A Distributed Cooperative Localization Algorithm for Mobile Multi-platforms Oriented to Unpredicted Communication Topology

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(Received 28 May 2018; revised 15 November 2018; accepted 20 November 2018)

Abstract: The cooperative localization (CL) is affected by the communication topology among the platforms. Based on the unscented Kalman filtering, the distributed CL (DCL) oriented to the unpredicted communication topology is investigated. To improve the adaptability, the character of the look-up Cholesky decomposition is exploited for the covariance matrix decomposing. Then, the distributed U transformation can be dynamically implemented according to the available communication topology. In the proposed algorithm, the global information is not required for the individual, and only the available information from the neighbor is used. Each platform's state can be estimated independently. The error covariance of the state estimates can be updated in the single platform. The algorithm is adaptive to any serial communication topologies where the measuring to the measured platform is a starting path. The applicability of the proposed algorithm to unpredicted communication topology is improved, remaining equivalent localization performance to free connection communication.

Key words: mobile multi-platforms; cooperative localization; unscented Kalman filtering; communication topology

CLC number: U674.7 **Document code:** A **Article ID:** 1005-1120(2019)02-0224-08

Nomenclature

N	Total number of platforms
x_i, y_i	Actual position scalar of platform i
$\boldsymbol{x}_i$	Actual position vector of platform i
$\bar{\boldsymbol{x}}_i$	Predictive position estimate of platform i
$\hat{\boldsymbol{x}}_i$	Posteriori position estimate of platform i
V_{mi}	Measurement linear velocity
ν_i	Measurement noise of linear velocity
z_{ij}	Relative measurement between platform i,j
n_{ij}	Noise of relative measurement z_{ij}
P_{ij}	Error covariance of the estimates between platform i,j
$\boldsymbol{x}$	Whole actual position vector, $\boldsymbol{x}=[\boldsymbol{x}_1,\cdots,\boldsymbol{x}_N]$
P	Whole error covariance of position estimate
k	Time step
δ	Sample interval
$\bar{\Theta}(k)$	Predictive estimate of the variable Θ
$\hat{\Theta}$	Posteriori estimate
$[\Theta_1;\Theta_2]$	Column stacking of the variables Θ_1,Θ_2

0 Introduction

Multi-platforms cooperation has great prospects in the military and the civil field, for example, environment exploring, coordination attacking[1-2]. Accurate self-localization is a premise for these tasks. Only relying on the inertial navigation unit or the integrated navigation system, the localization error will increase with time. This issue can be alleviated by the cooperative localization (CL), where the inter-platform measurements are utilized to improve the localization accuracy.

The information flowing among the platforms is the impetus for CL. However, in reality, the flowing may be constrained by the problems as follows: (1) Some platforms fail accidentally and cannot serve as the communication nodes; (2) various interferences affect the communication link; (3) the

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How to cite this article: WANG Leigang, CUI Jianling, KONG Depei, et al. A Distributed Cooperative Localization Algorithm for Mobile Multi-platforms Oriented to Unpredicted Communication Topology[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2019,36(2):224-231.
http://dx.doi.org/10.16356/j.1005-1120.2019.02.005

geometric distance is beyond the communication range. Consequently, the free communication topology cannot be guaranteed. Instead, it may be time-varying and un-prescribed, i. e., the unpredicted communication topology. The applicability of CL is thus restricted.

The CL has been investigated by several techniques, e. g., geometrical pattern[3], probabilistic reasoning[4-5], filtering[6-8]. Consider that the first two techniques are unfavorable in the calculation and storage cost. Additionally, the centralized architecture is easy to be disabled. Hence, the distributed CL (DCL) based on the filtering is hot. In early works, how to improve the localization accuracy is focused, and usually assume that each platform can get what it needs freely. In recent years, the influence from the communication topology has been gradually considered. In Ref.[9], a DCL algorithm suitable for the fan-shaped communication topology is proposed. In Ref.[10], a DCL algorithm adapted to the tree-like communication topology is addressed. Ref. [11] proposed a DCL algorithm adapted to a fixed ring structure. In the above algorithms, the extended Kalman filtering (EKF) or its inverse form is adopted for the non-linearity issue. While it has a few inherent defects in the accuracy and calculation cost.

Considering the advantages of unscented Kalman filtering (UKF) for the nonlinear system[12], the UKF is employed. Based on the UKF, we attempt to improve the applicability of the DCL algorithm to the unpredicted communication topology. In this process, two innovations are achieved:

(1) The recursion character of the look-up Cholesky decomposition is exploited for the covariance matrix decomposing.

(2) The algorithm is self-adaptive to the unpredicted communication topology. The unfixed serial communication topologies taken measurer-measuree platform as starting path can implement the algorithm.

1 Problem Statement

Assume that N platforms move in a two-dimensional area where a fixed reference frame is set. The position of each platform is denoted as the vector $\boldsymbol{x}_i(k)=[x_i(k); y_i(k)]$. ϕ denotes the motion heading. According to the linear velocity measurement $V_{mi}(k)$. The predictive position estimate of platform i can be expressed as

$$\bar{\boldsymbol{x}}_i(k+1)=\hat{\boldsymbol{x}}_i(k)+\begin{bmatrix}\delta V_{mi}(k)\cos\phi_{mi}\\ \delta V_{mi}(k)\sin\phi_{mi}\end{bmatrix} \tag{1}$$

At a certain moment k, assume that platform i detects platform j and obtains the relative measurement between them (e.g., relative distance, relative azimuth). The relative measurement can be expressed as

$$\boldsymbol{z}_{ij}(k)=\boldsymbol{h}_{ij}(\boldsymbol{x}_i(k),x_j(k))+n_{ij}(k) \tag{2}$$

where $n_{ij}\sim N(0,\boldsymbol{R}_{ij})$.

To this point, the CL based on the filtering can be stated as follow: at the moment k, based on the data of the predictive estimate $\{\bar{\boldsymbol{x}}_i(k)\}_{i=1}^{N}$, the corresponding error covariance $\bar{\boldsymbol{P}}(k)$ and the relative measurement $\boldsymbol{z}_{ij}(k)$, how to obtain the much credible posteriori estimate $\{\hat{\boldsymbol{x}}_i(k)\}_{i=1}^{N}$ as well as the corresponding covariance $\hat{\boldsymbol{P}}(k)$ for the single platform.

2 CL Model Based on UKF

As the usual KF, two stages, i.e., the time update and the observation update, are included in UKF. However, the position estimate is updated by the U transformation rather than the linearization as EKF. Here, how to implement the U transformation in a distributed manner, especially under the non-free communication, is crucial.

2.1 Time update

The data of proprioceptive measurement (e.g., velocity) and the relative measurement are essential for the time update and the observation update respectively. In CL, the frequency of relative measurement is lower than the proprioceptive one [7]. Thus, the time update and observation update are not carried out alternatively. The former always runs and the latter does not.

At each moment, since Eq.(1) is linear, the time update runs as the common KF. It has been proved that this update can be done independently in the single platform without communicating each other—see, e.g., Refs.[6,10].

When no relative measurement occurs, to unify the formula, after each time update, let

$$\hat{\boldsymbol{x}}_i(k+1)=\bar{\boldsymbol{x}}_i(k+1) \tag{3}$$

$$\hat{\boldsymbol{P}}_{ij}(k+1)=\bar{\boldsymbol{P}}_{ij}(k+1) \tag{4}$$

2.2 Observation update

Assume that at the moment k, platform m detects platform n and obtains the relative measurement $z_{mn}(k)$. To clear out the necessary elements for the single platform, first we analyze the distributed observation update of the centralized cooperative localization (CCL) where all data are centrally processed. Then, we address how to ensure that the single platform obtains the necessary elements.

The CCL provides a gold-standard benchmark for other algorithms. In CCL, the whole predictive position and the error covariance can be expressed as $\bar{\boldsymbol{x}}(k)=[\bar{x}_1(k),\cdots,\bar{x}_N(k)]$ and $\bar{\boldsymbol{P}}(k)=\begin{bmatrix}\bar{P}_{11}(k) & \cdots & \bar{P}_{1N}(k)\\ \vdots & \ddots & \vdots \\ \bar{P}_{N1}(k) & \cdots & \bar{P}_{NN}(k)\end{bmatrix}\in\mathbf{R}^{2N\times 2N}$, respectively.

For the observation update, the first step is using $\bar{\boldsymbol{x}}(k)$ and $\bar{\boldsymbol{P}}(k)$ to calculate $4N+1$ sets of the σ point by the U transformation, i.e.

$$\begin{cases}\bar{\boldsymbol{\xi}}_k^{(0)}=\bar{\boldsymbol{x}}(k)\\ \bar{\boldsymbol{\xi}}_k^{(p)}=\bar{\boldsymbol{x}}(k)+\sqrt{(2N+\lambda)\bar{\boldsymbol{P}}(k)}\,(:,p)\\ \qquad p=1,2,\cdots,2N\\ \bar{\boldsymbol{\xi}}_k^{(p)}=\bar{\boldsymbol{x}}(k)-\sqrt{(2N+\lambda)\bar{\boldsymbol{P}}(k)}\,(:,p-n)\\ \qquad p=n+1,n+2,\cdots,4N\end{cases} \tag{5}$$

where λ is a constant and $\boldsymbol{\Theta}(:,p)$ the pth column of matrix $\boldsymbol{\Theta}$. Then, each set of the σ point can be propagated by Eq.(2), i.e.

$$\bar{\zeta}_{mn}^{(p)}(k)=h(\bar{\xi}_k^{(p)}(m),\bar{\xi}_k^{(p)}(n)),p=0,1,\cdots,4N \tag{6}$$

$$\bar{z}_{mn}(k)=\sum_{p=0}^{4N}\omega_{(p)}^{m}\bar{\zeta}_{mn}^{(p)}(k) \tag{7}$$

where $\bar{\xi}_k^{(p)}(i)\in\mathbf{R}^{2\times 1}$ is the ith block in vector $\bar{\xi}_k^{(p)}$. $\bar{z}_{mn}(k)$ is the predictive relative measurement.

According to Eq.(7), the variance of the predictive measurement can be obtained as

$$\begin{aligned}\bar{\boldsymbol{P}}_{\bar{z}}(k)=\sum_{p=0}^{4N}\{&\omega_p^c(\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{z}_{mn}(k))\times\\ &(\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{z}_{mn}(k))^{\mathrm{T}}+\boldsymbol{R}_{mn}(k)\}\end{aligned} \tag{8}$$

The error covariance between the predictive relative measurement and the predictive estimate is given as

$$\begin{aligned}\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}(k)=\sum_{p=0}^{4N}&\omega_{(p)}^c(\bar{\xi}^{(p)}(k)-\bar{\boldsymbol{x}}(k))(\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\\ &\bar{z}_{mn}(k))^{\mathrm{T}}\end{aligned} \tag{9}$$

where $\boldsymbol{\omega}_p^c=\boldsymbol{\omega}_p^m=0.5/(2N+\lambda)$.

By Eqs.(8), (9), according to the method of KF observation update, the whole predictive estimate can be updated as

$$\hat{\boldsymbol{x}}(k)=\bar{\boldsymbol{x}}(k)+\boldsymbol{K}(\boldsymbol{z}_{mn}(k)-\bar{z}_{mn}(k)) \tag{10}$$

$$\hat{\boldsymbol{P}}(k)=\bar{\boldsymbol{P}}(k)+\boldsymbol{K}\bar{\boldsymbol{P}}_{\bar{z}}(k)\boldsymbol{K}^{\mathrm{T}} \tag{11}$$

$$\boldsymbol{K}=\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}(k)/\bar{\boldsymbol{P}}_{\bar{z}}(k) \tag{12}$$

Inspecting Eq.(10), the local measurement $\boldsymbol{z}_{mn}$ would update all the predictive estimates, and two types of elements are required:

(1) The elements related to the measuring and measured platform m,n, i.e.

$$\begin{aligned}\boldsymbol{S}_m(k)=\{&z_{mn}(k)-\bar{z}_{mn}(k),\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\\ &\bar{z}_{mn}(k),\bar{\boldsymbol{P}}_{\bar{z}}(k)\}\end{aligned} \tag{13}$$

which is called the source location information.

(2) The elements that determine how much the source location information is utilized, i.e., $\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}(k)$. Any block element in $\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}(k)$ can be expressed as

$$\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}^{i}(k)=f(\bar{\xi}_i^{(p)}(k)-\bar{\boldsymbol{x}}_i(k),\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{z}_{mn}(k)) \tag{14}$$

which is called the update weight, being only related to the source location information and the single platform i.

As Eq.(5), to obtain the σ points, the whole covariance matrix $\bar{\boldsymbol{P}}(k)$ and $\bar{\boldsymbol{x}}(k)$ are required. Both exist at the center in CCL. However, in DCL, confined by the communication condition, the whole covariance matrix $\bar{\boldsymbol{P}}(k)$ may be unavailable for each platform. Accordingly, two questions arise. One question is that the single platform cannot carry out the U-transformation to obtain the corresponding σ point as Eq.(5). The other is that as Eq.(11) is updated, along with the predictive estimates, the corresponding covariance also should be updated while it depends on the whole $\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}(k)$ and $\bar{\boldsymbol{P}}(k)$.

3 UKF-DCL Algorithm

3.1 Distributed U transformation

In some cases, the platform may be only receive its neighbors' information, instead of the whole elements $\boldsymbol{X}, \boldsymbol{P}$. Here, how the single platform obtains the equivalent σ points as the CCL is the prerequisite. Inspecting Eq.(5), it can be found that, to obtain the corresponding to the element $\xi_k^{(p)}(i)$, how the single platform i obtains the ith row elements in the matrix $\sqrt{\bar{\boldsymbol{P}}(k)}$ is the key for the distributed U transformation. Considering the symmetry of the matrix $\boldsymbol{P}$, it can be decomposed into a lower triangular matrix $\boldsymbol{A}$, i.e., $\boldsymbol{P}=\boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}$ and $\sqrt{\boldsymbol{P}}=\boldsymbol{A}$. The look-up Cholesky decomposition is employed to obtain the matrix $\boldsymbol{A}$. Let $\boldsymbol{A}=[\boldsymbol{A}_1,\cdots,\boldsymbol{A}_N]$ and $\boldsymbol{A}_i=[\boldsymbol{A}_{i1},\cdots,\boldsymbol{A}_{ii}]\in\mathbf{R}^{2\times 2i}$. Each block $\boldsymbol{A}_{ij}$ can be obtained by

$$\begin{bmatrix} \boldsymbol{A}_{11} & \cdots & 0 \\ \vdots & \ddots & 0 \\ \boldsymbol{A}_{(i-1)1} & \cdots & \boldsymbol{A}_{(i-1)(i-1)} \end{bmatrix}\begin{bmatrix} \boldsymbol{A}_{i1}^{\mathrm{T}} \\ \vdots \\ \boldsymbol{A}_{i(i-1)}^{\mathrm{T}} \end{bmatrix}=\begin{bmatrix} \boldsymbol{P}_{i1}^{\mathrm{T}} \\ \vdots \\ \boldsymbol{P}_{i(i-1)}^{\mathrm{T}} \end{bmatrix} \tag{15}$$

$$\boldsymbol{A}_{ii}\boldsymbol{A}_{ii}^{\mathrm{T}}=\boldsymbol{P}_{ii}-[\boldsymbol{A}_{i1},\cdots,\boldsymbol{A}_{i(i-1)}][\boldsymbol{A}_{i1},\cdots,\boldsymbol{A}_{i(i-1)}]^{\mathrm{T}} \tag{16}$$

Inspecting Eqs.(15), (16), the elements $\boldsymbol{A}_j\,(j>i)$ and $\boldsymbol{P}_{ij}^{\mathrm{T}}\,(j>i)$ supplied by platform j are not required for the row block $\boldsymbol{A}_i$. Assume that platform i receives the message from its neighbor, meanwhile an up-neighbor set $\mathrm{neigh}_i^u=\{1,2,\cdots,i-1\}$ is also available, which records the platform IDs of the message passing by. Then, for the receiver i, according to neigh_i^u, it can get the corresponding element $\boldsymbol{A}_i$. The first row $\boldsymbol{A}_{11}$ is obtained as $\boldsymbol{A}_{11}=\sqrt{\boldsymbol{P}_{11}}$. The successive row block can be obtained by the recursion. It means if the element blocks in the matrix $P$are virtually adjusted accordance with the communication order and decomposed, then each block $\boldsymbol{A}_i\,(i=1,\cdots,N)$ can be sequentially obtained.

Since the block $\bar{\boldsymbol{P}}_{ij}$ represents the error covariance between the predictive estimates of platforms i,j, the position adjustment of $\bar{\boldsymbol{P}}$ has no effect on its value. Thus, when one relative measurement occurs between the measuring platform m and measured platform n, the corresponding covariance item would be dynamically adjusted to the first position of the matrix $\boldsymbol{P}$ and forms a new covariance matrix as follows

$$\begin{bmatrix} \bar{\boldsymbol{P}}_{11}(k) & \cdots & \bar{\boldsymbol{P}}_{1N}(k) \\ \vdots & \ddots & \vdots \\ \bar{\boldsymbol{P}}_{N1}(k) & \cdots & \bar{\boldsymbol{P}}_{NN}(k) \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{\boldsymbol{P}}_{mm}(k) & \bar{\boldsymbol{P}}_{mn}(k) & * \\ \bar{\boldsymbol{P}}_{nm}(k) & \bar{\boldsymbol{P}}_{nn}(k) & * \\ * & * & * \end{bmatrix} \tag{17}$$

According to Eq.(14), only using its inherent element $\bar{\boldsymbol{P}}_{mm}(k)$, the platform m can obtain the block $\boldsymbol{A}_{11}$. Then it is added to the source location information $\boldsymbol{S}_m(k)$ and transmitted to the measured platform n. As the receiver n, according to Eqs.(13), (14), using $\boldsymbol{A}_{11}$ and $\bar{\boldsymbol{P}}_{mn}(k)$, it obtains $\boldsymbol{A}_{21}$, $\boldsymbol{A}_{22}$ and forms the secondary location information $S_n(k)$ and sends to its neighbor $a\,(a\notin \mathrm{neigh}_n^u)$. Based on the dynamic decomposition strategy of the covariance matrix, during the U transformation, the block element $\bar{\boldsymbol{\xi}}_i^{(p)}(k)\in\mathbf{R}^{2\times 1}$ in the vector $\bar{\boldsymbol{\xi}}^{(p)}(k)$ corresponding to the platform i can be expressed as

$$\bar{\xi}_i^{(p)}(k)=\begin{cases} \bar{x}_i+\sqrt{(n+\lambda)}\,\boldsymbol{A}_i(:,p) & 0<p\leqslant 2i \\ \bar{x}_i & p=0,p>2i \end{cases} \tag{18}$$

Substituting Eq.(18) into Eq.(7), the predictive relative measurement can be obtained. In platform i, utilizing the $\bar{\xi}_i^{(p)}(k)$ points, the covariance item between the predictive estimate and the predictive relative measurement can be expressed as

$$\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}^{i}(k)=\sum_{p=0}^{4N}\omega_p^{(c)}(\bar{\boldsymbol{\xi}}_i^{(p)}(k)-\bar{\boldsymbol{x}}_i(k))(\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{\boldsymbol{z}}_{mn}(k))^{\mathrm{T}} \tag{19}$$

Substituting Eq.(18) into Eq.(19), we have

$$\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}^{i}(k)=\sum_{p=1}^{2|\mathrm{neigh}_i^u|}\omega_p^{(c)}(\bar{\boldsymbol{\xi}}_i^{(p)}(k)-\bar{\boldsymbol{x}}_i(k))(\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{\boldsymbol{z}}_{mn}(k))^{\mathrm{T}} \tag{20}$$

where $|\mathrm{neigh}_i^u|$ denotes the number of the up-neighbors of platform i. Inspecting Eq.(20), the update weight $\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}^{i}(k)$ is only related to the element $\bar{\boldsymbol{\zeta}}_{mn}^{(p)}(k)-\bar{\boldsymbol{z}}_{mn}(k)$ from the measuring platform and the element $\bar{\boldsymbol{\xi}}_i^{(p)}(k)-\bar{\boldsymbol{x}}_i(k)$ in itself. The elements in the platforms $i\ (i\notin \mathrm{neigh}_i^u)$ are not required.

By the update weight $\bar{\boldsymbol{P}}_{\bar{x}\bar{z}}^{i}(k)$, the source location information $\boldsymbol{S}_m(k)$ can be used to update the predictive position estimate $\bar{\boldsymbol{x}}_i(k)$ in a linear addition manner.

$$\hat{x}_i(k)=\bar{x}_i(k)+\bar{P}^i_{\bar{x}\bar{z}}(k)/\bar{P}_{\bar{z}}(k)(z_{mn}(k)-\bar{z}_{mn}(k)) \tag{21}$$

3.2 Local covariance update

Inspecting Eq.(11), along with the whole position estimate being updated, the whole covariance matrix is also updated and the update increment is the second term of Eq.(11): $K\bar{P}_{\bar{z}}(k)K^{\mathrm{T}}$. Expanding $K\bar{P}_{\bar{z}}(k)K^{\mathrm{T}}$, each block element is obtained as

$$\Delta P_{ij}(k)=\bar{P}^i_{\bar{x}\bar{z}}(k)\bar{P}_{\bar{z}}(k)(\bar{P}^j_{\bar{x}\bar{z}}(k))^{\mathrm{T}} \tag{22}$$

Hence, each covariance item is updated as follows

$$\hat{P}_{ij}(k)=\bar{P}_{ij}(k)-\Delta P_{ij}(k) \tag{23}$$

For each platform i, once receiving the location information, all the update weights $\bar{P}^j_{\bar{x}\bar{z}}(k)$, $j\in \text{neigh}^u_i$ are available. Then, the covariance update increment $\Delta P_{ij}(k)$ can be obtained by Eq.(22), and the covariance terms $\bar{P}_{ij}(k)$ are updated as Eq.(23). However, for the covariance items $\bar{P}_{ij}(k)(j\notin \text{neigh}^u_i)$, they are unchanged. It should be pointed out that the update results need not to be feedback to the up-neighbors. For example, in the case of the communication topology as: $1\rightarrow 3\rightarrow 2\rightarrow 4$, the update progress of the covariance element is shown in Fig.1, where for each platform, the elements in the dashed line remain unchanged. For symmetry, i.e., $P_{ij}(k)=P^{\mathrm{T}}_{ji}(k)$, when the current unchanged elements are required in the future, they can be obtained by the transposition.

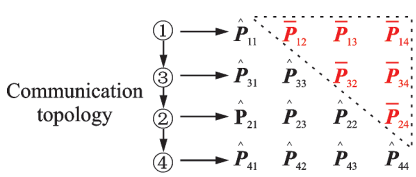

Fig.1 Relation between the covariance element update and communication topology

3.3 Algorithm procedure

For each platform, the procedure throughout CL is shown in Fig.2.

Assume that platform 1 detects platform 2 and obtains the relative measurement z_{12}. By communicating with platform 2 (Fig.3), platform 1 generates the source location information $S_1(k)$. The complete procedure is provided in Algorithm 1.

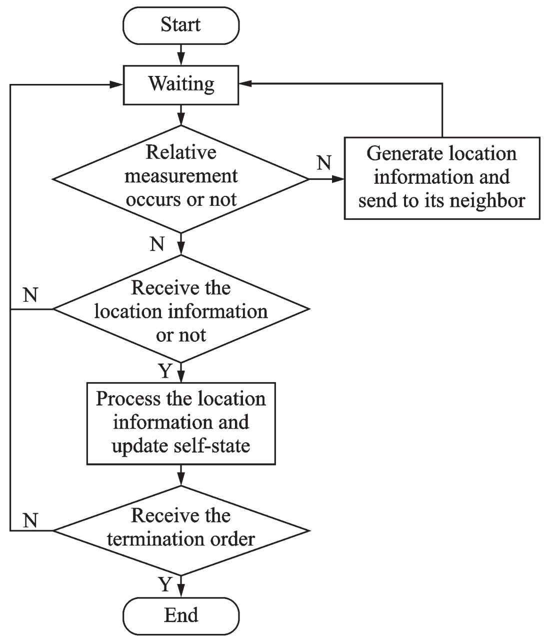

Fig.2 Flow chart of CL

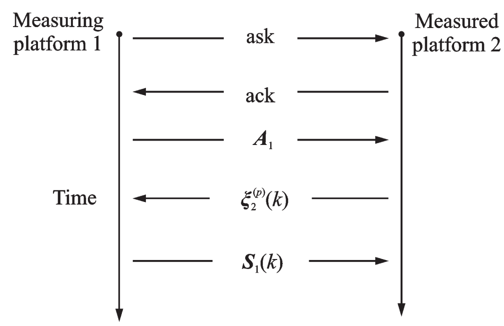

Fig.3 Mutual communication progress between the measuring and measured platform for the source location information generating

Algorithm 1 Generating the source location information (in the measuring platform 1)

(1) Calculate A_{11} according to $A_{11}=\sqrt{P_{11}}$; Calculate $\bar{\xi}^{(p)}_1(k)$ according to Eq.(18).

(2) When the link to the measured platform 2 is established, platform 1 send the block element A_{11} to platform 2.

(3) Receive $\bar{\xi}^{(p)}_2(k)$ corresponding to platform 2.

(4) Calculate $\bar{\zeta}^{(p)}_{12}(k)$, $\bar{z}_{12}(k)$ by Eqs.(6), (7), respectively.

(5) Calculate $\bar{P}_{\bar{z}}(k)$, $\bar{P}^1_{\bar{x}\bar{z}}(k)$ by Eqs. (8), (14), respectively.

(6) Update the predictive estimate $\bar{x}_1$ according to Eq.(21); update the local covariance item according to Eqs.(22), (23).

(7) Pack the source location information $\boldsymbol{S}_1(k)$ and send to platform 2.

For the receiver i, when it receives the location information S_{i_n} from its neighbor i_n, its predictive position estimate is updated as Algorithm 2.

Algorithm 2 Utilizing the location information (in the receiver i)

(1) Calculate $\bar{\boldsymbol{\xi}}_i^{(p)}(k)$ according to Eq.(18).

(2) Calculate $\bar{\boldsymbol{P}}^i_{\bar{x}\bar{z}}(k)$ according to Eq.(20).

(3) Update the predictive state $\bar{x}_i$ according to Eq.(21); update the local covariance item according to Eqs.(22),(23).

(4) Add $\boldsymbol{A}_i$, $\bar{\boldsymbol{P}}^i_{\bar{x}\bar{z}}(k)$ to $\boldsymbol{S}_{i_n}$. Pack the location information S_i and send to its neighbor.

The UKF-DCL algorithm is constituted by Algorithms 1 and 2, where the progress is given from the perspectives of the sender and receiver, respectively. Since the usability of the location information is not confined to a specific object, the sender does not need to know which platform receives its location information. Correspondingly, the receiver can also handle the location information from any platforms. It means that any serial communication topologies where the path from the measuring platform to the measured platform acts as the starting is enough for the UKF-DCL. As shown in Fig.4, for a group of N platforms, theoretically, there are $(N-2)\cdot(N-1)\cdot\cdots\cdot 2\cdot 1$ serial communication topologies suitable for the UKF-DCL. Each topology needs not to be prescribed in advance, and the UKF-DCL is self-adaptive.

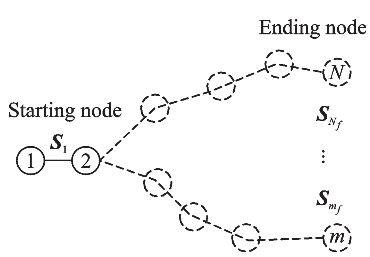

Fig.4 Illustration of all the communication topologies suitable for UKF-DCL

4 Simulation

The Matlab is adopted for the simulation. Assume that four platforms A_1, A_2, A_3, A_4 move in the same area along four ideal circulars. A_1, A_3, A_4 are counterclockwise and A_2 is clockwise. The layout of four platforms is shown as Fig.5, where the initial positions of the platforms are known and marked as "•". The simulation parameters are set as Table 1.

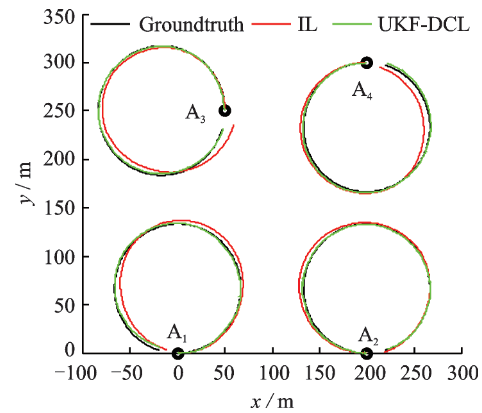

Fig.5 Motioning layout of four platforms

Table 1 Simulation parameter setting

Item	Value
Simulation time / s	400
Sample step / s	0.5
Linear velocity / $(\mathrm{m\cdot s^{-1}})$	1
Measurement noise of linear velocity / $(\mathrm{m\cdot s^{-1}})^2$	$N(0,(0.5)^2)$
Rotational velocity / $(\mathrm{rad\cdot s^{-1}})$	0.015
Measurement noise of rotational velocity / $(\mathrm{rad\cdot s^{-1}})^2$	$N(0,(10^{-3})^2)$
Measurement noise of relative distance/ m^2	$N(0,1)$

In addition, the simulation scheme is designed as follows: (1) At each moment, only one relative distance measurement probably occurs among the platforms and the occurrence probability is set as $p=0.5$. (2) The setting of communication topology is that the relative distances among the platforms are changing. Assume the platform can only communicate with its closest neighbor (it can be calculated by the distance formula). The communication topology is dynamically generated.

Three aspects are verified by the simulation, that is, whether the UKF-DCL has the basic cooperative capacity, whether the proposed UKF-DCL is equal to the centralized one, and whether the UKF-DCL is self-adaptive to the time-varying communication topology.

The results of the estimated trajectories under different algorithms are contrasted in Fig. 5, where the black line represents the actual trajectory, the red one represents the estimated trajectory from the independent localization (IL), and the green one represents the estimated trajectory from the UKF-DCL algorithm. It can be found that the estimated trajectories based on UKF-DCL are much closer to the actual trajectories than IL. It indicates that the proposed UKF-DCL has the cooperative capacity, that is, the local relative measurement improves the whole localization accuracy.

In Fig. 6, under IL, UKF-DCL and UKF-CCL, the distance errors between the estimated position $(\hat{x}_i, \hat{y}_i)$ and the actual position (x_i, y_i), i. e., $\mathrm{Err\_}d = \sqrt{(\hat{x}_i(k) - x_i(k))^2 + (\hat{x}_i(k) - y_i(k))^2}$, are compared for the four platforms. The error curves of UKF-DCL and UKF-CCL are completely coincident, which indicates that the proposed UKF-DCL has the same localization performance with UKF-CCL (the gold-standard benchmark).

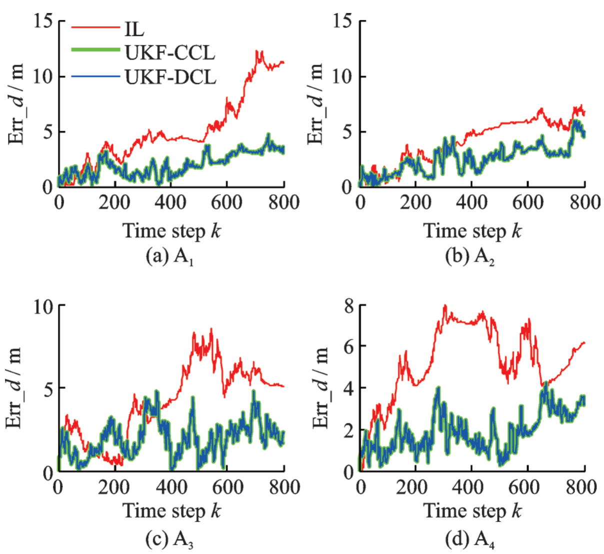

Fig. 6 Comparison of distance errors of four platforms under different algorithms

In Fig. 7, with different probabilities of relative measurement occurring, the average distance error is contrasted. The overall positioning accuracy is improved with the increasing number of the relative measurements.

In Fig. 8, the communication topologies (here,

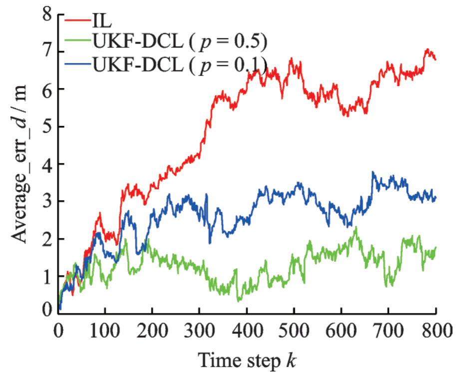

Fig. 7 Comparison of the average of distance errors about four platforms with different measurement probabilities

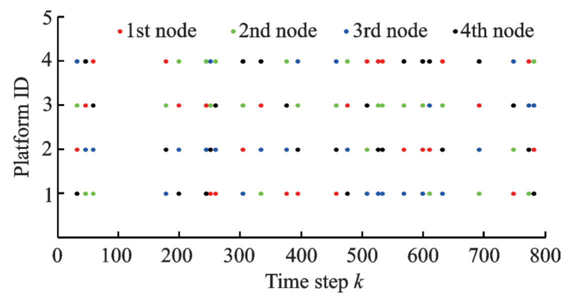

Fig. 8 Illustration of one time-varying communication topology suitable for UKF-DCL

determined by the distance) at each moment is shown, where the probability of the relative measurements occurring is set as $p = 0.05$. The communication topology among the platforms is not fixed. The proposed UKF-DCL algorithm can self-adapt to the dynamic communication topology.

5 Conclusions

Based on the UKF framework, how to implement DCL oriented to the unpredicted communication topology is studied. The character of the look-up Cholesky decomposition is exploited for the covariance matrix decomposing, which is key for the distributed U transformation. By this method, the distributed U transformation can be completed in each platform successively according to the available communication topology. By the proposed method, the requirement of the single platform on the global information is avoided. Each platform can update its position estimate and the covariance item related to itself independently. Hence, the demand on the communication path is reduced. The proposed algorithm is adaptive to any serial communication topologies where the path from the measuring platform to the measured platform acts as the starting path. So the adaptability to the bad communication environment is improved.

It should be noted that the derivation is premised on the scene where only one relative measurement occurs at a certain moment. For the case where multiple relative measurements happen simultaneously, it should be further investigated.

References

[1] ACEVEDO J J, ARRUE B C, MAZA I, et al. Cooperative large area surveillance with a team of aerial mobile robots for long endurance missions[J]. Journal of Intelligent & Robotic Systems, 2013, 70(70): 329-345.

[2] MAZA I, CABALLERO F, CAPTIAN J, et al. Firemen monitoring with multiple UAVs for search and rescue missions[C]//International Workshop on Safety Security and Rescue Robotics. Bremen: IEEE, 2010: 1-6.

[3] BAROOAH P, HESPANHA J P. Estimation on graphs from relative measurements[J]. Control Systems, 2007, 27(4): 57-74.

[4] FOX D, BURGARD W, KRUPPA H, et al. A probabilistic approach to collaborative multi-robot localization[J]. Autonomous robots, 2000, 8(3): 325-344.

[5] WAN J, ZHONG L, ZHANG F. Cooperative localization of multi-UAVs via dynamic nonparametric belief propagation under GPS signal loss condition[J]. International Journal of Distributed Sensor Networks, 2014(3): 83-105.

[6] WANG X G. The application of nonlinear filtering to UAV relative navigation[D]. Harbin: Harbin Institute of Technology, 2010. (in Chinese)

[7] ROUMELIOTIS S I, BEKEY G A. Distributed multi-robot localization[J]. IEEE Transactions on Robotics and Automation, 2002, 18(5): 781-795.

[8] WANG L G. Research on distributed cooperative localization for multi-mobile platforms under communication constraints[D]. Beijing: Tsinghua University, 2017. (in Chinese)

[9] KIA S S, ROUNDS S F, MARTINEZ S. A centralized-equivalent decentralized implementation of Extended Kalman Filters for cooperative localization [C]//2014 International Conference on Intelligent Robots and Systems, Chicago, IL: IEEE, 2014: 3761-3766.

[10] WANG L G, ZHANG T, GAO F F. Distributed cooperative localization with lower communication path requirements[J]. Robotics & Autonomous Systems, 2016, 79: 26-39.

[11] MU H. Decentralized algorithm of cooperative navigation for mobile platform[D]. Changsha: National University of Defense Technology, 2010. (in Chinese)

[12] HAN C Z, ZHU H Y, DUAN Z S. Multi-source information fusion[M]. 2nd Edition. Beijing: Tsinghua University Press, 2006. (in Chinese)

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Competing interests The authors declare no competing interests.

(Executive Editor: Zhang Tong)