Controllability of Spacecraft Attitude and Its Application in Reconfigurability Analysis

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Abstract: We review the controllability research on spacecraft attitude based on nonlinear geometry control theory. The existing studies on attitude controllability are mostly concerning the global controllability and small time local controllability (STLC). A presentation of study methods and connotation in both aspects is briefly carried out. As a necessary condition of reconfigurability, the controllability of the faulty attitude control system is studied. Moreover, two reconfigurability conditions based on controllability results that consider the actuator faults for a pyramid configuration spacecraft are provided.

Key words: controllability; spacecraft attitude control; geometry control theory; fault-tolerant control

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0 Introduction

Spacecraft has attracted increasing attention thanks to its successful applications in national defense, communication, meteorology, reconnaissance, etc. As an important component of spacecraft control system, attitude control[1] is a process of spacecraft orientation in predetermined direction that is realized by applying torques to the rigid body of the spacecraft. The control problem can be classified into two kinds. First, as required in pointing to a fixed ground for a communication satellite, attitude stabilization is used to hold the attitude accurately; Second, attitude maneuver is used in various cases: tracking a target for an orbiting astronomical observatory and attitude adjustment before orbit change for a prober, for example. In both kinds, we only consider, for simplicity, active control in this paper, that is, the torques applied to spacecraft are merely caused by actuators such as gas jets, reaction wheels (RW) and control moment gyros (CMG).

As a basic question, controllability of the attitude for a spacecraft means that we can transfer one attitude of the spacecraft to another by a suitable chosen motion of the actuators in a finite time interval. The studies on this can be divided into linear based[2] and nonlinear based methods. Since the motion of spacecraft is in practice nonlinear, the controllability conditions of linearized system maybe fail[3], compared with nonlinear based methods. The development of the spacecraft controllability benefits from nonlinear controllability results[4-6] based on nonlinear geometric theory. These results were mostly developed during 1970s—1980s, which can be summarily acquainted in Ref.[7]. More specifically, only the nonlinear geometric theory based results of the controllability for spacecrafts will be introduced hereinafter.

According to the existing literatures[3,8-16] on the controllability of the spacecraft, the study of this problem can be concluded into two fields: global controllability and STLC.

For global controllability, Crouch firstly employed global controllability results that it is sufficient to satisfy the Lie algebra rank condition for

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Poisson stable system and gave sufficient conditions for the controllability of the spacecraft using gas jet actuators and momentum exchange devices, respectively[3]. Then Kuang-Yow[8] derived a weaker condition for global controllability that the requirement of Poisson stability is replaced by weak positive Poisson stability. And Bhat extended the global controllability results into spacecraft described by time-varying system using magnetic actuation actuators or CMGs, in which the conditions for accessibility and strong accessibility were also given[9,10].

For STLC, an early result[12] showed that there will not be STLC for a spacecraft if there is only one actuator, which is actually caused by the missing of the convex hull of the control torques. Krishnan et al. proved that at any equilibrium the reduced dynamic spacecraft system actuated by two wheel actuators is STLC[13]. Then most of related studies[14-16] were carried out for the spacecraft using CMGs. Bayadi et al. attested that it is STLC for dynamics if the total angular momentum of the spacecraft is less than that of the CMGs array[14]. Gui et al. studied this topic further and gave a second-order sufficient condition of STLC for a spacecraft adopting various configurations[15-16].

Controllability, as a fundamental property of a spacecraft, has been mainly used for two aspects as follows:

(1) It served as an analytical criterion to decide the actuators placement[17-18] for a spacecraft. In detail, the controllability was a criterion that could be used for quantitative comparison of various locations to select the most favorable actuators configuration.

(2) It provided a fundamental guideline for astro-engineers to design control laws[3,11,13,15]. Especially, in some unusual situations, it answered whether the control law can be designed for the under-actuated system.

The spacecraft is prone to fail due to the complex mission environment and sophisticated components. Because of the high cost and the particularity of the task, it is a great concern how to avoid the loss of life and property in events of components of the spacecraft. Fault-tolerant control (FTC) systems for spacecrafts have been widely studied[19-20] to address this issue.

Inspired by the both above-mentioned applications of controllability, we believe that the controllability of the faulty system is promising to be a criterion to judge whether the FTC law can be designed, which is actually called reconfigurability or recoverability in some FTC analysis studies[21-22]. In this paper, based on the controllability of the faulty spacecraft attitude control system, two preliminary reconfigurability results are deduced.

1 Model

In this section, we introduce the mathematic models of the spacecraft based on 3×3 orthogonal matrix including kinematic and dynamic equations according to Refs.[23-24]. The kinematic equation describes the relation of angular position and angular velocity. And the evolution of the angular velocity related to actuators is described by dynamic equation. It is worth mentioning that, unlike kinematic equation, the dynamic equation depends on the method of generating control torque. There are two kinds of actuators to generate control torque: external torque actuator which refers to gas jet, and internal torque devices such as RW and CMG.

1.1 Kinematic equation

The attitude for the rigid body of spacecraft related to external space is described by the relationship between two coordinate systems: one is selected as an inertial reference, denoted by $\boldsymbol{S}_{\mathrm{r}}$, for example, fixed on the earth; another $\boldsymbol{S}_{\mathrm{b}}$ is fixed at the rigid body with the center of mass being the origin. The relationship can be represented by a 3×3 direction cosine matrix $\boldsymbol{A}$ in which each element a_{ij} denotes the corresponding relation between the ith coordinate of $\boldsymbol{S}_{\mathrm{b}}$ and the jth coordinate of $\boldsymbol{S}_{\mathrm{r}}$, $1\leqslant i,j\leqslant 3$. Then the kinematic equation is defined as

$$\dot{\boldsymbol{A}}=\boldsymbol{S}(\omega_b)\boldsymbol{A} \tag{1}$$

where $\boldsymbol{\omega}_{\mathrm{b}}\in\boldsymbol{R}^3$ is the angular velocity measured in $\boldsymbol{S}_{\mathrm{b}}$ and $\boldsymbol{S}(\boldsymbol{\omega}_{\mathrm{b}})$ is a map: $\boldsymbol{R}^3\rightarrow SO(3)$, in which $SO(3)$ denotes three-dimensional special matrix group

$$S(\boldsymbol{\omega}_b)=\begin{bmatrix} 0 & \omega_{b3} & -\omega_{b2} \\ -\omega_{b3} & 0 & \omega_{b1} \\ \omega_{b2} & -\omega_{b1} & 0 \end{bmatrix} \tag{2}$$

Actually, for any given time $\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}=\boldsymbol{I}$, which means $\boldsymbol{A}$ belongs to $SO(3)$, where $\boldsymbol{I}$ denotes the identity matrix. And the map $\boldsymbol{S}(\cdot)$ holds that $\boldsymbol{S}(a)b=b\times a$.

1.2 Dynamic equation

To describe the relation of the angular velocities and control torques. Some relative notations are defined as follows: $\boldsymbol{J}_{\mathrm{b}}$ denotes the moment of inertia of the vehicle in the body coordinates and it is assumed that the mass distribution of the vehicle is fixed so that $\boldsymbol{J}_{\mathrm{b}}$ will be constant; $\boldsymbol{h}_{\mathrm{r}}$ denotes the angular momentum measured in the inertial coordinates. And we have

$$\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}=\boldsymbol{J}_{\mathrm{b}}\boldsymbol{\omega}_{\mathrm{b}} \tag{3}$$

Distinguishing the sources of the change of $\boldsymbol{h}_{\mathrm{r}}$, we give the dynamic equations with respect to external torque actuator and internal torque actuator as follows.

1.2.1 External torque actuator

When the control torques are caused by external torque actuators, the total angular momentum will be changed in the control process. Taking gas jet actuators for example, let b_i be the axes about which the corresponding control torque $\|b_i\|u_i$ caused by actuators, $\|\cdot\|$ the norm in Euclidean space $\boldsymbol{R}^3$ throughout the paper. Taking the derivative of $\boldsymbol{\omega}_{\mathrm{b}}$ in Eq.(3) and combining Eq.(1), we have

$$\boldsymbol{J}_{\mathrm{b}}\dot{\boldsymbol{\omega}}_{\mathrm{b}}=\boldsymbol{S}(\boldsymbol{\omega}_{\mathrm{b}})\boldsymbol{J}_{\mathrm{b}}\boldsymbol{\omega}_{\mathrm{b}}+\sum_{i=1}^{m}\boldsymbol{b}_i u_i \tag{4}$$

where m denotes the pairs number of the actuators in the array of the gas jets.

1.2.2 Internal torque actuator

When the control torques are caused by internal torque actuators, the total angular momentum is conserved in the control process. Taking CMGs for example, let $\boldsymbol{J}_{\mathrm{b}}^{*}$ be the moment of inertia of the spacecraft without actuators, $\boldsymbol{J}_{ci}$ the moment of inertia of the ith CMG, v_i the angular velocity of the wheel of the ith CMG. Then we have

$$\sum_{i=1}^{m}\boldsymbol{J}_{ci}(\boldsymbol{\omega}_{\mathrm{b}}+v_i)+\boldsymbol{J}_{\mathrm{b}}^{*}\boldsymbol{\omega}_{\mathrm{b}}=\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}},\ \dot{h}_{\mathrm{r}}=0 \tag{5}$$

By similar derivation in Eq.(4), we get the dynamic equation using internal torque actuators as

$$\boldsymbol{J}_{\mathrm{b}}\dot{\boldsymbol{\omega}}_{\mathrm{b}}=\boldsymbol{S}(\boldsymbol{\omega}_{\mathrm{b}})\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}(0)+\sum_{i=1}^{m}b_i u_i \tag{6}$$

To sum up, the model of the attitude motion of spacecraft can be described by Eqs.(1), (4) for external torque actuator case and Eqs.(1), (6) for internal torque actuator case. And the attitude motion is summarily represented as

$$\begin{gathered}\dot{A}=\boldsymbol{S}(\boldsymbol{\omega}_{\mathrm{b}})\boldsymbol{A} \\ \dot{\boldsymbol{\omega}}_{\mathrm{b}}=\boldsymbol{J}_{\mathrm{b}}^{-1}g(\boldsymbol{A},\ \boldsymbol{\omega}_{\mathrm{b}})+\boldsymbol{J}_{\mathrm{b}}^{-1}\sum_{i=1}^{m}b_i u_i\end{gathered} \tag{7}$$

where $g(\boldsymbol{A},\ \boldsymbol{\omega}_{\mathrm{b}})$ is decided by Eq.(4) or Eq.(6).

2 Controllability of Spacecraft Attitude

2.1 Preliminaries

Consider a smooth vector field $\boldsymbol{X}$ on the smooth and connected manifold $SO(3)$. Let ϕ^{X} denote its flow

$$\phi^{X}: \boldsymbol{R}\times SO(3)\to SO(3);\ (t,x)\mapsto \phi_t^{X}(x)$$

The reachable set of the system from $x\in SO(3)$ at time $T\geqslant 0$ is the set $\boldsymbol{R}(x,T)$ of all angular positions that can be reached at T by following flows $\phi_T^{X}(x)=\phi_{t_k}^{X_k}(\phi_{t_{k-1}}^{X_{k-1}}(\cdots\phi_{t_1}^{X_1}(x)))$ of the vector field $\{X_1,\cdots,X_k\}$ of system (7) that starts at $t=0$ and $T=\sum_{i=1}^{k}t_i$. The reachable set from x is defined as $\boldsymbol{R}(x)=\bigcup_{T\geqslant 0}\boldsymbol{R}(x,T)$. Then some concepts of controllability can be defined using reachable sets. The system is accessible if $\mathrm{int}(\boldsymbol{R}(x))\neq\varnothing$ or strongly accessible if $\mathrm{int}(\boldsymbol{R}(x,T))\neq\varnothing$ for every $T\geqslant 0$. If $\boldsymbol{R}(x)=SO(3)$ for every x, the system is globally controllable. And the STLC at x meets if there exists a $T\geqslant 0$ such that $x\in\mathrm{int}(\boldsymbol{R}(x,t))$ for every $t\in(0,T)$.

2.2 Global controllability

The global controllability results in Ref s.[3, 8, 10] are used to the models in Eq.(7), accompanied by the connotations and applications of these results in this subsection. The global controllability is concluded if the uncontrolled motion of the spacecraft is weakly positively Poisson stable (WPPS) and the system is accessible, that is, Lie algebra rank condition is satisfied. We first introduce the both necessary conditions of global controllability.

2.2.1 Weak positive Poisson stability

Definition 1 A vector field $\boldsymbol{X}$ is WPPS if, for every open set $U \subseteq SO(3)$ and every $t > 0$, there exists $T > t$ such that $\phi_T^X(U) \cap U \neq \varnothing$.

The uncontrolled motion of dynamic equations in Eqs.(4), (6) can take the form

$$\boldsymbol{J}_\mathrm{b}\dot{\boldsymbol{\omega}}_\mathrm{b} = \boldsymbol{S}(\boldsymbol{\omega}_\mathrm{b})(\boldsymbol{J}_\mathrm{b}\boldsymbol{\omega}_\mathrm{b} + \boldsymbol{v}) \tag{8}$$

where $\boldsymbol{v}$ is a constant. From Theorem 2 in Ref. [3], the vector fields in Eqs.(4), (6) are Poisson stable corresponding to uncontrolled motion (8). Further, under this circumstance, Eq.(3) can be rewritten as

$$\boldsymbol{A}\boldsymbol{h}_\mathrm{r} = \boldsymbol{J}_\mathrm{b}\boldsymbol{\omega}_\mathrm{b} + \boldsymbol{v} \tag{9}$$

And Eq.(1) can be replaced by

$$\dot{\boldsymbol{A}} = \boldsymbol{S}\{\boldsymbol{J}_\mathrm{b}^{-1}(\boldsymbol{A}\boldsymbol{h}_\mathrm{r} - \boldsymbol{v})\}\boldsymbol{A} \tag{10}$$

where $\boldsymbol{v}$ is constant. Define a differential form Ω on $SO(3)$ by $\Omega_R(\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{V}_3) = \boldsymbol{v}_1^\mathrm{T}(\boldsymbol{v}_2 \times \boldsymbol{v}_3)$, where $R \in SO(3)$, $\boldsymbol{V}_i$ is the tangent vector fields on $SO(3)$ and $\boldsymbol{v}_i = \boldsymbol{S}^{-1}(\boldsymbol{V}_i)$, $i = 1,2,3$. According to Poincare's Recurrence Theorem, it is sufficient to verify that the flow of vector field in Eq.(10) preserves the volume form Ω on $SO(3)$, that is, $L_{\boldsymbol{S}\{J_\mathrm{b}^{-1}(Ah_\mathrm{r}-v)\}A}\Omega \equiv 0$, where $L_a b$ denotes the Lie derivative of b on a. The proof can be found in Ref. [10] and following proposition is obtained.

Proposition 1 If $L_{\boldsymbol{S}\{J_\mathrm{b}^{-1}(Ah_\mathrm{r}-v)\}A}\Omega \equiv 0$, for every $\boldsymbol{h}_\mathrm{r} \in \boldsymbol{R}^3$ system (10) is WPPS on $SO(3)$.

2.2.2 Accessibility

The definition of accessibility has been already given and we discuss the sufficient condition of this property directly. To obtain this conclusion, an algebra $\boldsymbol{L}$ called accessible algebra is constituted by (1) all Lie brackets $[\boldsymbol{X}_i, \boldsymbol{X}_j]$, $[\boldsymbol{X}_i, [\boldsymbol{X}_j, \boldsymbol{X}_k]]$, etc; (2) all sums of $\sum_{i=1}^{n} \lambda_i \boldsymbol{X}_i$, where $\boldsymbol{X}_i, \boldsymbol{X}_j, \cdots, \boldsymbol{X}_n$ are the vector, fields of system (7) and $\lambda_i \in \boldsymbol{R}$. Then the sufficient condition is given as follows.

Proposition 2 System (7) is accessible if $\dim(\boldsymbol{L}) = \dim\{SO(3) \times \boldsymbol{R}^3\}$.

Remark 1 The accessible algebra actually contains all of the directions that the state at this moment, and attitude in this paper can go with. And the rank condition means that the entire tangent space of the state space $SO(3)$ can be spanned by the vector fields of the system.

Combining Proposition 1 and Proposition 2, we have the sufficient condition of the global controllability for spacecraft attitude.

Theorem 1 If Eq.(10) is WPPS and the rank condition $\dim(\boldsymbol{L}) = \dim\{SO(3) \times \boldsymbol{R}^3\}$ is satisfied, system (7) is globally controllable.

Remark 2 Global controllability means that we can transfer any attitude of the spacecraft to any attitude by a suitable chosen motion of the actuators in a finite time interval. Obviously, it is a rigorous condition in control problem.

2.3 STLC

In this subsection, we introduce the STLC results stated in Refs.[12-16] and only the angular positions will be considered in this subsection. First, we give an attitude reduced equation like Eq.(10)

$$\dot{\boldsymbol{A}} = \boldsymbol{S}\{\boldsymbol{J}_\mathrm{b}^{-1}(\boldsymbol{A}\boldsymbol{h}_\mathrm{r} - \boldsymbol{v})\}\boldsymbol{A}, \quad \boldsymbol{v} = \int \sum_{i=1}^{m} b_i u_i \,\mathrm{d}t \tag{11}$$

where $\boldsymbol{v}$ is the changed angular momentum caused by actuators.

Then, according to Refs. [14-15], STLC is decided by the relation between the convex hull of $\boldsymbol{v}$, denoted by $\mathrm{conv}(\boldsymbol{v})$, and $\boldsymbol{h}_\mathrm{r}$. We give the result directly and specifically analyze the connotation about this result.

Theorem 2 System (11) is

(1) STLC for all $A \in SO(3)$, if $\boldsymbol{A}\boldsymbol{h}_\mathrm{r} \in \mathrm{int}(\mathrm{conv}(\boldsymbol{v}))$ and $\boldsymbol{A}\boldsymbol{h}_\mathrm{r} = \boldsymbol{v}$.

(2) Not STLC at $A \in SO(3)$, if $\boldsymbol{A}\boldsymbol{h}_\mathrm{r} \notin \mathrm{conv}(\boldsymbol{v})$.

Remark 3 It is obvious that the system (11) is at equilibrium if $\boldsymbol{A}\boldsymbol{h}_\mathrm{r} = \boldsymbol{v}$, which tells us that STLC holds at equilibriums merely. The condition $\boldsymbol{A}\boldsymbol{h}_\mathrm{r} \in \mathrm{int}(\mathrm{conv}(\boldsymbol{v}))$ actually means that only if angular momentum provided by actuators is greater than angular momentum of total system the STLC maybe holds. And this property implies the existence of local asymptotically stabilization control law, as illustrated in Ref. [25].

3 Reconfigurability Analysis Based on Controllability Results

In this section, we consider a spacecraft using 4-CMGs with a Pyramid configuration. The model with actuator faults is given, followed by the controllability analysis for the faulty system, which is called reconfigurability in this paper.

3.1 Model of spacecraft with CMGs faults

Single gimbal CMG consists of a rotor spinning around the principal axis to provide angular momentum and a gimbal frame which can change the direction of the angular momentum. We consider the configuration with 4-single gimbal CMGs arranged as a Pyramid, as shown in Fig.1.

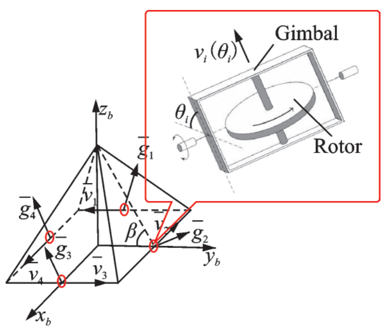

Fig. 1 Pyramid configuration with 4-single gimbal CMGs

According to the structure of the single gimbal CMG, the faults maybe take place in both gimbal and rotor. For simplicity, we only consider the multiplicative rotor faults. In this case, the total failures and partial failures of rotors are handled. The CMGs model with these faults can be represented as

$$v(\boldsymbol{\theta})=\sum_{i=1}^{4}k_i h_0(\theta_i)=$$

$$h_1\begin{bmatrix}-\cos\beta\sin\theta_1\\ \cos\theta_1\\ \sin\beta\sin\theta_1\end{bmatrix}+h_2\begin{bmatrix}-\cos\theta_2\\ -\cos\beta\sin\theta_2\\ \sin\beta\sin\theta_2\end{bmatrix}+$$

$$h_3\begin{bmatrix}\cos\beta\sin\theta_3\\ -\cos\theta_3\\ \sin\beta\sin\theta_3\end{bmatrix}+h_4\begin{bmatrix}\cos\theta_4\\ \cos\beta\sin\theta_4\\ \sin\beta\sin\theta_4\end{bmatrix} \tag{12}$$

where $\boldsymbol{\theta}=[\theta_1\ \theta_2\ \theta_3\ \theta_4]^{\mathrm{T}}$ denotes the gimbal angular vector; h_0 the normal angular momentum provided by the spinning rotor; $k_i\in[0,1]$ is the effective coefficient of the i th rotor; 0 denotes total failure while 1 denotes normality. Selecting $\dot{\boldsymbol{\theta}}$ as the input and combining Eq.(11), the attitude control system of this spacecraft is

$$\begin{aligned}\dot{A}&=\boldsymbol{S}\{\boldsymbol{J}_{\mathrm{b}}^{-1}(A\boldsymbol{h}_{\mathrm{r}}-\boldsymbol{v}(\boldsymbol{\theta}))\}A\\ \dot{\boldsymbol{\theta}}&=\boldsymbol{u}\end{aligned} \tag{13}$$

3.2 Reconfigurability analysis

In this subsection, we analyze the reconfigurability using global controllability and STLC results introduced in Section 2. Naturally, reconfigurability is defined by the above analysis.

Definition 2 The faulty system is reconfigurable if the faulty system maintains global controllability and asymptotic stability reconfigurable if the faulty system is STLC, that is, there exists a local asymptotic stability control law.

We achieve the reconfigurability result by analyzing the global controllability for the faulty system.

Theorem 3 System (13) is always reconfigurable on $SO(3)$ for every $\boldsymbol{h}_{\mathrm{r}}\in\boldsymbol{R}^3$, if $\sum_{i=1}^{4}k_i>0$.

Proof According to Theorem 1, the global controllability in Eq.(13) can be proven in two aspects, i.e. the WPPS of Eq.(13) when $\dot{\boldsymbol{\theta}}=0$ and the accessibility.

First, when $\dot{\boldsymbol{\theta}}=0$, let $\boldsymbol{h}_{\mathrm{r}}\in\boldsymbol{R}^3$, $\boldsymbol{\theta}\in\boldsymbol{R}^4$, and consider the vector field $\boldsymbol{f}$ on $SO(3)$ defined by $\boldsymbol{f}(A)=\boldsymbol{S}\left\{\boldsymbol{J}_{\mathrm{b}}^{-1}(A\boldsymbol{h}_{\mathrm{r}}-\boldsymbol{v}(\boldsymbol{\theta}))\right\}A$. Define a Ω as stated in Section 2.2.1. According to Proposition 1, it is sufficient to show that $L_f\Omega\equiv 0$. To compute $L_f\Omega$, let $A\in SO(3)$, and $V_i\in T_ASO(3)$, $i=1,2,3$. On $SO(3)$, left invariant vector fields $\xi_i, i=1,2,3$, hold that $\xi_i(A)=V_i$. Define a map $y(A)=\Omega_A(\xi_1(A),\xi_2(A),\xi_3(A))$. Then we have

$$\begin{aligned}(L_f\Omega)_A(V_1,V_2,V_3)=&(L_f y)(A)-\\ &\Omega_A([\boldsymbol{f},\xi_1](A),\xi_2(A),\xi_3(A))-\\ &\Omega_A(\xi_1(A),[\boldsymbol{f},\xi_2](A),\xi_3(A))-\\ &\Omega_A(\xi_1(A),\xi_2(A),[\boldsymbol{f},\xi_3](A))\end{aligned} \tag{14}$$

The left invariant property of Ω and ξ_i implies that, for every A, $\Omega_A(\xi_1(A),\xi_2(A),\xi_3(A))=\Omega_A(\xi_1(I),\xi_2(I),\xi_3(I))$, which means that y is a constant function. Then the first term on the right side of Eq.(14) is zero. And the last terms on right side of Eq.(14) are also zero by them as an entirety and applying the property of Jacobian identity of Lie bracket and cross product, which can be found in Ref.[10] in detail. So Eq.(13) is WPPS when $\dot{\boldsymbol{\theta}}=0$.

Second, we show that the rank condition of Lie algebra of Eq. (13) is satisfied. Define $f_0=(\boldsymbol{S}\{\boldsymbol{J}_{\mathrm{b}}^{-1}(\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}-\boldsymbol{v}(\boldsymbol{\theta}))\}\boldsymbol{A},0), f_i=(0,e_i)$, where $e_i\in\boldsymbol{R}^4$ and the ith element of e_i is one while others are zero, $i=1,2,3,4$. Define Lie brackets $\boldsymbol{\zeta}_i=[f_0,f_i]$, $\boldsymbol{\zeta}_{2i}=[f_i,\boldsymbol{\zeta}_i]$, $\boldsymbol{\zeta}_{3i}=[\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_{2i}]$, we have

$$\boldsymbol{\zeta}_i=(\boldsymbol{S}\{\boldsymbol{J}_{\mathrm{b}}^{-1}(-v'_i(\theta_i))\}\boldsymbol{A},0)$$
$$\boldsymbol{\zeta}_{2i}=(\boldsymbol{S}\{\boldsymbol{J}_{\mathrm{b}}^{-1}(v''_i(\theta_i))\}\boldsymbol{A},0)$$
$$\boldsymbol{\zeta}_{3i}=(\boldsymbol{S}\{\boldsymbol{J}_{\mathrm{b}}^{-1}(v''_i(\theta_i))\times\boldsymbol{J}_{\mathrm{b}}^{-1}(-v'_i(\theta_i))\}\boldsymbol{A},0)$$

where $v'_i(\theta_i)$ and $v''_i(\theta_i)$ are the first and the second derivatives of v_i to θ_i, respectively. According to Eq.(12), $v'_i(\theta_i)$ is perpendicular to $v_i(\theta_i)$ and $v''_i(\theta_i)$ parallels to $v_i(\theta_i)$. As a result, $\boldsymbol{\zeta}_i,\boldsymbol{\zeta}_{2i},\boldsymbol{\zeta}_{3i},\boldsymbol{\zeta}_{3i}$ are linear independent if $k_i\neq0, i=1,2,3,4$, that is $\dim(\boldsymbol{L})=\dim\{SO(3)\}=3$. Through the above analysis, the rank condition holds only if $\sum_{i=1}^{4}k_i>0$.

Remark 4 This theorem tells us that the reconfigurability always holds for the spacecraft even with only one normal rotor. And the judgement of the reconfigurability can be concluded as the test of the independent Lie brackets.

Define r the radius of the inscribed sphere of the convex hull of $\boldsymbol{v}(\boldsymbol{\theta})$, Theorem 4 is reached using the results of STLC in Theorem 2.

Theorem 4 System (13) is always locally asymptotic stability reconfigurable when $\|\boldsymbol{h}_{\mathrm{r}}\|<r$ and $\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}=\boldsymbol{v}(\boldsymbol{\theta})$.

Proof When the convex hull generated by CMGs have the inscribed sphere, it is obvious that the points in the inscribed sphere are the interior points of the conv($\boldsymbol{v}$). If $\|\boldsymbol{h}_{\mathrm{r}}\|<r$, we have $\|\boldsymbol{A}\|\|\boldsymbol{h}_{\mathrm{r}}\|=\|\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}\|<r$ which is a sufficient condition that $\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}$ is in the inscribed sphere centered at 0 with radius r. And it is obtained that $\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}\in\mathrm{int}(\mathrm{conv}(\boldsymbol{v}(\boldsymbol{\theta})))$. For the other condition $\boldsymbol{A}\boldsymbol{h}_{\mathrm{r}}=\boldsymbol{v}(\boldsymbol{\theta})$, according to Theorem 2, the results have been proven.

Remark 5 The judgement of this kind reconfigurability involves the relation between the convex hull of angular momentum caused by CMGs array and the total momentum of the spacecraft. The result can be tested by whether the inclusion relation holds after faults.

3.3 Example

In this subsection, we discuss an example in which $\beta=53.1^{\circ}$ and two opposite rotors of the CMGs totally fail: 1 and 3. Then an under-actuated system is obtained in this case, modeled by Eq.(13) with $\boldsymbol{v}(\boldsymbol{\theta})$

$$\boldsymbol{v}(\boldsymbol{\theta})=\sum_{i=1}^{4}k_ih_0(\theta_i)=$$
$$h_2\begin{bmatrix}-\cos\theta_2\\-\cos\beta\sin\theta_2\\\sin\beta\sin\theta_2\end{bmatrix}+h_4\begin{bmatrix}\cos\theta_4\\\cos\beta\sin\theta_4\\\sin\beta\sin\theta_4\end{bmatrix}\tag{15}$$

According to Theorem 3, the under-actuated system is reconfigurable when $k_2=k_4\neq0$. To judge whether it is locally asymptotic stability reconfigurable, we study the convex hull of Eq.(15). Actually, this convex hull can be understood easily by plane geometry, as shown in Fig.2, there is an inscribed sphere with radius $0.96h_0$ centered at 0 in it.

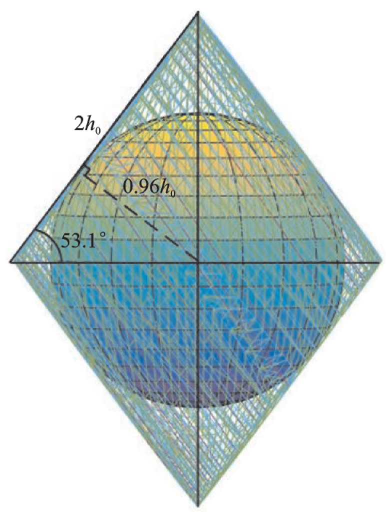

Fig. 2 Convex hull of Eq. (15) with an inscribed sphere

By Theorem 4, the system is locally asymptotic stability reconfigurable at equilibriums when $\|\boldsymbol{h}_{\mathrm{r}}\|<0.96h_0$, which means there exists a locally asymptotic stability control law for the under-actuated system under suitable angular velocity with respect to the inertial coordinates.

4 Conclusions

As an important embodiment of national economic development, spacecraft has recently attracted increasing attention. A briefly review on the controllability study of spacecraft has been presented in this paper. The existing studies in controllability for spacecraft, which are divided into global controllability and STLC, are outlined. Further, we expanded these results to the reconfigurability analysis for a spacecraft using CMGs with rotor faults. It is promising to get the quantitative results after we have obtained only the qualitative results in this paper.

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