,\mathrm{sign}(\tilde{\boldsymbol{\omega}})$. By using the aforementioned adaptive fault estimation observer, the resulting state estimation error dynamic is

$$\dot{\tilde{\boldsymbol{\omega}}}=-\boldsymbol{L}-\boldsymbol{J}^{-1}(\boldsymbol{\phi}(\boldsymbol{\omega},t)-\boldsymbol{\phi}(\hat{\boldsymbol{\omega}},t))+\boldsymbol{J}^{-1}\boldsymbol{DU}\tilde{\boldsymbol{e}}-\boldsymbol{J}^{-1}\boldsymbol{\kappa} \tag{13}$$

Hence, the following results for the aforementioned error dynamic can be obtained.

Theorem 1 Under Assumption 1—2, an adaptive sliding mode fault estimation observer is designed in the form of Eq. (11) for the faulty spacecraft dynamic Eq. (5). Then the error dynamic system of fault estimation observer is asymptotical stable by selecting a sufficiently large matrix $\boldsymbol{L}$. Meanwhile, the actuators' efficiency factors can be accurately estimated by the adaptive estimation law Eq.(12).

Proof Considering the following Lyapunov function

$$V_1=\frac{1}{2}\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\tilde{\boldsymbol{\omega}}+\frac{1}{2\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\tilde{\boldsymbol{e}}+\frac{1}{2\gamma v}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}\lambda,$$

the derivative of V_1 along the trajectory of the augmented state error dynamic Eq.(13) can be written as

$$\dot{V}_1=-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{L}\tilde{\boldsymbol{\omega}}-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{J}^{-1}(\boldsymbol{\phi}(\boldsymbol{\omega},t)-\boldsymbol{\phi}(\hat{\boldsymbol{\omega}},t))+\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{J}^{-1}\boldsymbol{DU}\tilde{\boldsymbol{e}}+\frac{1}{\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\dot{\tilde{\boldsymbol{e}}}-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{J}^{-1}\boldsymbol{\kappa}+\frac{1}{2\gamma v}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}\dot{\lambda} \tag{14}$$

Substituting adaptive update law Eq.(12) into Eq. (14) yields

$$\dot{V}_1=-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{L}\tilde{\boldsymbol{\omega}}-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{J}^{-1}(\boldsymbol{\phi}(\boldsymbol{\omega},t)-\boldsymbol{\phi}(\hat{\boldsymbol{\omega}},t))-\tilde{\boldsymbol{\omega}}^{\mathrm{T}}\boldsymbol{J}^{-1}\boldsymbol{\kappa}+\frac{\lambda}{\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\hat{\boldsymbol{e}}-\frac{1}{2\gamma}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}\lambda \tag{15}$$

Note that

$$\frac{\lambda}{\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\hat{\boldsymbol{e}}=-\frac{\lambda}{\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\tilde{\boldsymbol{e}}+\frac{\lambda}{\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\boldsymbol{e}\leqslant-\frac{\lambda}{2\gamma}\tilde{\boldsymbol{e}}^{\mathrm{T}}\tilde{\boldsymbol{e}}+\frac{\lambda}{2\gamma}\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} \tag{16}$$

Substituting Eq. (16) into Eq. (15), the inequality (15) can be transformed into

$$\dot{V}_1\leqslant-(\|L\|+\beta\|\boldsymbol{J}^{-1}\|)\|\tilde{\boldsymbol{\omega}}\|^2-\frac{1}{2\gamma v}\|\tilde{\boldsymbol{e}}\|^2$$

By selecting the appropriate positive definite matrix $\boldsymbol{L}$, we have $\dot{V}_1\leqslant 0$. Since $\dot{V}_1\leqslant 0$, V_1 is bounded. Note that V_1 is positive definite and radially unbounded. By LaSalle-Yoshizawa theorem [12], it follows that $\lim_{t\to\infty}V_1(0)=0$, implying that $\tilde{\boldsymbol{\omega}}\to 0$ and $\tilde{\boldsymbol{e}}\to 0$ as $t\to\infty$. Therefore, the system of adaptive fault estimation observer is globally asymptotically stable. This proof is completed.

Remark 2 In this paper, the designed sliding mode fault estimation observer has good robustness to external disturbance. Compared with the fault estimation observer design methods proposed in Refs. [13] and [14], the adaptive fault estimation laws in Eq. (12) made a significant improvement. Namely, the adaptive coefficient λ added in the adaptive fault estimation law given in Eq.(12) can effectively improve the accuracy and rapidity of fault estimation.

3 Adaptive Sliding Mode Fault-Tolerant Control Design

In this section, an adaptive fault-tolerant controller is designed for the rigid spacecraft systems based on backstepping sliding mode control method. To develop the robust attitude control scheme, we define

$$\boldsymbol{e}_1=\boldsymbol{\sigma}-\boldsymbol{\sigma}_{\mathrm{d}},\ \boldsymbol{e}_2=\boldsymbol{\omega}-\boldsymbol{\omega}_{\mathrm{d}}$$

For the attitude angle loop, an integrated sliding mode surface is formed as

$$\boldsymbol{S}_1=\boldsymbol{e}_1+\boldsymbol{K}_1\int_0^t\boldsymbol{e}_1\,\mathrm{d}t \tag{17}$$

where $\boldsymbol{K}_1=\mathrm{diag}\{k_1,k_1,k_1\}$, and k_1 is a positive odd integer. Taking the time differentiating of $\boldsymbol{S}_1$ in

Eq. (17), we have

$$\dot{\boldsymbol{S}}_1 = \dot{\boldsymbol{e}}_1 + \boldsymbol{K}_1\boldsymbol{e}_1 = \boldsymbol{e}_2 + \boldsymbol{\omega}_{\mathrm{d}} + \boldsymbol{F}\boldsymbol{\sigma} - \dot{\boldsymbol{\sigma}}_{\mathrm{d}} + \boldsymbol{K}_1\boldsymbol{e}_1 \quad (18)$$

We select the exponential rate as the asymptotical reaching law of sliding mode $\boldsymbol{S}_1$, namely

$$\dot{\boldsymbol{S}}_1 = -\upsilon_1\boldsymbol{S}_1 - \varepsilon_1\,\mathrm{sgn}(\boldsymbol{S}_1) \quad (19)$$

where υ_1 and ε_1 are two positive constant scalars. According to Eqs. (18) and (19), the virtual control input $\boldsymbol{\omega}_{\mathrm{d}}$ is selected as

$$\boldsymbol{\omega}_{\mathrm{d}} = \dot{\boldsymbol{\sigma}}_{\mathrm{d}} - \boldsymbol{F}\boldsymbol{\sigma} - \boldsymbol{K}_1\boldsymbol{e}_1 - \upsilon_1\boldsymbol{S}_1 - \varepsilon_1\,\mathrm{sgn}(\boldsymbol{S}_1) \quad (20)$$

Based on the angular velocity error $\boldsymbol{e}_2$, the second integrated sliding mode surface is designed for the inner loop systems, namely

$$\boldsymbol{S}_2 = \boldsymbol{e}_2 + \boldsymbol{K}_2\int_0^t \boldsymbol{e}_2\,\mathrm{d}t \quad (21)$$

where $\boldsymbol{K}_2 = \mathrm{diag}\{k_2, k_2, k_2\}$, and k_2 is a positive odd integer. Taking the time differentiating of $\boldsymbol{S}_2$ in Eq. (21), we have

$$\dot{\boldsymbol{S}}_2 = \dot{\boldsymbol{e}}_2 + \boldsymbol{K}_2\boldsymbol{e}_2 = \boldsymbol{J}^{-1}\boldsymbol{F}_{\omega} + \boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{E}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d}(t) - \dot{\boldsymbol{\omega}}_{\mathrm{d}} + \boldsymbol{K}_2\boldsymbol{e}_2 \quad (22)$$

Here, the exponential reaching law is given as

$$\dot{\boldsymbol{S}}_2 = -\nu_2\boldsymbol{S}_2 - \varepsilon_2\,\mathrm{sgn}(\boldsymbol{S}_2) \quad (23)$$

where υ_2 and ε_2 are two positive constant scalars. Using the output of parameter approximation, the control input τ is chosen as

$$\boldsymbol{\tau} = -(\boldsymbol{J}^{-1}\boldsymbol{D}\hat{\boldsymbol{E}})^{+}(\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega} + \boldsymbol{J}^{-1}\hat{\boldsymbol{d}} - \dot{\boldsymbol{\omega}}_{\mathrm{d}} + \boldsymbol{K}_2\boldsymbol{e}_2 + \boldsymbol{\nu}_2\boldsymbol{S}_2 + \boldsymbol{\varepsilon}_2\mathrm{sgn}(\boldsymbol{S}_2)) \quad (24)$$

Here $\hat{\boldsymbol{E}} = \mathrm{diag}\{\hat{\boldsymbol{e}}_1, \hat{\boldsymbol{e}}_2, \hat{\boldsymbol{e}}_3, \hat{\boldsymbol{e}}_4\}$, which can be obtained from Eq. (12). $(\boldsymbol{J}^{-1}\boldsymbol{D}\hat{\boldsymbol{E}})^{+} = (\boldsymbol{J}^{-1}\boldsymbol{D}\hat{\boldsymbol{E}})^{\mathrm{T}}[\boldsymbol{J}^{-1}\boldsymbol{D}\hat{\boldsymbol{E}}(\boldsymbol{J}^{-1}\boldsymbol{D}\hat{\boldsymbol{E}})^{\mathrm{T}}]^{-1}\tilde{\boldsymbol{d}} = \bar{d}\boldsymbol{\varrho} - \hat{\boldsymbol{d}}$, $\hat{\boldsymbol{d}}$ is the estimated value of $\bar{d}\boldsymbol{\varrho}$, and $\boldsymbol{\varrho} = [1, 1, 1]^{\mathrm{T}}$. The parameter updated law of $\hat{\boldsymbol{d}}$ is designed as

$$\dot{\hat{\boldsymbol{d}}} = \frac{1}{\Gamma}\boldsymbol{J}^{-\mathrm{T}}\boldsymbol{S}_2 \quad (25)$$

Theorem 2 For the attitude control systems of spacecraft Eq. (4) in faulty case, it is supposed that Assumptions 1－2 are satisfied, the proposed virtual control input $\boldsymbol{\omega}_{\mathrm{d}}$ and tracking control law τ described in Eqs. (20) and (24) guarantees that all the signals of the whole closed-loop attitude systems are uniformly ultimately bounded and the attitude angle $\boldsymbol{\sigma}$ could asymptotically track the desired attitude command $\boldsymbol{\sigma}_{\mathrm{d}}$.

Proof For the whole closed-loop attitude control system, a Lyapunov function is firstly given by

$$V = \frac{1}{2}\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1 + \frac{1}{2}\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2 + \frac{1}{2}\tilde{\boldsymbol{d}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{d}} \quad (26)$$

Taking the time derivative of $\boldsymbol{V}$, it can be obtained that

$$\dot{V} = \boldsymbol{S}_1^{\mathrm{T}}(\boldsymbol{e}_2 + \boldsymbol{\omega}_{\mathrm{d}} + \boldsymbol{F}\boldsymbol{\sigma} - \dot{\boldsymbol{\theta}}_{\mathrm{d}} + \boldsymbol{K}_1\boldsymbol{e}_1) + \boldsymbol{S}_2^{\mathrm{T}}(\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega} + \boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{E}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d}(t) - \dot{\boldsymbol{\omega}}_{\mathrm{d}} + \boldsymbol{K}_2\boldsymbol{e}_2) + \tilde{\boldsymbol{d}}^{\mathrm{T}}\Gamma\dot{\tilde{\boldsymbol{d}}} \quad (27)$$

Substituting the virtual control input $\boldsymbol{\omega}_{\mathrm{d}}$ described in Eq. (20) into Eq. (27), we have

$$\dot{V} = \boldsymbol{S}_1^{\mathrm{T}}(-\upsilon_1\boldsymbol{S}_1 - \varepsilon_1\,\mathrm{sgn}(\boldsymbol{S}_1) + \boldsymbol{e}_2) + \boldsymbol{S}_2^{\mathrm{T}}(\boldsymbol{J}^{-1}\boldsymbol{F}_{\omega} + \boldsymbol{J}^{-1}\boldsymbol{D}\boldsymbol{E}\boldsymbol{\tau} + \boldsymbol{J}^{-1}\boldsymbol{d}(t) - \dot{\boldsymbol{\omega}}_{\mathrm{d}} + \boldsymbol{K}_2\boldsymbol{e}_2) + \tilde{\boldsymbol{d}}^{\mathrm{T}}\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{d}}} \quad (28)$$

Substituting the passive fault tolerant controller τ described in Eq. (24) into Eq. (28), we have

$$\dot{V} \leqslant -\boldsymbol{S}_1^{\mathrm{T}}\upsilon_1\boldsymbol{S}_1 - \varepsilon_1\boldsymbol{S}_1^{\mathrm{T}}\,\mathrm{sgn}(\boldsymbol{S}_1) + \boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{e}_2 - \varepsilon_2\boldsymbol{S}_2^{\mathrm{T}}\,\mathrm{sgn}(\boldsymbol{S}_2) - \boldsymbol{S}_2^{\mathrm{T}}\upsilon_2\boldsymbol{S}_2 + \boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{J}^{-1}(\boldsymbol{d} - \bar{d}\boldsymbol{\varrho} + \bar{d}\boldsymbol{\varrho} - \hat{\boldsymbol{d}}) + \tilde{\boldsymbol{d}}^{\mathrm{T}}\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{d}}} \leqslant -\boldsymbol{S}_1^{\mathrm{T}}\big(\upsilon_1\boldsymbol{S}_1 + \varepsilon_1\mathrm{sgn}(\boldsymbol{S}_1)\big) + \boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{e}_2 - \boldsymbol{S}_2^{\mathrm{T}}(\upsilon_2\boldsymbol{S}_2 + \varepsilon_2\,\mathrm{sgn}(\boldsymbol{S}_2) - \boldsymbol{J}^{-1}(\boldsymbol{d} - \bar{d}\boldsymbol{\varrho}) - \boldsymbol{J}^{-1}\tilde{\boldsymbol{d}} + \tilde{\boldsymbol{d}}^{\mathrm{T}}\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{d}}} \quad (29)$$

According to Lemma 1, the following inequalities can be obtained

$$\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{e}_2 \leqslant \frac{1}{2}\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1 + \frac{1}{2}\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{e}_2 \leqslant \frac{1}{2}(\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1 + \boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2) \quad (30)$$

Substituting Eq. (25) into Eq. (30), we have

$$\dot{V} \leqslant -(\upsilon_1 - \frac{1}{2})\boldsymbol{S}_1^{\mathrm{T}}\boldsymbol{S}_1 - \varepsilon_1\boldsymbol{S}_1^{\mathrm{T}}\,\mathrm{sgn}(\boldsymbol{S}_1) - (\upsilon_2 - \frac{1}{2})\boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{S}_2 - \varepsilon_2\boldsymbol{S}_2^{\mathrm{T}}\mathrm{sgn}(\boldsymbol{S}_2) + \boldsymbol{S}_2^{\mathrm{T}}\boldsymbol{J}^{-1}(\boldsymbol{d} - \bar{d}\boldsymbol{\varrho}) \leqslant -(\upsilon_1 - \frac{1}{2})\|\boldsymbol{S}_1\|^2 - (\upsilon_2 - \frac{1}{2})\|\boldsymbol{S}_2\|^2 - \varepsilon_1\|\boldsymbol{S}_1\| - \big(\varepsilon_2 - \|\boldsymbol{J}^{-1}(\boldsymbol{d} - \bar{d}\boldsymbol{\varrho})\|\big)\|\boldsymbol{S}_2\|$$

Selecting sufficiently large ε_i and υ_i, such that $\varepsilon_1 > 1/2$, $\varepsilon_2 > 1/2$, $\upsilon_1 > 0$, $\upsilon_2 > \|\boldsymbol{J}^{-1}(\boldsymbol{d} - \bar{d}\boldsymbol{\varrho})\|$, we will have $\dot{V} \leqslant 0$. Since $\dot{V} \leqslant 0$, $V(t)$ is bounded. Note that $\dot{V} = 0$ implies that $\boldsymbol{S}_1 = \boldsymbol{0}$ and $\boldsymbol{S}_2 = \boldsymbol{0}$, which further implies that $\boldsymbol{e}_1 = 0$ and $\boldsymbol{e}_2 = 0$. Hence, by LaSalle's Invariance principle [13], it follows that $e_1(t) \to 0$ and $e_2(t) \to 0$, as $t \to \infty$. According to Lyapunov stability theory, it can be seen that the closed loop attitude system in actuator faulty case is asymptotically stable. This completes the proof of Theorem 2.

Remark 3 In terms of singular perturbation theory, the inner angular rate loop sliding mode dynamics in Eq. (21) must be much faster than the outer attitude angular loop sliding mode dynamics in Eq. (17) to preserve sufficient time-scale separation between the two loops. The parameters of controllers should satisfy $\upsilon_2 \geqslant 3\upsilon_1$ and $\varepsilon_2 \geqslant 3\varepsilon_1$.

Remark 4 In order to avoid eliminating the chattering phenomenon in simulation verification phase, the discontinuous terms $\mathrm{sgn}(\boldsymbol{S}_1)$ in Eq. (19) and $\mathrm{sgn}(\boldsymbol{S}_2)$ in Eq. (23) will be replaced by the saturation functions $\mathrm{sat}(\boldsymbol{S}_1/\vartheta)$ and $\mathrm{sat}(\boldsymbol{S}_2/\varpi)$, respectively. $\vartheta > 0$ and $\varpi > 0$ are the boundary layer thickness of saturation function.

$$\mathrm{sat}\left(\frac{S_{1i}}{\vartheta_i}\right)=\begin{cases}1 & S_{1i} > \vartheta_i \\ S_{1i}/\vartheta_i & S_{1i} \leqslant \vartheta_i \\ -1 & S_{1i} < -\vartheta_i\end{cases}$$

$$\mathrm{sat}\left(\frac{S_{2i}}{\varpi_i}\right)=\begin{cases}1 & S_{2i} > \varpi_i \\ S_{2i}/\varpi_i & S_{2i} \leqslant \varpi_i \\ -1 & S_{2i} < -\varpi_i\end{cases}$$

In the process of stability proof, the saturation terms $\mathrm{sat}(\boldsymbol{S}_1/\vartheta)$ and $\mathrm{sat}(\boldsymbol{S}_2/\varpi)$ are the same as in the function of the symbolic terms. It guarantees not only the positivity of the terms $\boldsymbol{S}_1^{\mathrm{T}}\mathrm{sat}(\boldsymbol{S}_1/\vartheta)$ and $\boldsymbol{S}_2^{\mathrm{T}}\mathrm{sat}(\boldsymbol{S}_2/\varpi)$, but also the continuity from the -1 to the 1 process.

Remark 5 Although the known upper boundary of the external disturbance is required in the design of faults detection and estimation observer, the boundary of the external disturbance is not directly utilized to design fault tolerant controller. Furthermore, the main drawback of the sliding mode control is that it requires the upper bound of the external disturbance and uncertainty for selection of the sliding gains. To overcome this difficulty, an adaptive law is introduced to adapt the upper bound, yielding a proposed novel adaptive sliding mode fault tolerant control.

Remark 6 In Ref. [8], a robust fault diagnosis scheme is proposed for the satellite attitude control system by using a bank of adaptive unknown input observers. It is to be noted that the result developed in Ref. [8] cannot be solved to the fault tolerant tracking control problem in this study effectively. To solve the fault tolerant control tracking problem of spacecraft with actuator faults, an adaptive sliding mode control based fault tolerant tracking scheme is designed and the result obtained in this study can be regarded as the supplement and extension of Ref. [8].

4 Numerical Simulations

In this section, numerical simulations are carried out to verify the effectiveness of the proposed control technique. The same physical parameters considered in Ref. [8] are used, which are

$$\boldsymbol{J}=\begin{bmatrix}18 & 0 & 0\\ 0 & 21 & 0\\ 0 & 0 & 24\end{bmatrix},\boldsymbol{D}=\begin{bmatrix}1 & 0 & 0 & 1/\sqrt{3}\\ 0 & 1 & 0 & 1/\sqrt{3}\\ 0 & 0 & 1 & 1/\sqrt{3}\end{bmatrix}$$

$$\boldsymbol{d}(t)=10^{-5}\cdot\begin{bmatrix}3\cos(\omega_0 t)+1\\ 1.5\cos(\omega_0 t)+3\cos(\omega_0 t)\\ 3\cos(\omega_0 t)+1\end{bmatrix}$$

The initial values are chosen as $\varphi_0=2.5°$, $\theta_0=1.5°$, $\psi_0=2°$ and $\omega_{x0}=\omega_{y0}=\omega_{z0}=0$. The orbital angular velocity $\omega_0=0.0012\mathrm{rad/s}$. $\boldsymbol{y}_{\mathrm{d}}=\boldsymbol{\sigma}_{\mathrm{d}}$ is the desired system output signals. The desired system output signals $\boldsymbol{\sigma}_{\mathrm{d}}=[\varphi_{\mathrm{d}},\theta_{\mathrm{d}},\psi_{\mathrm{d}}]$ are given by

$$\varphi_{\mathrm{d}}=2°,\ \theta_{\mathrm{d}}=1°,\ \psi_{\mathrm{d}}=3°$$

The gain parameters of virtual controller ω_d and actual controller δ are chosen as

$$\boldsymbol{K}_1=\mathrm{diag}\{1,1,1\},\ \boldsymbol{K}_2=\mathrm{diag}\{3,3,3\}$$

$$\upsilon_1=1.5,\ \upsilon_2=5,\ \varepsilon_1=1.2,\ \varepsilon_2=4$$

It is assumed that the unknown actuator faults occurr at different time in the simulation, namely,

$$\begin{cases}e_1(t)=0.8,\ e_2(t)=0.6 & t\geqslant 1\ \mathrm{s}\\ e_3(t)=0.4,\ e_4(t)=0.2 & t\geqslant 2\ \mathrm{s}\end{cases}$$

To show the effectiveness of the FTC control scheme, the necessary simulation comparisons are given. Firstly, we use the normal backstepping methood developed in Ref. [15] to do simulations in actuator fault free case and faulty case, and the attitude angle tracking responses are shown in Figs. 2—3, respectively. It can be seen that the closed-loop attitude system has the satisfactory dynamic performance when the actuator is in a healthy case. When an actuator loss of effectiveness fault occurs, it is not difficult to find that the normal backstepping method does not guarantee the stability of the closed

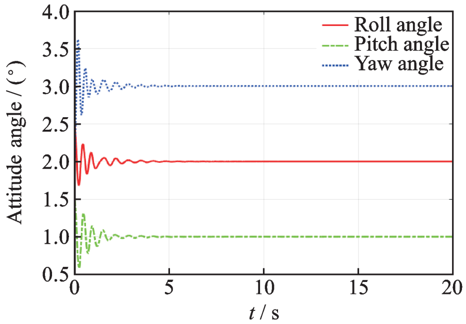

Fig. 2　The attitude angle output responses in healthy case using classical backstepping control

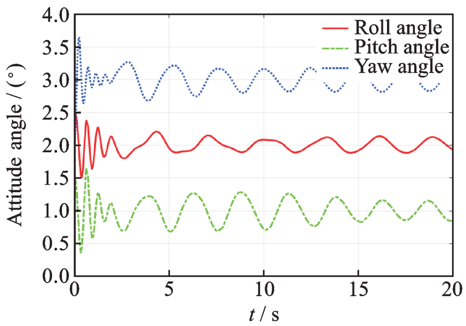

Fig. 3　The attitude angle output responses in faulty case using classical backstepping control

loop attitude system.

In the following two FTC methods simulation results are given. By utilizing the passive FTC approach developed in Ref. [16], the attitude angle and control input curves of faulty attitude systems are depicted in Figs. 4 and 5. It needs more than 10 s to make the system to track the desired command. By using adaptive fault estimation observer, it can be seen that loss of effectiveness fault could be estimated accurately, which is depicted in Fig. 6. The attitude angle curve of faulty attitude systems is depicted in Fig. 7 with the implement of active fault tolerant controller given in Eq.(24). It is not difficult to notice that the attitude angle output can still track the desired attitude commands no more than 6 s. And Fig. 8 shows control input curve using FTC developed in this paper. Compared with the passive FTC method presented in Ref. [16], the proposed FTC approach has better fault tolerant capability and can relax the requirement of a prior knowledge of fault. From the above simulation results, it is seen that the proposed fault tolerant control scheme is effective for the faulty attitude tracking control system of the spacecraft.

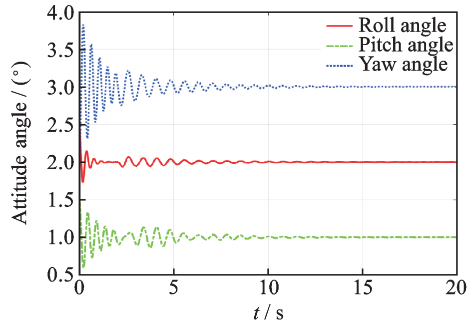

Fig. 4　The attitude angle output responses in faulty case using passive FTC developed in Ref. [16]

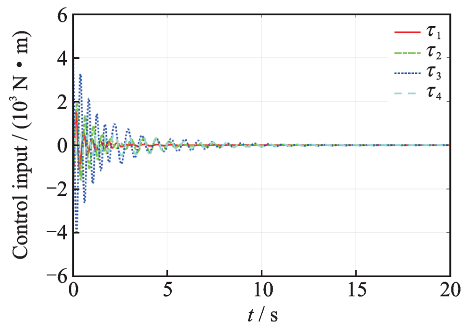

Fig. 5　The control input τ response in faulty case using passive FTC developed in Ref. [16]

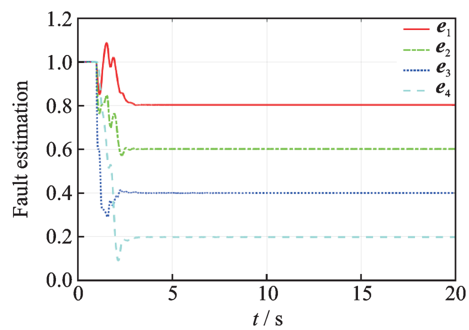

Fig. 6　The actuator fault estimation $\hat{e}$ curve

5　Conclusions

In this study, an adaptive fault-tolerant tracking control scheme has been designed for the nonlinear attitude system of spacecraft with external disturbance and loss of actuator effectiveness. For the faulty dynamic equation of spacecraft, a nonlinear fault detection and estimation observer is proposed to monitor the state changes. By using the estima-

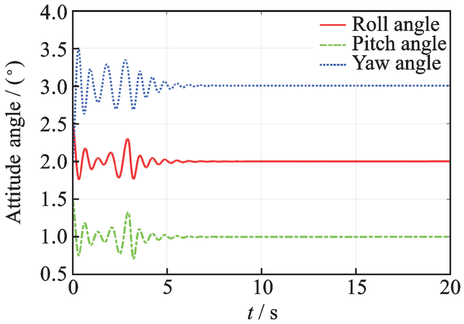

Fig. 7 The attitude angle output response in faulty case using fault-tolerant control

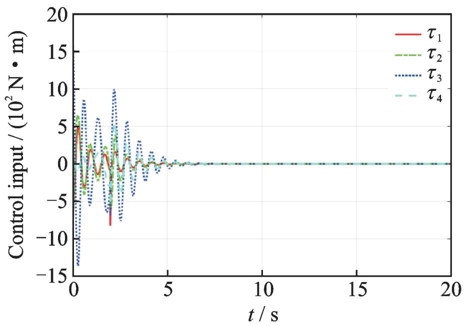

Fig. 8 The control input τ response in faulty case using fault-tolerant control

tion of fault information and sliding mode control scheme, an adaptive fault-tolerant controller is proposed. Meanwhile, the stability of the whole closed-loop attitude systems is analyzed using the Lyapunov theory. The simulation example and comparisons with the previous method are provided to show the effectiveness of the control approach.

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