Dimensional Variation Modeling of Aircraft Compliant Part Assembly Considering Clamping Force Change

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(Received 18 September 2017; revised 21 March 2018; accepted 4 April 2019)

Abstract: Compliant parts are widely applied to aircraft structures. Due to the ease of deformation of compliant parts in assembly, the prediction of assembly variation is especially important for assembly quality control. A dimensional variation model considering the clamping force change in assembly is proposed based on the method of influence coefficient (MIC). First, the assembly process is decomposed into several steps including positioning, clamping, joining, and spring-back. Then, the force-displacement relationship is formulated according to the varied force conditions on the parts in each assembly step. Finally, two examples are illustrated to validate the proposed assembly variation model. The results show the impact of clamping force change is significant on the assembly variation, and the proposed model can predict the assembly variation more accurately than the referred method without clamping force correction at the over-constrained locating points of fixture.

Key words: aircraft; dimensional variation; compliant parts; clamping force change; method of influence coefficient (MIC)

CLC number: V262.4 **Document code:** A **Article ID:** 1005-1120(2019)02-0298-08

0 Introduction

To achieve the high structural efficiency of aircraft, thin-walled compliant parts including sheet metals are widely applied to aircraft manufacturing. For certain type of aircraft, the sheet metals can even account for over 50% of the total amount of aircraft structural parts[1]. In compliant part assembly, two or more parts are fixtured, and then joined together[2]. Due to the existence of part and fixture errors, the misalignment between the mating parts before joining will be inevitable. To reduce the dimensional variation of assembly, over-constraint fixturing scheme is usually employed in aircraft assembly. By this scheme, clamping forces will be exerted to eliminate the shape misalignment of low-stiffness parts by introducing extra locators and clamps of fixture before joining. However, this forced deformation of parts will introduce adverse residual stress in assembly system, which may produce dimensional variation of assembly after the clamping release of fixture.

According to the experimental data of flexible sheet metal assembly, Takezawa[3] figured out the stack-up- tolerance analysis with the underlying assumption of rigid body assembly is inapplicable to compliant sheet metals. Liu and Hu[4] proposed a one-dimensional offset beam element model for predicting the assembly variation of compliant sheet metals. The impacts of part errors, tooling errors and assembly sequences on assembly variation were evaluated using the developed model. Further, Liu and Hu[5] developed the method of influence coefficient (MIC) to analyze the assembly variation. A typical assembly process is decomposed into four steps, as shown in Fig.1: (1) Part loading and locating; (2) part holding; (3) part joining; (4) part

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How to cite this article: TAN Changbai, ZHANG Wei, WANG Zhiguo. Dimensional Variation Modeling of Aircraft Compliant Part Assembly Considering Clamping Force Change[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2019,36(2):298-305.

http://dx.doi.org/10.16356/j.1005-1120.2019.02.012

unloading from the fixture, part spring-back.

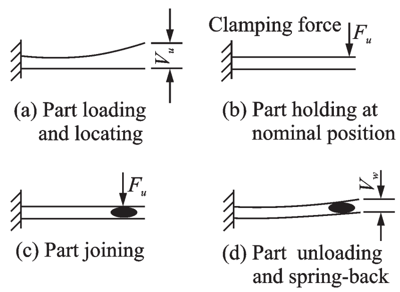

Fig.1 Compliant part assembly

And then, the force-displacement relationship is formulated for each assembly step by the sensitivity analysis incorporating FEA. Thus, assembly variation can be approximately described as a linear function of part errors by concatenating the force-displacement equations of these assembly steps. MIC produced a profound impact in this study, since the proposed analytic model is high efficient compared with the traditional FEA method for assembly variation analysis. Based on MIC, Camelio et al.[6] developed a method for the prediction of sheet metal assembly variation using the components of geometric covariance. Yu et al.[7] investigated the impact of material errors of parts on the assembly variation using MIC, and developed the compliant assembly variation model considering both geometric and material errors of parts. Long[8] studied the impact of tool (fixture and welding gun) errors and developed an assembly variation model incorporating the part error and tool error using MIC. Hu et al.[9] introduced the variation simulation models for compliant assembly and the application of these models in robust design and adaptive control of assembly quality. Tian et al.[10] investigated the propagation, transformation and accumulation of part variations in multi-station assembly with state space model. Dahlström et al.[11] proposed an assembly variation model considering the contact of sheet metal in assembly. Xing et al.[12] defined sheet metal assembly as six steps: (1) "3-2-1" part positioning onto fixture; (2) applying extra clamps on parts; (3) part assembly; (4) welding gun releasing; (5) clamp releasing; (6) fixture locator releasing. They proposed an assembly variation model considering the different patterns of fixture releasing. Tan et al.[13] proposed an approach of geometric covariance modeling based on hybrid polynomial approximation and spectrum analysis, which could obtain the surface variation of compliant parts and its correlation. Chen et al.[14] proposed a method of statistical variation analysis for compliant part assembly coupling the geometric and material errors based on the perturbation theory and the finite element method. Zhang et al.[15] proposed a new method for assembly variation optimization of aircraft compliant parts based on a concept of active fixture locating compensation.

In aircraft assembly, compliant parts (e. g., wing and fuselage panel) are usually located by over-constrained fixturing scheme for the purpose of reducing assembly variation. The clamping forces at over-constrained locating points will vary in different assembly steps of part loading, clamping, joining and unloading. The existing methods have little consideration of this force change in the force-displacement analysis of assembly variation, which may have significant effect on the assembly variation. Based on MIC, this study aims to develop an improved model for dimensional variation analysis in compliant part assembly considering the clamping force change of fixture at over-constrained points. This model can provide more reliable and accurate prediction for the dimensional variation in aircraft assembly.

1 Variation Modeling of Compliant Part Assembly

To formulate the assembly variation, a simple assembly model including two sheet metals will be considered. First, the method proposed by Liu and Hu[5] is used to decompose the riveting process of aircraft assembly into four steps: Part locating, part clamping; joining; releasing and spring-back. Due to the existence of part errors and tool errors in assembly system, residual stress will be introduced after the parts are clamped to the nominal position before joining. Consequently, the spring-back and the dimensional variation of assembly will be introduced after the constraints of fixture are released. Subsequently, the key measurement points will be selected to quantify the dimensional variation of assembly. In this parper, the joining points and the key product characteristic (KPC) points of parts will be defined as the key measurement points, because these points have important effect on the assembly quality. Finally, the linear mechanical equation depicting the force-displacement relationship of each assembly step is formulated through the sensitivity analysis of MIC.

1.1 Part locating

The over-constrained locating of aircraft riveting can be divided into two sub-steps. First, the deterministic locating will be applied to each part to constrain the DOFs of rigid motion (i.e., "3-2-1" locating scheme). Second, additional locating will be applied to the assembly to eliminate the off-plane deviation of compliant parts at the locating points (i.e., "N-2-1" locating scheme).

(1) "3-2-1" locating scheme

Only the rigid motion of part is concerned in the "3-2-1" locating. The geometric error of part in the assembly system caused in this sub-step can be formulated by using the deterministic analysis of the locating errors. Generally, we assume the locating errors by the "3-2-1" locating at the over-constrained points and joining points are denoted as $\boldsymbol{V}_L^A$, $\boldsymbol{V}_L^B$ and $\boldsymbol{V}_J^A$, $\boldsymbol{V}_J^B$. The locating errors at KPC points are $\boldsymbol{V}_K^A$, $\boldsymbol{V}_K^B$ on Part A and Part B, respectively. As shown in Fig.2, Part A and Part B are located by "3-2-1" locating scheme.

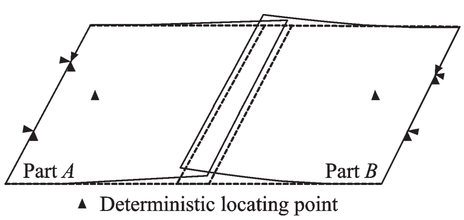

Fig.2 "3-2-1" locating of parts

(2) "N-2-1" locating scheme

The geometric errors at the over-constrained locating points of fixture are supposed to be $\boldsymbol{V}_F^A$ and $\boldsymbol{V}_F^B$ for Part A and Part B, respectively. Because of the clamping force enforced by fixture, the misalignment between the locator and the part will be eliminated by the deformation of compliant part. According to FEM, the force-displacement relationship can be depicted as follows

$$[\boldsymbol{F}_{L1}^A, 0, 0]^{\mathrm{T}} = \boldsymbol{K}_A[\boldsymbol{V}_F^A - \boldsymbol{V}_L^A, \boldsymbol{V}_{J1}^A, \boldsymbol{V}_{K1}^A]^{\mathrm{T}} \tag{1}$$

$$[\boldsymbol{F}_{L1}^B, 0, 0]^{\mathrm{T}} = \boldsymbol{K}_B[\boldsymbol{V}_F^B - \boldsymbol{V}_L^B, \boldsymbol{V}_{J1}^B, \boldsymbol{V}_{K1}^B]^{\mathrm{T}} \tag{2}$$

where $\boldsymbol{F}_{L1}^A$ and $\boldsymbol{F}_{L1}^B$ are the forces at the over-constrained locating points of Part A and Part B, respectively; $\boldsymbol{K}_A$ and $\boldsymbol{K}_B$ the stiffness matrices achieved from the FEA model under the boundary conditions of "3-2-1" positioning where the over-constrained locating points, joining points and KPC points are concerned; $\boldsymbol{V}_{J1}^A$ and $\boldsymbol{V}_{J1}^B$ the displacement at joining points, while $\boldsymbol{V}_{K1}^A$ and $\boldsymbol{V}_{K1}^B$ the displacement at KPC points of Part A and Part B, respectively. Fig.3 illustrates the "N-2-1" locating of Part A and Part B, where $\boldsymbol{F}_{L1}^A$ and $\boldsymbol{F}_{L1}^B$ represent the vectors of $[\boldsymbol{F}_{L11}^A, \boldsymbol{F}_{L12}^A, \boldsymbol{F}_{L13}^A]^{\mathrm{T}}$ and $[\boldsymbol{F}_{L11}^B, \boldsymbol{F}_{L12}^B, \boldsymbol{F}_{L13}^B]^{\mathrm{T}}$, respectively. The locating errors of Part A are illustrated in Fig. 4.

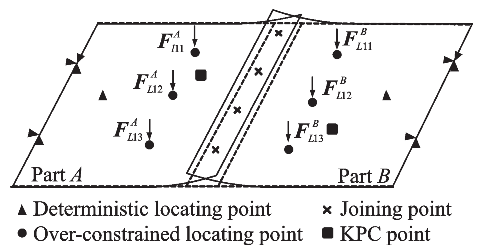

Fig.3 "N-2-1" locating of parts

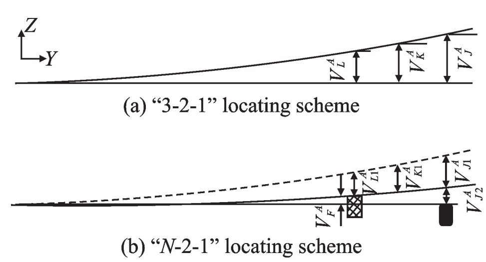

Fig.4 Locating errors of Part A

1.2 Part clamping

The parts will deform subject to the clamping forces, and the joining points will move to their nominal positions. Moreover, the forces on the over-constrained points will change to achieve the new equilibrium. Considering the deformation and the force change, the force-displacement equations are represented as follows

$$[\Delta F_{L1}^{A}, F_{C}^{A}, 0]^{\mathrm{T}} = K_{A}[0, -V_{J2}^{A}, V_{K2}^{A}]^{\mathrm{T}} \tag{3}$$

$$[\Delta F_{L1}^{B}, F_{C}^{B}, 0]^{\mathrm{T}} = K_{B}[0, -V_{J2}^{B}, V_{K2}^{B}]^{\mathrm{T}} \tag{4}$$

where F_C^A and F_C^B are the clamping forces at the joining points of Part A and Part B, respectively. As shown in Fig. 5, clamping forces are loaded before joining, where F_C^A represents the vector of $[F_{C1}^A, F_{C2}^A, F_{C3}^A, F_{C4}^A]^{\mathrm{T}}$. ΔF_{L1}^A and ΔF_{L1}^B are the force variations at the over-constrained locating points of Part A and Part B, respectively; V_{J2}^A and V_{J2}^B the position errors of joining points from their nominal positions after the "N-2-1" locating; V_{K2}^A and V_{K2}^B the displacement at KPC points of Part A and Part B, respectively. Thus, the clamping forces at the over-constrained points will become

$$F_{L2}^{A} = F_{L1}^{A} + \Delta F_{L1}^{A} \tag{5}$$

$$F_{L2}^{B} = F_{L1}^{B} + \Delta F_{L1}^{B} \tag{6}$$

The clamping forces at the joining points will become

$$F_{C} = F_{C}^{A} + F_{C}^{B} \tag{7}$$

The initial deviation before spring-back at KPC points of Part A and Part B are

$$U_{K0}^{A} = V_{K}^{A} - V_{K1}^{A} - V_{K2}^{A} \tag{8}$$

$$U_{K0}^{B} = V_{K}^{B} - V_{K1}^{B} - V_{K2}^{B} \tag{9}$$

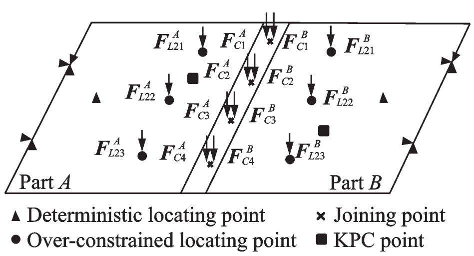

Fig. 5 Applied clamping force

1.3 Joining

The joining points of two parts will be kept in their nominal positions by the clamping force, and then they will be joined by riveting. Technically, the distortion of assembly by the riveting will be ignored in this paper, because its impact on the dimensional variation is localized.

1.4 Releasing and spring-back

(1) Clamping release at the joining points

The deformation of parts will introduce the residual stress into assembly system. Consequently, spring-back will happen to reach a new equilibrium once the clamping at the joining points is removed. According to the conclusion by Liu and Hu[4], resilience force is approximately equal to the reaction force of clamping force. Based on the assumption of linearly elastic and small deformation of materials, the relationship between spring-back displacement and resilience force can be represented as

$$[\Delta F_{L2}^{A}, \Delta F_{L2}^{B}, F_{RC}, 0, 0]^{\mathrm{T}} = K_{R}[0, 0, U_{J1}, U_{K1}^{A}, U_{K1}^{B}]^{\mathrm{T}} \tag{10}$$

where ΔF_{L2}^A and ΔF_{L2}^B are the force variations at the over-constrained points after clamping is released at the joining points; U_{J1} is the spring-back displacement at the joining points; U_{K1}^A and U_{K1}^B are the spring-back displacement at the KPC points of Part A and Part B; K_R is the stiffness matrix of assembly from the FEA model established under the boundary condition of Part A and Part B being "3-2-1" located where the over-constrained locating points, joining points and KPC points are concerned; and F_{RC} the vector of the resilience forces $[F_{RC1}, F_{RC2}, F_{RC3}, F_{RC4}]^{\mathrm{T}}$ at joining points, as shown in Fig.6. Then the forces at the over-constrained points will be

$$F_{L3}^{A} = F_{L2}^{A} + \Delta F_{L2}^{A} \tag{11}$$

$$F_{L3}^{B} = F_{L2}^{B} + \Delta F_{L2}^{B} \tag{12}$$

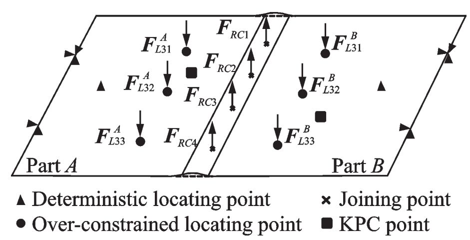

Fig.6 Clamping release at the joining points

(2) Clamping release at the over-constrained points

Releasing the over-constrained locating will further cause spring-back of assembly as shown in Fig.7. The relationship between the resilience force and the displacement can be formulated as

$$[F_{RL}^{A}, F_{RL}^{B}, 0, 0, 0]^{\mathrm{T}} = K_{R}[U_{LR}^{A}, U_{LR}^{B}, U_{J2}, U_{K2}^{A}, U_{K2}^{B}]^{\mathrm{T}} \tag{13}$$

where U_{LR}^A and U_{LR}^B are the assembly variations of

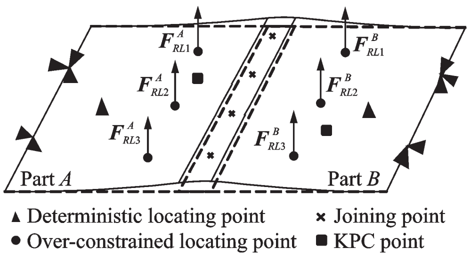

Fig.7 Clamping release at the over-constrained points

Part A and Part B at the over-constrained locating points after spring-back; $\boldsymbol{U}_{J2}$ is the spring-back displacement of joining points; $\boldsymbol{U}_{K2}^{A}$ and $\boldsymbol{U}_{K2}^{B}$ are the spring-back displacement at KPC points of Part A and Part B, respectively; $\boldsymbol{F}_{RL}^{A}$ and $\boldsymbol{F}_{RL}^{B}$ the resilience forces of Part A and Part B at the over-constrained points, respectively, which have the same magnitude with but are in the opposite directions of $\boldsymbol{F}_{L3}^{A}$ and $\boldsymbol{F}_{L3}^{B}$, respectively.

Thus, after the clamping release of riveter and the over-constrained locating of fixture, the spring-back of joining points and KPC points are

$$\boldsymbol{U}_{J}=\boldsymbol{U}_{J1}+\boldsymbol{U}_{J2} \tag{14}$$

$$\boldsymbol{U}_{K}^{A}=\boldsymbol{U}_{K0}^{A}+\boldsymbol{U}_{K1}^{A}+\boldsymbol{U}_{K2}^{A} \tag{15}$$

$$\boldsymbol{U}_{K}^{B}=\boldsymbol{U}_{K0}^{B}+\boldsymbol{U}_{K1}^{B}+\boldsymbol{U}_{K2}^{B} \tag{16}$$

where $\boldsymbol{U}_{K}^{A}$ and $\boldsymbol{U}_{K}^{B}$ are the position errors at KPC points of Part A and Part B. Thus, the assembly variation is

$$\boldsymbol{U}=[\boldsymbol{U}_{J},\boldsymbol{U}_{K}^{A},\boldsymbol{U}_{K}^{B}]^{\mathrm{T}} \tag{17}$$

2 Case Study

A sheet metal assembly and an aircraft fuselage panel assembly are used to validate the proposed assembly variation model. The proposed method considering clamping force change will be compared with the referred method without clamping force correction[5]. Both methods will be compared with the method of direct FEA simulation in terms of assembly variations.

2.1 Sheet metal assembly

Two rectangular steel sheets are supposed to have the same size (600 mm × 420 mm × 3 mm) and material parameters (Young's modulus $E=210$ GPa, Poisson's ratio $\upsilon=0.3$). The FEA model of this assembly is shown in Fig.8. The shell element type "S4R" is used to mesh each part, and the boundary condition is listed in Table 1. With the "3-2-1" locating, the initial locating errors of 1.5 mm and 2.5 mm exist for Part A at the over-constrained locating points (P_{11}, P_{12} and P_{13}) and for Part B at the over-constrained locating points (P_{21}, P_{22} and P_{23}), respectively. The initial misalignment errors of 4 mm and 7 mm exist for Part A and Part B, respectively.

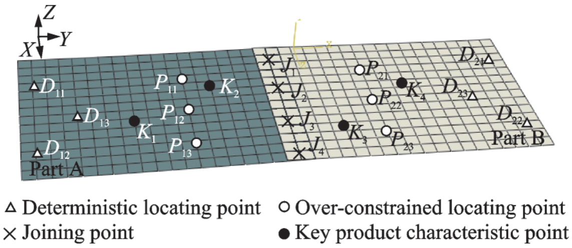

Fig.8 FEA model of sheet metal assembly

Table 1 Boundary condition of sheet metal

Restricting direction	Node
$X/Y/Z$	D_{11}, D_{21}
Y/Z	D_{12}, D_{22}
Z	D_{13}, P_{11}, P_{12}, P_{13} D_{23}, P_{21}, P_{22}, P_{23}

Based on this model, the super element stiffness matrices for each part and assembly can be achieved. Further, the assembly variations are calculated using our method and the referred method, respectively. A direct FEA simulation is also conducted to calculate the assembly variation which acts as the benchmark for two methods. All the results are listed in Table 2.

Table 2 Comparison of the two methods for sheet metal assembly

Case 1	Variation at joining point				Variation at KPC point				Mean absolute error
	J_1	J_2	J_3	J_4	K_1	K_2	K_3	K_4	
Abaqus simulation / mm	0.627	0.558	0.558	0.627	0.259	0.479	0.283	0.026	
Method 1 (proposed) / mm	0.630	0.560	0.560	0.630	0.259	0.480	0.283	0.026	0.001
Method 2 (referred) / mm	1.339	1.262	1.262	1.339	0.555	0.983	1.106	0.667	0.637

The assembly variation by the proposed method has a mean absolute error of 0.001 mm, compared with a mean absolute error of 0.637 mm by the referred method. Obviously, the proposed method is more accurate in the prediction of dimensional variation of sheet metal assembly.

2.2 Fuselage panel assembly

Fig.9 shows a typical 3D model of fuselage panel including skin, stringer, and clip. Due to the poor stiffness along the normal direction of skin, fuselage panel is easy to deform in this direction. To control the assembly variation, an assembly fixture is used to hold and locate the parts. Specially, the fixture board provides over-constrained locating of skin, stringer and clip along the normal direction of skin. In this example, the assembly variation along the normal direction of skin is analyzed.

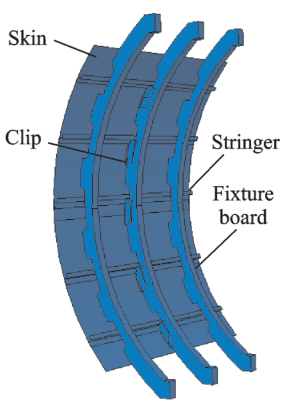

Fig.9 A typical 3D model of fuselage panel

To simplify the FEA model and assembly simulation, without loss of generality, we build a simplified panel structure including three fixture boards, two skins, five stringers and five clips. The radius and width of Skin A are 1 990 mm and 2 000 mm, respectively, and the radius and the width of Skin B are 2 000 mm and 2 000 mm, respectively. All the materials of skins, stringers and clips are supposed to be Al-Li alloy (Young's modulus $E=$ 73 GPa, Poisson's ratio $\upsilon=0.3$). The thickness of skin and clip are 2 mm while the thickness of stringer is 1.6 mm. To highlight the research focus, the locating errors of fixture will be neglected here.

The FEA model of assembly is shown in Fig.10. The shell element type "S4R" is used to mesh each part, and the boundary condition is listed in Table 3. "N" is the normal direction of skin at corresponding points. The initial locating errors are 1.5 mm at points P_{11}, P_{12}, P_{13} and 2.5 mm at points P_{14}, P_{15}, P_{16} for Skin A, respectively. The initial locating errors are 2.5 mm at points P_{21}, P_{22}, P_{23} and 4 mm at points P_{24}, P_{25}, P_{26} for Skin B, respectively. The initial alignment errors of 5 mm and 8.5 mm exist at points J_1, J_2, J_3 for Skin A and Skin B, respectively.

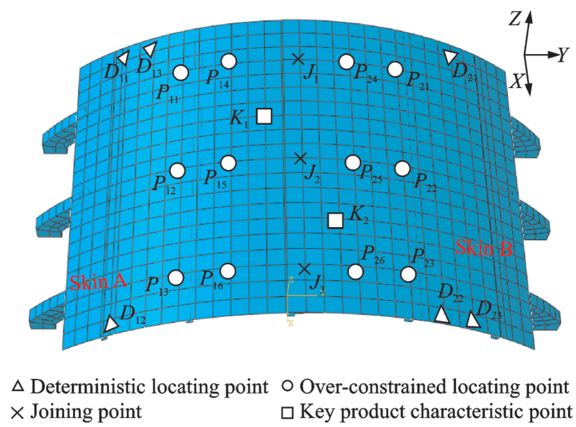

Fig.10 FEA model of fuselage panel assembly

Table 3 Boundary condition of fuselage panel

Restricting direction	Node
$X/Y/Z$	D_{11}, D_{21}
Y/Z	D_{12}, D_{22}
Z	D_{13}, D_{23}
N	P_{11}, P_{12}, P_{13}, P_{14}, P_{15}, $P_{16}P_{21}$, P_{22}, P_{23}, P_{24}, P_{25}, P_{26}

The normal dimensional variation of panel assembly by the proposed method, the referred method, and the direct FEA simulation are listed in Table 4.

It is observed the variation values at the joining points and the KPC points by the proposed method are much closer to the values by the simulation, compared with the referred method. The proposed method has a mean absolute error of 0.061 mm as well, compared with 1.202 mm by the referred method. From the results, it can be concluded that the clamping force change in the assembly has a significant impact on the assembly variation, and the

Table 4 Comparison of the two methods for fuselage panel assembly

Case 2	Variation at joining point			Variation at KPC point		Mean absolute error
	J_1	J_2	J_3	K_1	K_2	
Abaqus simulation / mm	0.956	0.898	0.778	0.906	1.251	
Method 1 (proposed) / mm	0.892	0.893	0.750	1.025	1.164	0.061
Method 2 (referred) / mm	2.643	2.443	2.493	1.383	1.836	1.202

proposed method considering clamping force change is more accurate than the referred method.

3 Conclusions

The aircraft manufacturing has extremely strict requirements on the dimensional quality. It is helpful to develop a method for assembly variation prediction. Based on MIC, we take an insight into force-displacement relationship of compliant assembly through the breakdown of the riveting assembly process. Further, an improved assembly variation model is proposed considering the clamping force change of fixture after the riveter is released. Case study is conducted to validate the proposed method. Through a comparative analysis of assembly variation with the referred method without clamping force correction and the direct FEA simulation, two conclusions are drawn as follows:

(1) The clamping force has a significant impact on the dimensional variation analysis of aircraft compliant assembly using MIC.

(2) By formulating the force change at fixture locating points, this proposed method can predict the assembly variation more accurately than the traditional MIC ignoring the clamping force change.

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Acknowledgement This work was funded by the National Natural Science Foundation of China (No.51275236).

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Author contributions Mr. **ZHANG Wei** completed the experiments, conducted the analysis, and wrote the manuscript. Dr. **TAN Changbai** designed the study and guided the experiments. Dr. **WANG Zhiguo** contributed to the discussion and background for the study.

Competing interests The authors declare no competing interests.

(Production Editor: Xu Chengting)