Robust Adaptive Attitude Maneuvering and Vibration Reducing Control of Flexible Spacecraft with Prescribed Performance

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Abstract: A robust adaptive control scheme with prescribed performance is proposed for attitude maneuver and vibration suppression of flexible spacecraft, in which the parametric uncertainty, external disturbances and unmeasured elastic vibration are taken into account simultaneously. On the basis of the prescribed performance control (PPC) theory, the prescribed steady state and transient performance for the attitude tracking error can be guaranteed through the stabilization of the transformed system. This controller does not need the knowledge of modal variables. The absence of measurements of these variables is compensated by appropriate dynamics of the controller which supplies their estimates. The method of sliding mode differentiator is introduced to overcome the problem of explosion of complexity inherent in traditional backstepping design. In addition, the requirements of knowing the system parameters and the unknown bound of the lumped uncertainty, including external disturbance and the estimate error of sliding mode differentiator, have been eliminated by using adaptive updating technique. Within the framework of Lyapunov theory, the stability of the transformed system is obtained. Finally, numerical simulations are carried out to verify the effectiveness of the proposed control scheme.

Key words: prescribed performance; flexible spacecraft; attitude maneuver; vibration suppression; modal observer; sliding mode differentiator

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0 Introduction

With the development of aerospace technology, flexible spacecraft in modern space missions often carries large flexible structures, and will be expected to achieve high pointing accuracy and fast attitude maneuvering. However, the attitude maneuvering will introduce certain levels of vibration to flexible appendages due to rigid-flex coupling effect, which will deteriorate pointing performance and make the dynamical model of spacecraft highly nonlinear. Besides, flexible spacecraft is unavoidably subjected to various external disturbances from practical space environment and parameter perturbation. Therefore, attitude maneuver and vibration control strategies robust to parametric uncertainties and external disturbance, and also suppressive to the induced vibration are in great demand in future space missions.

Recently, considerable work has been found for designing nonlinear attitude controller in the presence of above stated issues[1-2], in which optimal and nonlinear control systems for the control of flexible spacecraft have been developed. Variable structure controllers for flexible spacecraft with large space structures were designed in Refs.[3-4] because of their insensitivity to system uncertainty and external disturbance.

However, design methods in these studies required perfect knowledge of the system parameters

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and prior information on the bounds of disturbances for the computation of control gains. Unlike these methods, nonlinear adaptive control methods, including an adaptation mechanism for tuning the controller gains, were called for. A variety of adaptive attitude controllers have been developed. For example, a new adaptive system in Ref.[5] was designed for rotational maneuver and vibration suppression of an orbiting spacecraft with flexible appendages. Then, an adaptive output regulation of the closed-loop system was accomplished in spite of large parameter uncertainties and disturbance input. New variable structure control approaches in Refs.[4, 6] were proposed for vibration control of flexible spacecraft during attitude maneuvering, and the adaptive version of the proposed controller was achieved through releasing the limitation of knowing the bounds of the uncertainties and perturbations in advance.

The relevant drawback of these control strategies is either the extra necessity to measure the modal variables or to treat the effects of the flexible dynamics on the rigid motion as the additional disturbance acting on a rigid structure. With regard to the latter situation, a weighted homogeneous extended state observer in Ref.[7] was designed to estimate and thus to attenuate the total disturbance in finite time, including external disturbance torque and coupling effect. As a result, the prior knowledge of the total disturbance was not required.

Unfortunately, the availability of the measured modal variables is an unrealistic hypothesis in some cases, due to the impracticability of using appropriate sensors or the economical requisite. An interesting solution for the attenuation of the flexible oscillations induced by spacecraft maneuvers is to reconstruct the unmeasured modal position and velocity by means of appropriate dynamics. This is an advantageous aspect in case of sensor failures or the reduction of the structure complexity and control system design costs.

For the circumstance that modal variables are unavailable, a class of nonlinear controllers[8-12] was derived for spacecraft with flexible appendages. It did not ask for measures of the modal variables, but only used the parameters describing the attitude and the spacecraft angular velocity. The derived controller then used estimates of the modal variables and its rate to avoid direct measurement.

However, the above control methodologies cannot satisfy certain prescribed transient and steady tracking performance. To deal with this issue, a prescribed performance control (PPC) method was proposed for the feedback linearizable nonlinear systems using one kind of transformation functions[13-14]. In the method the prescribed performance bound can characterize the convergence rate and maximum overshoot of the tracking errors. Using the appropriate performance function and error transformation, the tracking errors can converge to an arbitrarily small residual set with a convergence rate no less than a predefined value and the maximum overshoot less than a sufficiently small specified constant.

In this paper, a robust adaptive controller with prescribed performance for attitude maneuver and vibration suppression of flexible spacecraft is proposed. Using the PPC technique, the constrained original attitude control system is transformed to an unconstrained one. Then the stabilization of the unconstrained system can ensure the prescribed performance bounds of the original system. During the control design, a mode observer is constructed to supply elastic modal estimates by utilizing the inherent physical properties of flexible appendages. In order to deal with the problem of explosion of complexity inherent in traditional backstepping design, the first order sliding mode differentiator (FOSD) is used. In addition, an adaptive law is derived to estimate the unknown items, thus the prior information of system parameters and the upper bound of the lumped uncertainty are no longer needed.

1 Mathematical Model and Problem Statement

1.1 Mathematical model of a flexible spacecraft

The mathematical model of a flexible spacecraft is briefly recalled in the section. The kinematic equation in terms of modified Rodrigues parameters

(MRPs) is expressed as[15]

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{G}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{1}$$

$$\boldsymbol{G}(\boldsymbol{\sigma}) = \frac{1}{4}\left[(1-\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{\sigma})\boldsymbol{I}_3 + 2\boldsymbol{S}(\boldsymbol{\sigma}) + 2(\boldsymbol{\sigma}\boldsymbol{\sigma}^{\mathrm{T}})\right] \tag{2}$$

Under the assumption of small elastic displacements, the dynamic equations of spacecraft with flexible appendages can be found in Ref.[16] and given by

$$\boldsymbol{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\delta}^{\mathrm{T}}\ddot{\boldsymbol{\eta}} = -\boldsymbol{\omega}^{\times}(\boldsymbol{J}\boldsymbol{\omega} + \boldsymbol{\delta}^{\mathrm{T}}\dot{\boldsymbol{\eta}}) + \boldsymbol{u} + \boldsymbol{d} \tag{3}$$

$$\ddot{\boldsymbol{\eta}} + \boldsymbol{C}\dot{\boldsymbol{\eta}} + \boldsymbol{K}\boldsymbol{\eta} = -\boldsymbol{\delta}\dot{\boldsymbol{\omega}} \tag{4}$$

where $\boldsymbol{J}$ is the total inertia matrix, $\boldsymbol{\delta}$ the coupling matrix between flexible and rigid dynamics, $\boldsymbol{\eta}$ the modal coordinate vector, $\boldsymbol{u}$ the control input, $\boldsymbol{d}$ the external disturbance, and the damping and stiffness matrices are expressed as $\boldsymbol{C} = \mathrm{diag}\{2\xi_i\omega_{ni}\}$ and $\boldsymbol{K} = \mathrm{diag}\{\omega_{ni}^2\}(i=1,\cdots,N)$, respectively, in which N is the number of elastic modes considered, ω_{ni} the natural frequencies, and ξ_i the corresponding damping.

Through introducing the auxiliary variable $\boldsymbol{\psi} = \dot{\boldsymbol{\eta}} + \boldsymbol{\delta}\boldsymbol{\omega}$ which represents the total angular velocity expressed in modal variables, the dynamics of the flexible spacecraft from Eqs.(3), (4) can be further expressed as

$$\begin{bmatrix}\dot{\boldsymbol{\eta}}\\ \dot{\boldsymbol{\psi}}\end{bmatrix} = \begin{bmatrix}0 & \boldsymbol{I}\\ -\boldsymbol{K} & -\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{\eta}\\ \boldsymbol{\psi}\end{bmatrix} + \begin{bmatrix}-\boldsymbol{I}\\ \boldsymbol{C}\end{bmatrix}\boldsymbol{\delta}\boldsymbol{\omega} \tag{5}$$

$$\begin{aligned}\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\omega}} = &-\boldsymbol{\omega}^{\times}(\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega} + \boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi}) + \\ &\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\psi} + \boldsymbol{K}\boldsymbol{\eta} - \boldsymbol{C}\boldsymbol{\delta}\boldsymbol{\omega}) + \boldsymbol{u} + \boldsymbol{d}\end{aligned} \tag{6}$$

where $\boldsymbol{J}_{\mathrm{mb}} = \boldsymbol{J} - \boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$ with $\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$ as the contribution of the flexible parts to the total inertia matrix.

1.2 Preliminaries

To facilitate control system design, the following assumptions and lemmas are presented and will be used in the subsequent developments.

Assumption 1 The components of external disturbance vector $\boldsymbol{d}$ in system Eq.(6) are assumed to be bounded by a set of unknown bounded constants, that is

$$|d_i| \leqslant d_{Mi} \quad i=1,2,3 \tag{7}$$

Assumption 2 The unknown inertia matrix $\boldsymbol{J}_{\mathrm{mb}}$ satisfies

$$J_{\mathrm{mb},ij,\min} \leqslant J_{\mathrm{mb},ij} \leqslant J_{\mathrm{mb},ij,\max} \quad i,j=1,2,3 \tag{8}$$

The inertia matrix $\boldsymbol{J}_{\mathrm{mb}}$ in Eq.(6) cannot be accurately measured, then adaptation technique is required to tackle these bounded unknown system parametres. To facilitate the construction of adaptive law, define a linear operator $\boldsymbol{L}(\cdot): \mathrm{R}^3 \rightarrow \mathrm{R}^{3\times 6}$ acting on arbitrary vector $\boldsymbol{\xi} = [\xi_1 \quad \xi_2 \quad \xi_3]^{\mathrm{T}}$ such that

$$\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\xi} = \boldsymbol{L}(\boldsymbol{\xi})\boldsymbol{\theta}_{\mathrm{mb}} \tag{9}$$

$$\boldsymbol{L}(\boldsymbol{\xi}) = \begin{bmatrix}\xi_1 & 0 & 0 & \xi_2 & \xi_3 & 0\\ 0 & \xi_2 & 0 & \xi_1 & 0 & \xi_3\\ 0 & 0 & \xi_3 & 0 & \xi_1 & \xi_2\end{bmatrix} \tag{10}$$

$$\boldsymbol{\theta}_{\mathrm{mb}}^{\mathrm{T}} = [J_{\mathrm{mb},11}, J_{\mathrm{mb},22}, J_{\mathrm{mb},33}, J_{\mathrm{mb},23}, J_{\mathrm{mb},13}, J_{\mathrm{mb},12}] \tag{11}$$

According to Eq.(8), $\boldsymbol{\theta}_{\mathrm{mb}}$ satisfies

$$\boldsymbol{\theta}_{\mathrm{mb}} \in \boldsymbol{\Omega}_{\theta_{\mathrm{mb}}} = \left\{\boldsymbol{\theta}_{\mathrm{mb}} \middle| \boldsymbol{\theta}_{\mathrm{mb},\min} \leqslant \boldsymbol{\theta}_{\mathrm{mb}} \leqslant \boldsymbol{\theta}_{\mathrm{mb},\max}\right\} \tag{12}$$

where parameter vectors $\boldsymbol{\theta}_{\mathrm{mb},\max}$ and $\boldsymbol{\theta}_{\mathrm{mb},\min}$ are known upper and lower bound of $\boldsymbol{\theta}_{\mathrm{mb}}$.

Lemma 1[17-18] The following inequality holds for any $\varepsilon > 0$ and $\eta \in \mathrm{R}$

$$0 \leqslant |\eta| - \eta\tanh\left(\frac{\eta}{\varepsilon}\right) \leqslant \delta\varepsilon \tag{13}$$

where δ is a constant that satisfies $\delta = \mathrm{e}^{-(\delta+1)}$, that is, $\delta = 0.278\,5$.

Lemma 2[19] The "first-order sliding mode differentiator(FOSD)" is designed as

$$\begin{cases}\dot{\varsigma}_0 = -\bar{\mu}_0|\varsigma_0 - l(t)|^{0.5}\mathrm{sign}(\varsigma_0 - l(t)) + \varsigma_1\\ \dot{\varsigma}_1 = -\bar{\mu}_1\mathrm{sign}(\varsigma_1 - \varsigma_0)\end{cases} \tag{14}$$

where ς_0 and ς_1 are the system states, $\bar{\mu}_0$ and $\bar{\mu}_1$ the designed parameters of FOSD, and $l(t)$ is the input function. $\dot{\varsigma}_0$ can estimate $\dot{l}(t)$ with an arbitrary precision in case that the initial error $\varsigma_0(t_0) - l(t_0)$ and $\dot{\varsigma}_0(t_0) - \dot{l}(t_0)$ are bounded.

1.3 Control problem formulation

The control objective of this work is to design a control law and a parameter adaptive law, such that:

(1) The attitude control errors achieve prescribed transient and steady-state performance.

(2) The vibration induced by the maneuver rotation is also suppressed in the presence of parametric uncertainties, external disturbances and input saturation constraints, i.e.

$$\lim_{t\to\infty}\boldsymbol{\eta} = 0, \lim_{t\to\infty}\dot{\boldsymbol{\eta}} = 0 \tag{15}$$

2 Robust Adaptive Backstepping Controller Design

To remove the hypothesis of the measurability of the modal position and velocity, an elastic mode estimator to supply their estimates is constructed as follows[20]

$$\begin{bmatrix} \dot{\hat{\boldsymbol{\eta}}} \\ \dot{\hat{\boldsymbol{\psi}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{K} & -\boldsymbol{C} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{C} \end{bmatrix} \boldsymbol{\delta\omega} \tag{16}$$

where $\hat{\boldsymbol{\eta}}$, $\hat{\boldsymbol{\psi}}$ are the estimates of modal variables, $\boldsymbol{e}_\eta = \boldsymbol{\eta} - \hat{\boldsymbol{\eta}}$ and $\boldsymbol{e}_\psi = \boldsymbol{\psi} - \hat{\boldsymbol{\psi}}$ the estimation errors.

From Eqs.(5), (16), the response of $\boldsymbol{e}_\eta, \boldsymbol{e}_\psi$ can be algebraically arranged as

$$\begin{bmatrix} \dot{\boldsymbol{e}}_\eta \\ \dot{\boldsymbol{e}}_\psi \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi \end{bmatrix} \tag{17}$$

Since matrix $\boldsymbol{A}$ is a Hurwitz matrix, the estimation errors $\boldsymbol{e}_\eta, \boldsymbol{e}_\psi$ will asymptotically converge to zero.

Next, a robust adaptive controller with prescribed performance is derived in the presence of parametric uncertainties, external disturbances and unmeasured elastic vibration.

According to PPC theory, the attitude control error should be confined in the prescribed bounds, shown as

$$\underline{\delta}\,\rho(t) < \boldsymbol{\sigma}_i < \bar{\delta}\rho(t) \quad i = 1,2,3 \tag{18}$$

where $\underline{\delta}$ and $\bar{\delta}$ are the chosen positive constants. To achieve the control objective, the prescribed performance function $\rho(t)$ is chosen as[21]

$$\rho(t) = (\rho_0 - \rho_\infty)\mathrm{e}^{-\beta t} + \rho_\infty \tag{19}$$

where the constant ρ_∞ is the maximum amplitude of the control error at the steady state. The decreasing rate $\mathrm{e}^{-\beta t}$ of $\rho(t)$ represents the desired convergence speed of $\boldsymbol{\sigma}_i (i = 1,2,3)$. Therefore, the appropriate choice of the performance function $\rho(t)$ and the design constant impose bounds on the control error trajectory.

Error transformation is used to transform the original system with constrained control error into an equivalent unconstrained one[21]. An error transformation[22] is defined as

$$\boldsymbol{\varepsilon}_i = \boldsymbol{S}^{-1}\left(\frac{\boldsymbol{\sigma}_i}{\rho}\right) = \tan\left(\frac{\pi}{2} \cdot \frac{\boldsymbol{\sigma}_i}{\rho}\right) \tag{20}$$

where $\boldsymbol{\varepsilon}$ is transformed error.

Differentiating $\boldsymbol{\varepsilon}_i$ with respect to time, we obtain

$$\dot{\boldsymbol{\varepsilon}}_i = \frac{\partial \boldsymbol{S}^{-1}}{\partial\left(\frac{\boldsymbol{\sigma}_i}{\rho}\right)} \cdot \frac{1}{\rho}\left(\dot{\boldsymbol{\sigma}}_i - \frac{\dot{\rho}}{\rho}\boldsymbol{\sigma}_i\right) \tag{21}$$

Denote

$$\boldsymbol{R} = \operatorname{diag}\{r_1, r_2, r_3\}, \boldsymbol{v} = [v_1, v_2, v_3]^{\mathrm{T}}$$

$$r_i = \frac{\partial \boldsymbol{S}^{-1}}{\partial\left(\frac{\boldsymbol{\sigma}_i}{\rho}\right)} \cdot \frac{1}{\rho},\ v_i = -\frac{\dot{\rho}}{\rho}\boldsymbol{\sigma}_i,\ i = 1,2,3 \tag{22}$$

Therefore, the transformed system model is provided by

$$\begin{cases} \dot{\boldsymbol{\varepsilon}} = \boldsymbol{R}(\boldsymbol{G}(\boldsymbol{\sigma})\boldsymbol{\omega} + \boldsymbol{v}) \\ \begin{bmatrix} \dot{\boldsymbol{\eta}} \\ \dot{\boldsymbol{\psi}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{K} & -\boldsymbol{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\psi} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{C} \end{bmatrix} \boldsymbol{\delta\omega} \\ \boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times(\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega} + \boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi}) + \\ \qquad \boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C\psi} + \boldsymbol{K\eta} - \boldsymbol{C\delta\omega}) + \boldsymbol{u} + \boldsymbol{d} \end{cases} \tag{23}$$

It is established that the tracking error can be guaranteed within PPB as long as the stability of the transformed system is ensured. Then the stabilization of the transformed system can ensure the control objective of the original system.

A robust adaptive control algorithm with PPC (RAC-PPC) is presented. The transformed system (Eq.(23)) is a strict feedback system, and hence backstepping is the suitable approach.

Considering $\boldsymbol{\omega}$ as the virtual control variable, the tracking error is defined as

$$\boldsymbol{z} = \boldsymbol{\omega} - \boldsymbol{\alpha} \tag{24}$$

where $\boldsymbol{\alpha}$ is a virtual control input to be designed later.

The first Lyapunov candidate function is chosen as

$$\begin{aligned} V_1 = {} & \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon} + \\ & \frac{1}{2}[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{K}_1 \begin{bmatrix} 2\boldsymbol{K} + \boldsymbol{C}^2 & \boldsymbol{C} \\ \boldsymbol{C} & 2\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}} \end{bmatrix} + \\ & \frac{1}{2}[\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{K}_2 \begin{bmatrix} 2\boldsymbol{K} + \boldsymbol{C}^2 & \boldsymbol{C} \\ \boldsymbol{C} & 2\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi \end{bmatrix} \end{aligned} \tag{25}$$

where the positive definite matrices $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ are partitioned as

$$\boldsymbol{K}_1 = \begin{bmatrix} k_{11}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & k_{12}\boldsymbol{I} \end{bmatrix}, \boldsymbol{K}_2 = \begin{bmatrix} k_{21}\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & k_{22}\boldsymbol{I} \end{bmatrix} \tag{26}$$

Using Eqs.(16), (17), the time derivative of Eq.(25) along the system trajectories (Eq.(23)) is given by

$$\dot{V}_1=[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG}+(k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C}-2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}](\boldsymbol{z}+\boldsymbol{\alpha})+\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv}-[\hat{\boldsymbol{\eta}}^{\mathrm{T}}\quad\hat{\boldsymbol{\psi}}^{\mathrm{T}}]\begin{bmatrix}k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K}\\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix}-[\boldsymbol{e}_\eta^{\mathrm{T}}\quad\boldsymbol{e}_\psi^{\mathrm{T}}]\begin{bmatrix}k_{21}\boldsymbol{CK} & -2k_{21}\boldsymbol{K}\\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} \tag{27}$$

Define the stabilizing function $\boldsymbol{\alpha}(\boldsymbol{\varepsilon},\hat{\boldsymbol{\eta}},\hat{\boldsymbol{\psi}})$ as

$$\boldsymbol{\alpha}=-\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}}+\boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}}-2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right] \tag{28}$$

Based on what is mentioned above, Eq.(27) becomes

$$\dot{V}_1=[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG}+(k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C}-2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\boldsymbol{z}+\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv}-[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG}+(k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C}-2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}}+\boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}}-2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right]-[\hat{\boldsymbol{\eta}}^{\mathrm{T}}\quad\hat{\boldsymbol{\psi}}^{\mathrm{T}}]\begin{bmatrix}k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K}\\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\eta}}\\ \hat{\boldsymbol{\psi}}\end{bmatrix}-[\boldsymbol{e}_\eta^{\mathrm{T}}\quad\boldsymbol{e}_\psi^{\mathrm{T}}]\begin{bmatrix}k_{21}\boldsymbol{CK} & -2k_{21}\boldsymbol{K}\\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_\eta\\ \boldsymbol{e}_\psi\end{bmatrix} \tag{29}$$

Take the derivate of $\boldsymbol{z}$ left-multiplied by inertia matrix $\boldsymbol{J}_{\mathrm{mb}}$, then we have

$$\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{z}}=-\boldsymbol{\omega}^{\times}(\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega}+\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi})+\boldsymbol{u}+\boldsymbol{d}-\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\alpha}}+\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C\psi}+\boldsymbol{K\eta}-\boldsymbol{C\delta\omega}) \tag{30}$$

To avoid the complex computation of $\dot{\boldsymbol{\alpha}}$, an FOSD based on Lemma 2 is used to approximate it[23], that is

$$\begin{cases}\dot{\boldsymbol{\chi}}=-\boldsymbol{K}_{\alpha1}|\boldsymbol{\chi}-\boldsymbol{\alpha}|^{0.5}\operatorname{sign}(\boldsymbol{\chi}-\boldsymbol{\alpha})+\boldsymbol{\zeta}\\ \dot{\boldsymbol{\zeta}}=-\boldsymbol{K}_{\alpha2}\operatorname{sign}(\boldsymbol{\zeta}-\dot{\boldsymbol{\chi}})\end{cases} \tag{31}$$

where $\boldsymbol{\chi}$ and $\boldsymbol{\zeta}$ are the states of the system Eq.(31), and $\boldsymbol{K}_{\alpha1},\boldsymbol{K}_{\alpha2}$ are the positive definite design matrixes.

According to Eq.(30), we have

$$\dot{\boldsymbol{\alpha}}=\dot{\boldsymbol{\chi}}+\Delta\dot{\boldsymbol{\alpha}} \tag{32}$$

where $\Delta\dot{\boldsymbol{\alpha}}$ denotes the estimate error. Obviously, we have

$$|\Delta\dot{\boldsymbol{\alpha}}|\leqslant\boldsymbol{\lambda}_i,\ \boldsymbol{\lambda}_i>0\qquad i=1,2,3 \tag{33}$$

Then, Eq.(30) can be rewritten as

$$\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{z}}=-\boldsymbol{\omega}^{\times}(\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega}+\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi})-\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\chi}}+\boldsymbol{u}+\bar{\boldsymbol{d}}+\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C\psi}+\boldsymbol{K\eta}-\boldsymbol{C\delta\omega}) \tag{34}$$

where $\bar{\boldsymbol{d}}$ is the lumped uncertainty, including external disturbance and the estimate error of sliding mode differentiator, i.e., $\bar{\boldsymbol{d}}=\boldsymbol{d}-\boldsymbol{J}_{\mathrm{mb}}\Delta\dot{\boldsymbol{\alpha}}$. According to Assumption 1 and Eq.(33), the boundness of $\bar{\boldsymbol{d}}$ is ensured, that is

$$|\bar{\boldsymbol{d}}_i|\leqslant\boldsymbol{\rho}_i\qquad i=1,2,3 \tag{35}$$

where $\boldsymbol{\rho}_i$ is the unknown upper bound of $\bar{\boldsymbol{d}}_i$ and satisfies $\boldsymbol{\rho}_i\geqslant\boldsymbol{d}_{Mi}+\boldsymbol{\lambda}_i(i=1,2,3)$. Since $\boldsymbol{\rho}$ is unknown, an adaptive law will be designed to estimate it online, and $\hat{\boldsymbol{\rho}}$ will be employed to denote the estimation.

By recalling the definitions of linear operator in Eq.(9), the parametric linearization of terms $\boldsymbol{\omega}^{\times}\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega}$ and $\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\chi}}$ in Eq.(34) can be given as

$$\boldsymbol{\omega}^{\times}\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{\omega}=\boldsymbol{\omega}^{\times}L(\boldsymbol{\omega})\boldsymbol{\theta}_{\mathrm{mb}},\ \boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{\chi}}=L(\dot{\boldsymbol{\chi}})\boldsymbol{\theta}_{\mathrm{mb}} \tag{36}$$

Then Eq.(34) can be developed from Eq.(36) as

$$\boldsymbol{J}_{\mathrm{mb}}\dot{\boldsymbol{z}}=\boldsymbol{F}\boldsymbol{\theta}_{\mathrm{mb}}-\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\psi}+\boldsymbol{u}+\bar{\boldsymbol{d}}+\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C\psi}+\boldsymbol{K\eta}-\boldsymbol{C\delta\omega}) \tag{37}$$

where $\boldsymbol{F}=-\boldsymbol{\omega}^{\times}L(\boldsymbol{\omega})-L(\dot{\boldsymbol{\chi}})$.

The design procedure can be summarized in the following theorem.

Theorem 1 Consider the flexible spacecraft system governed by Eqs.(1), (5) and (6) with Assumptions 1—2. If the control law is designed by

$$\boldsymbol{u}=-\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}}+\boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}}-2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right]+\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{C\delta\omega}+[\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\hat{\boldsymbol{\psi}}-\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\hat{\boldsymbol{\psi}}+\boldsymbol{K}\hat{\boldsymbol{\eta}})]-\frac{1}{2}(\boldsymbol{\delta\omega}^{\times})^{\mathrm{T}}(\boldsymbol{\delta\omega}^{\times}\boldsymbol{z})-\frac{1}{2}(\boldsymbol{C\delta})^{\mathrm{T}}(\boldsymbol{C\delta z})-\frac{1}{2}(\boldsymbol{K\delta})^{\mathrm{T}}(\boldsymbol{K\delta z})-\boldsymbol{F}\hat{\boldsymbol{\theta}}_{\mathrm{mb}}-\boldsymbol{K}_3\boldsymbol{z}-\tanh(\boldsymbol{z})\hat{\boldsymbol{\rho}}-\frac{\boldsymbol{z}(1+k)|\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv}|}{\|\boldsymbol{z}\|^2+b} \tag{38}$$

and the adaptive control is selected as

$$\begin{cases}\dot{\hat{\theta}}_{\mathrm{mb}}=\mathrm{Proj}_{\hat{\theta}_{\mathrm{mb}}}(\Gamma_1\boldsymbol{F}^{\mathrm{T}}\boldsymbol{z})\\ \dot{\hat{\rho}}=\Gamma_2(\tanh(\boldsymbol{z})\boldsymbol{z}-k_\rho\hat{\boldsymbol{\rho}})\end{cases} \tag{39}$$

where $k\in L_\infty$[24-25] satisfies

$$\dot{k}=\begin{cases}\dfrac{a}{k}\cdot\dfrac{k\|\boldsymbol{z}\|^2-b_1}{\|\boldsymbol{z}\|^2+b}\cdot|\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv}| & k\neq0\\ b & k=0\end{cases} \tag{40}$$

with $b_1>b>0$ that are small constants. The projection operator in Eq.(39) is defined to avoid the parameter drift problem in Ref.[26], then the control objective shown in Section 1.3 can be achieved.

Proof Consider the composite Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\boldsymbol{J}_{\mathrm{mb}}\boldsymbol{z} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{\mathrm{mb}}^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\tilde{\boldsymbol{\theta}}_{\mathrm{mb}} + \frac{1}{2}\tilde{\boldsymbol{\rho}}^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\tilde{\boldsymbol{\rho}} + \frac{1}{2a}k^2 \tag{41}$$

In view of Eq.(29) and control law Eq.(38), taking the derivative of the above Lyapunov function along Eq.(37), it follows that

$$\begin{aligned}\dot{V}_2 = &-[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}} + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right] - \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\begin{bmatrix} k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K} \\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - \\ &[\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\begin{bmatrix} k_{21}\boldsymbol{CK} & -2k_{21}\boldsymbol{K} \\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C}\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} + \\ &(\tilde{\boldsymbol{\theta}}_{\mathrm{mb}}^{\mathrm{T}}\boldsymbol{\Gamma}_1^{-1}\dot{\hat{\boldsymbol{\theta}}}_{\mathrm{mb}} - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{F}\tilde{\boldsymbol{\theta}}_{\mathrm{mb}}) + \\ &(\boldsymbol{z}^{\mathrm{T}}\bar{\boldsymbol{d}} - \boldsymbol{z}^{\mathrm{T}}\mathrm{Tanh}(\boldsymbol{z})\hat{\boldsymbol{\rho}} + \tilde{\boldsymbol{\rho}}^{\mathrm{T}}\boldsymbol{\Gamma}_2^{-1}\dot{\hat{\boldsymbol{\rho}}}) + \\ &\left(\frac{1}{a}k\dot{k} + \boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv} - \frac{(1+k)\|\boldsymbol{z}\|^2|\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{Rv}|}{\|\boldsymbol{z}\|^2 + b}\right) - \\ &\frac{1}{2}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z}) - \frac{1}{2}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z}) - \\ &\frac{1}{2}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z}) - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{e}_\psi + \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{e}_\psi + \boldsymbol{K}\boldsymbol{e}_\eta)\end{aligned} \tag{42}$$

The last terms in Eq.(42) can be expanded as

$$\begin{cases} -\boldsymbol{z}^{\mathrm{T}}\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{e}_\psi \leqslant \frac{1}{2}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{\delta}\boldsymbol{\omega}^{\times}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\psi^{\mathrm{T}}\boldsymbol{e}_\psi \\ \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{e}_\psi \leqslant \frac{1}{2}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{C}\boldsymbol{\delta}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\psi^{\mathrm{T}}\boldsymbol{e}_\psi \\ \boldsymbol{z}^{\mathrm{T}}\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{e}_\eta \leqslant \frac{1}{2}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z})^{\mathrm{T}}(\boldsymbol{K}\boldsymbol{\delta}\boldsymbol{z}) + \frac{1}{2}\boldsymbol{e}_\eta^{\mathrm{T}}\boldsymbol{e}_\eta \end{cases} \tag{43}$$

Thus, by substituting Eq.(43) and the updating law Eqs.(39),(40) into Eq.(42), we have

$$\begin{aligned}\dot{V}_2 \leqslant &-[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}} + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right] - \boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} - \\ &\frac{k_\rho}{2}\|\tilde{\boldsymbol{\rho}}\|^2 + \frac{1}{2}\|\boldsymbol{\varphi}\|^2 + \frac{1}{2}(1+k_\rho)\|\boldsymbol{\rho}\|^2\end{aligned} \tag{44}$$

where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are given by

$$\boldsymbol{P}_1 = \begin{bmatrix} k_{11}\boldsymbol{CK} & -2k_{11}\boldsymbol{K} \\ 2k_{12}\boldsymbol{K} & k_{12}\boldsymbol{C}\end{bmatrix}$$

$$\boldsymbol{P}_2 = \begin{bmatrix} k_{21}\boldsymbol{CK} - \frac{1}{2}\boldsymbol{I} & -2k_{21}\boldsymbol{K} \\ 2k_{22}\boldsymbol{K} & k_{22}\boldsymbol{C} - \boldsymbol{I}\end{bmatrix}$$

If k_{1i}, k_{2i} $(i=1,2)$ is chosen such that $\boldsymbol{P}_1 > 0, \boldsymbol{P}_2 > 0$, the following inequality holds

$$\begin{aligned}\dot{V}_2 \leqslant &-[\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG} + (k_{12}\hat{\boldsymbol{\psi}}^{\mathrm{T}}\boldsymbol{C} - 2k_{11}\hat{\boldsymbol{\eta}}^{\mathrm{T}}\boldsymbol{K})\boldsymbol{\delta}]\cdot \\ &\left[(\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{RG})^{\mathrm{T}} + \boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}} - 2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})\right] - \\ &[\hat{\boldsymbol{\eta}}^{\mathrm{T}} \quad \hat{\boldsymbol{\psi}}^{\mathrm{T}}]\boldsymbol{P}_1\begin{bmatrix}\hat{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\psi}}\end{bmatrix} - [\boldsymbol{e}_\eta^{\mathrm{T}} \quad \boldsymbol{e}_\psi^{\mathrm{T}}]\boldsymbol{P}_2\begin{bmatrix}\boldsymbol{e}_\eta \\ \boldsymbol{e}_\psi\end{bmatrix} - \\ &\boldsymbol{z}^{\mathrm{T}}\boldsymbol{K}_3\boldsymbol{z} - \frac{k_\rho}{2}\|\tilde{\boldsymbol{\rho}}\|^2 + D\end{aligned} \tag{45}$$

where $D = \frac{1}{2}\|\boldsymbol{\varphi}\|^2 + \frac{1}{2}(1+k_\rho)\|\boldsymbol{\rho}\|^2$. This implies that $\boldsymbol{\varepsilon}$ and $\boldsymbol{z}$ are uniformly ultimately bounded. The boundedness of $\boldsymbol{\varepsilon}$ leads to the control objective of controller design by the property of PPC.

Remark From the performance analysis, we know that the proposed controller can achieve the prescribed performance. For arbitrary parametric uncertainty and external disturbances with bounded amplitudes, the controller can eliminate their influence, and therefore the system robustness is improved.

3 Numerical Simulations

The proposed robust adaptive control with prescribed performance (RAC-PPC) is verified through the numerical simulations. The spacecraft is characterized by a nominal main body inertia matrix

$$\boldsymbol{J} = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 280 & 10 \\ 4 & 10 & 190\end{bmatrix} \mathrm{kg}\cdot\mathrm{m}^2$$

and by the coupling matrix

$$\boldsymbol{\delta} = \begin{bmatrix} 6.456\,37 & 1.278\,14 & 2.156\,29 \\ -1.256\,19 & 0.917\,56 & -1.672\,64 \\ 1.116\,87 & 2.489\,01 & -0.836\,74 \\ 1.236\,37 & -2.658\,10 & -1.125\,03\end{bmatrix} \mathrm{kg}^{1/2}\mathrm{m/s}^2$$

Then matrix $\boldsymbol{J}_{\mathrm{mb}} = \boldsymbol{J} - \boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{\delta}$ is given by

$$\boldsymbol{J} = \begin{bmatrix} 303.961\,3 & -3.593\,0 & -9.697\,5 \\ -3.593\,0 & 264.263\,8 & 7.870\,9 \\ -9.697\,5 & 7.870\,9 & 180.586\,9\end{bmatrix} \mathrm{kg}\cdot\mathrm{m}^2$$

The first four elastic modes have been taken into account for the implemented spacecraft model resulting from the modal analysis of the structure, with natural frequency and damping presented in Table 1.

In the following simulations, the rest-to-rest slew maneuver is considered to bring a flexible

Table 1 Parameters of the flexible dynamics

Mode	Natural frequency/ $(\mathrm{rad} \cdot \mathrm{s}^{-1})$	Damping
1	1.097 3	0.050
2	1.276 1	0.060
3	1.653 8	0.080
4	2.289 3	0.025

spacecraft with any initial nonzero attitude to zero and then to keep it resting at zero attitude. The initial attitude and initial angular velocity are $\boldsymbol{\sigma}^{\mathrm{T}}(0)=[0.713\,2, -0.377\,6, 0.229\,8]$ and $\boldsymbol{\omega}^{\mathrm{T}}(0)=[0,0,0]$. In addition, the initial modal variables and its time derivative are given by $\boldsymbol{\eta}(0)=\boldsymbol{0}$ and $\boldsymbol{\psi}(0)=\dot{\boldsymbol{\eta}}(0)+\boldsymbol{\delta\omega}(0)=\boldsymbol{0}$.

To examine the robustness to external disturbance, simulation is carried on corresponding to the following periodic disturbance torque

$$\boldsymbol{d}(t)=\begin{bmatrix} 0.03\cos(0.01t)+0.1 \\ 0.015\sin(0.02t)+0.03\cos(0.025t) \\ 0.03\sin(0.01t)+0.01 \end{bmatrix}\mathrm{N}\cdot\mathrm{m}$$

To illustrate the effectiveness of the proposed control approach, we apply the control method of this paper (RAC-PPC) and the control method without prescribed performance (RAC). The RAC control law is given as follows

$$\begin{aligned}\boldsymbol{u}=&-[(\boldsymbol{\sigma}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{\sigma}))^{\mathrm{T}}+\boldsymbol{\delta}^{\mathrm{T}}(k_{12}\boldsymbol{C}\hat{\boldsymbol{\psi}}-2k_{11}\boldsymbol{K}\hat{\boldsymbol{\eta}})]+\\&\boldsymbol{\delta}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{\delta\omega}+[\boldsymbol{\omega}^{\times}\boldsymbol{\delta}^{\mathrm{T}}\hat{\boldsymbol{\psi}}-\boldsymbol{\delta}^{\mathrm{T}}(\boldsymbol{C}\hat{\boldsymbol{\psi}}+\boldsymbol{K}\hat{\boldsymbol{\eta}})]-\\&\frac{1}{2}(\boldsymbol{\delta\omega}^{\times})^{\mathrm{T}}(\boldsymbol{\delta\omega}^{\times}\boldsymbol{z})-\frac{1}{2}(\boldsymbol{C\delta})^{\mathrm{T}}(\boldsymbol{C\delta z})-\\&\frac{1}{2}(\boldsymbol{K\delta})^{\mathrm{T}}(\boldsymbol{K\delta z})-\boldsymbol{F}\hat{\boldsymbol{\theta}}_{\mathrm{mb}}-\boldsymbol{K}_3\boldsymbol{z}-\tanh(\boldsymbol{z})\hat{\boldsymbol{\rho}}\end{aligned} \tag{46}$$

The prescribed performances of attitude control error are set to steady-state error no more than $\rho_{\infty}=0.001$, minimum convergence speed $\mathrm{e}^{-\beta t}=\mathrm{e}^{-0.2t}$, and the parameter $\rho_0=1.213\,2$. The controller parameters are shown in Table 2.

Attitude control error responses and the prescribed performance function bounds are depicted in Figs. 1—3. Obviously, attitude control errors under RAC-PPC are confined in the prescribed bounds, i.e., the prescribed performance is achieved in spite of parametric uncertainties, external disturbances and unmeasured elastic vibration. However, the attitude control errors without prescribed performance violate the prescribed error bounds. Moreover, compared with RAC, RAC-PPC has faster convergence

Table 2 Controller parameters

Control scheme	Parameter and value
RAC	$k_{11}=k_{12}=0.01, k_{21}=4, k_{22}=10$ $\boldsymbol{K}_3=0.01\boldsymbol{I}_3$ $\boldsymbol{\Gamma}_1=0.01\boldsymbol{I}_6, \boldsymbol{K}_{\alpha1}=\boldsymbol{K}_{\alpha2}=\boldsymbol{I}_3$ $\boldsymbol{\Gamma}_2=0.01\boldsymbol{I}_3, k_{\rho}=0.01$
RAC-PPC	$k_{11}=k_{12}=0.01, k_{21}=4, k_{22}=10$ $\boldsymbol{K}_3=0.01\boldsymbol{I}_3$ $\boldsymbol{\Gamma}_1=0.01\boldsymbol{I}_6, \boldsymbol{K}_{\alpha1}=\boldsymbol{K}_{\alpha2}=\boldsymbol{I}_3$ $\boldsymbol{\Gamma}_2=0.01\boldsymbol{I}_3, k_{\rho}=0.01$ $a=0.001, b=0.1, b_1=0.5$ $k(0)=0.1$

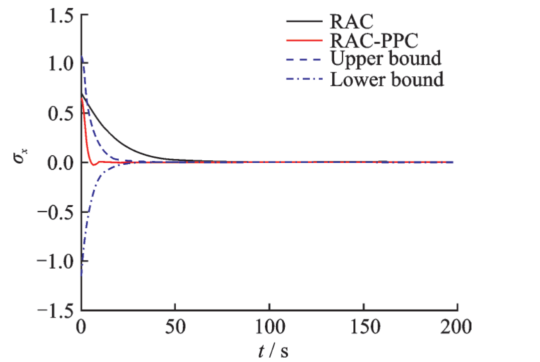

Fig.1 Attitude error $\boldsymbol{\sigma}_x$ and prescribed performance bounds

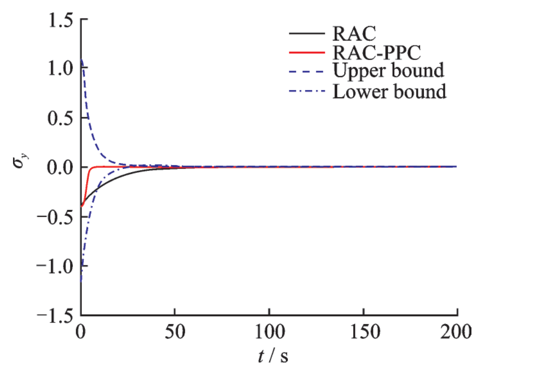

Fig.2 Attitude error $\boldsymbol{\sigma}_y$ and prescribed performance bounds

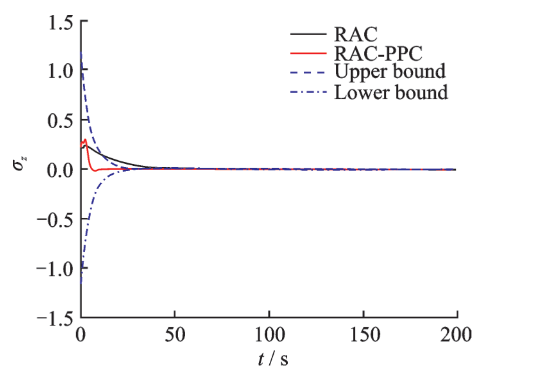

Fig.3 Attitude error $\boldsymbol{\sigma}_z$ and prescribed performance bounds

speed, shown as Fig.4.

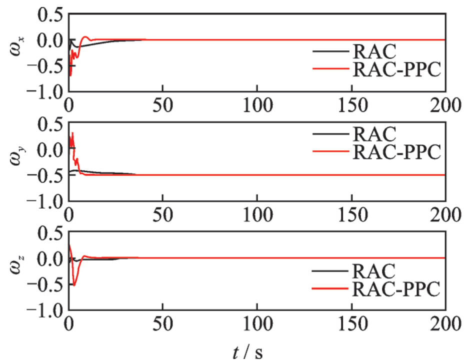

Fig.4 Time response of angular velocity

The steady control errors are presented in Table 3. As shown in Table 3, the attitude control errors under RAC-PPC are confined in the prescribed steady bounds. Furthermore, compared with RAC, RAC-PPC can achieve higher steady control precision.

Table 3 Steady error comparison

Variable	Steady error	
	RAC	RAC-PPC
σ_x	2.86E−4	4.41E−10
σ_y	2.52E−3	3.58E−9
σ_z	8.37E−3	5.05E−10
ω_x/(rad • s^{-1})	6.13E−5	1.95E−5
ω_y/(rad • s^{-1})	7.71E−5	1.05E−5
ω_z/(rad • s^{-1})	1.34E−4	1.14E−5

The behavior of the modal displacements and their estimates are given in Fig.5. It is noted that all the elastic vibrations and their rates approach zero at 80 s. It can be observed that not only the vibrations induced by attitude maneuver are effectively suppressed, but also the model displacements can be well estimated by the model observer, whose performance is explicitly demonstrated in Fig.6. The steady observation errors of the model observer in Eq.(16) are tabulated in Table 4.

The responses of estimated inertial parameters corresponding to update law of Eq.(39) are illustrated in Figs.7, 8. It is clear that the convergence of these estimated parameters can be achieved, but not to the true values. That is because sufficient frequency components in the tracking error states are not guaranteed. In other words, the persistent excitation

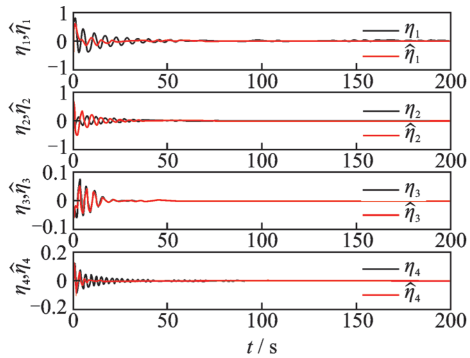

Fig.5 Time response of vibration displacements and their estimates

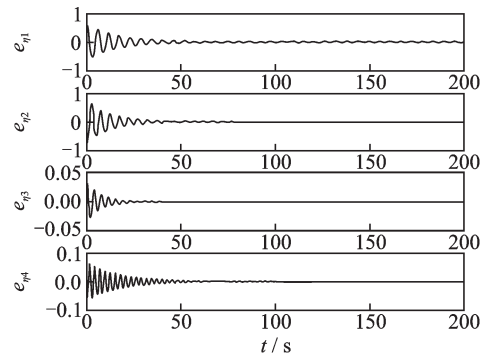

Fig.6 Time response of vibration estimate errors

Table 4 Steady observation errors of model observer

Mode	Steady observation error
Mode 1 $\lvert \eta_1 - \hat{\eta}_1 \rvert$	7.381E−6
Mode 2 $\lvert \eta_2 - \hat{\eta}_2 \rvert$	1.61E−7
Mode 3 $\lvert \eta_3 - \hat{\eta}_3 \rvert$	0
Mode 4 $\lvert \eta_4 - \hat{\eta}_4 \rvert$	3.92E−7

(PE) condition is not satisfied. Furthermore, time responses of the demand control torque are depicted in Fig.9.

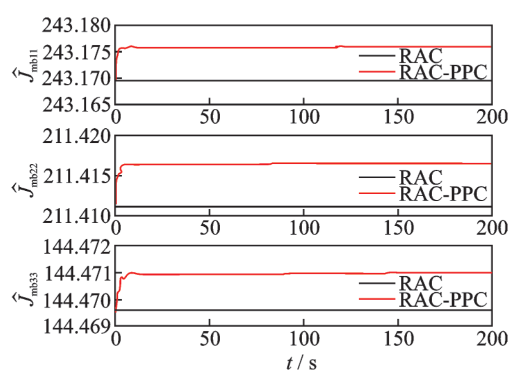

Fig.7 Time response of the estimated parameters of inertia

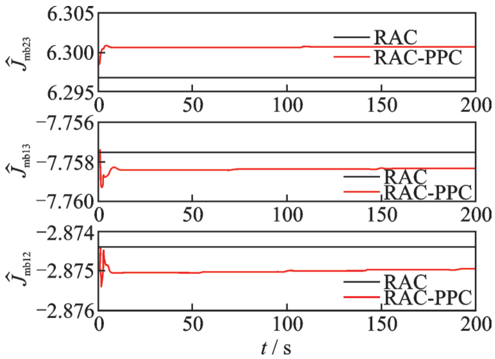

Fig. 8 Time response of the estimated parameters of the product of inertia

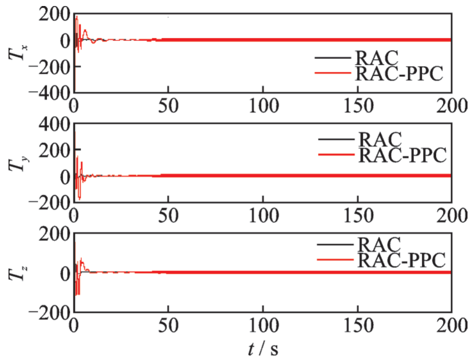

Fig.9 Time response of control torques

4 Conclusions

In this study, taking the parametric uncertainty, external disturbances and unmeasured elastic vibration are taken into account simultaneously, a guaranteed prescribed performance robust adaptive control scheme is proposed for attitude maneuver and vibration suppression of flexible spacecraft. Based on PPC theory, this novel control scheme can guarantee attitude errors to stratify the prescribed transient-steady performance by introducing the performance function. During the control design, a modal observer is constructed to supply elastic modal estimates by utilizing the inherent physical properties of flexible appendages. With the utilization of Sliding mode differentiator, the problem of explosion of complexity inherent in traditional backstepping design is also overcomed. In addition, an adaptive law is derived so that the requirements of knowing system parameters and the upper bound of the lumped uncertainty are eliminated. Finally, the stability is rigorously proved and the simulation results demonstrate the effectiveness and superiority of the proposed control scheme.

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Author contributions Mr. **TAO Jiawei** designed the study, complied the models, conducted the analysis, interpreted the results and wrote the manuscript. Prof. **ZHANG Tao** contributed to the discussion and background the study. Both authors commented on the manuscript draft and approved the final manuscript.

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