Operational Mode Identification Based on Sliding Time Window Method and Eigensystem Realization Algorithm

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(Received 7 July 2018; revised 18 January 2019; accepted 2 July 2019)

Abstract: The identification result of operational mode is eurychoric while operational mode identification is investigated under ambient excitation, which is influenced by the signal size and the time interval. The operational mode identification method, which is based on the sliding time window method and the eigensystem realization algorithm (ERA), is investigated to improve the identification accuracy and stability. Firstly, the theory of the ERA method is introduced. Secondly, the strategy for decomposition and implementation is put forward, including the sliding time window method and the filtration method of modes. At last, an example is studied, where the model of a cantilever beam is built and the white noise exciting is input. Results show that the operational mode identification method can realize the modes, and has high robustness to the signal to noise ratio and signal size.

Key words: mode identification; robust; eigensystem realization algorithm (ERA); operational mode; damping ratio

CLC number: O32 **Document code**: A **Article ID**: 1005-1120(2019)05-0838-07

0 Introduction

In the space engineering, the precise dynamic characteristic of the rocket is indicated for the design of dynamic load, guidance, navigation and control where the theory calculation and the ground experiment are taken to get the modal result. But the difference between the designed result with the real one when flying is not clear, which is unfavorable for the optimization and improvement. And measuring the input forces is very difficult for the operating structures. Therefore, the system mode identification only relying on response data has attracted considerable attention in recent years.

In the conventional modal analysis, impulse response functions calculated by inverse Fourier transforms of frequency response functions (FRFs) have been widely used with time-domain modal identification algorithms[1-2]. This approach is called a forced response technique since FRF is determined by using both input and output measurements from the forced response testing of structures. Then special modal identification methods are employed to estimate modal parameters only from output data, such as peak-picking from power spectral density (PSD) functions[3], autoregressive moving average (ARMA) models[4-5], stochastic subspace methods[6-7] based on random decrement processing with the Ibrahim time domain (ITD) technique[8-9], maximum entropy method (MEM)[10], least square curve fitting technique[11] and etc. In this paper, the expressions of cross-correlation functions between measured responses are derived under white noise excitations.

Recently, the operational mode identification method is used in the rocket design. Theodore, et al[12] performed the operational modal analysis on Ares I-X in-flight data. Since the dynamic system is not stationary due to propellant mass loss, the modal identification is only possible by analyzing the system as a series of linearized models over short periods of time via a sliding time-window of short time

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How to cite this article: WANG Liang, ZHANG Yan, CAI Yipeng, et al. Operational Mode Identification Based on Sliding Time Window Method and Eigensystem Realization Algorithm[J]. Transactions of Nanjing University of Aeronautics and Astronautics, 2019,36(5):838-844.

http://dx.doi.org/10.16356/j.1005-1120.2019.05.015

intervals. A time-domain zooming technique was also employed to enhance the modal parameter extraction. Results of this study demonstrated that free-decay time domain modal identification methods can be successfully applied to in-flight launch vehicle modal extraction. The dynamic properties of a solid rocket motor using data recorded during a ring test were identified by Coppotelli, et al[13]. The dynamic identification of the motor was provided by applying the operational modal analysis (OMA) methodologies to estimate the modal parameters of the structure undergoing its operative conditions. The sensitivity of the OMA approaches to deal with structures characterized by time-dependent parameters was evaluated through a numerical simulation. Moreover, a comparison between the estimates of different state-of-the-art approaches in OMA (operating in both time domain and frequency domain) was provided. James, et al[14] focused on recent efforts to utilize spacecraft flight data for extracting system parameters, with a special interest on modal damping. Their work utilized the analysis of correlation functions derived from a sliding window technique applied to the time record. Four different case studies were reported in the sequence that drove the authors' understanding. The insights derived from these four exercises were preliminary conclusions for the general state-of-the-art, but may be of specific utility to similar problems approached with similar tools. Coppotelli, et al[15] demonstrated the capability of the developed operational-modal-analysis methods to identify the dynamic properties of the solid rocket motor under its actual operative conditions, by using response data recorded during the firing test. The main objective was first to prove the applicability and then to evaluate the overall efficiency of different state-of-the-art approaches in operational modal analysis, so as to track changes in the natural frequencies, modal damping ratios, and mode shapes of the first stage of the Vega launch vehicle undergoing significant mass variation due to the burning propeller. Additionally, a sensitivity of the considered approaches to deal with structures characterized by time-dependent parameters was numerically carried out.

The identification results of the mode are eurychoric while the operational mode identification is investigated under ambient excitation, which is unstable with the selected signal. To solve the problem of the time-invariant systems, a method which combines with the sliding time window method, statistical method and eigenvalue realization algorithm (ERA) is put forward to improve the precision and stability of the identified result.

1 Modal Identification Method

ERA is a time domain modal identification method which consists of two major parts as basic formulation of the minimum-order realization and modal parameter identification.

For an n-dimension linear system, the vibration equation can be expressed as

$$\begin{cases}\dot{y}(t)=\boldsymbol{A}y(t)+\boldsymbol{B}f(t)\\ z(t)=\boldsymbol{G}\dot{y}(t)\end{cases}\tag{1}$$

After the response is discrete by sampling time Δt, and the time $t=t_0+k\Delta t$, the response can be obtained from

$$\dot{y}[(k+1)\Delta t]=\boldsymbol{A}\boldsymbol{e}^{A\Delta t}y(k\Delta t)+ \boldsymbol{A}\int_0^T \boldsymbol{e}^{As}\mathrm{d}s\boldsymbol{B}f(k\Delta t)+\boldsymbol{B}f[(k+1)\Delta t]\tag{2}$$

The transfer function of the z transform is

$$\boldsymbol{H}(z)=\sum_{k=0}^{\infty}\boldsymbol{h}(k)z^{-k}\tag{3}$$

Eq. (1) is transformed as

$$\boldsymbol{H}(z)=z^{-2}\boldsymbol{G}\boldsymbol{A}_2(\boldsymbol{I}-z^{-1}\boldsymbol{A}_1)^{-1}\boldsymbol{B}_1+ z^{-1}\boldsymbol{G}\boldsymbol{B}_2+\boldsymbol{G}\boldsymbol{B}\tag{4}$$

Tidy up as

$$\boldsymbol{h}(0)=\boldsymbol{G}\boldsymbol{B},\boldsymbol{h}(k)=\boldsymbol{G}\boldsymbol{A}\boldsymbol{A}_1^{k-1}\boldsymbol{B}_1\tag{5}$$

Hankel matrix is formed as

$$\boldsymbol{H}(k-1)=\begin{bmatrix}\boldsymbol{h}(k) & \boldsymbol{h}(k+1) & \cdots & \boldsymbol{h}(k+\beta-1)\\ \boldsymbol{h}(k+1) & \boldsymbol{h}(k+2) & \cdots & \boldsymbol{h}(k+\beta)\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{h}(k+\alpha-1) & \boldsymbol{h}(k+\alpha) & \cdots & \boldsymbol{h}(k+\alpha+\beta-2)\end{bmatrix}\tag{6}$$

Tidy up as

$$\boldsymbol{H}(k-1)=\boldsymbol{P}\boldsymbol{A}_1^{k-1}\boldsymbol{Q}\tag{7}$$

where $\boldsymbol{P}=[\boldsymbol{G}\boldsymbol{A}\quad \boldsymbol{G}\boldsymbol{A}\boldsymbol{A}_1\quad\cdots\quad \boldsymbol{G}\boldsymbol{A}\boldsymbol{A}_1^{\alpha-1}]^{\mathrm{T}}$, $\boldsymbol{Q}=[\boldsymbol{B}_1\quad \boldsymbol{A}_1\boldsymbol{B}_1\quad\cdots\quad \boldsymbol{A}_1^{\beta-1}\boldsymbol{B}_1]$, and α,β are the coefficients of controllability and observability, respectively.

Let $k=1$, singular value decomposition of $\boldsymbol{H}(0)$ can be got as

$$\boldsymbol{H}(0)=\boldsymbol{U\Sigma V}^{\mathrm{T}} \tag{8}$$

Then

$$\boldsymbol{h}(k+1)=$$
$$\boldsymbol{E}_M^{\mathrm{T}}\boldsymbol{U\Sigma}^{1/2}(\boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{H}(1)\boldsymbol{\Sigma}^{-1/2})\boldsymbol{\Sigma}^{1/2}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{E}_L \tag{9}$$

where $\boldsymbol{E}_M^{\mathrm{T}}=[\boldsymbol{I}_M,\boldsymbol{0}_M,\cdots,\boldsymbol{0}_M]$, $\boldsymbol{E}_L=[\boldsymbol{I}_L,\boldsymbol{0}_L,\cdots,\boldsymbol{0}_L]^{\mathrm{T}}$.

Let $\boldsymbol{A}_1=\boldsymbol{\Sigma}^{-1/2}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{H}(1)\boldsymbol{\Sigma}^{-1/2}$, $\boldsymbol{B}_1=\boldsymbol{\Sigma}^{1/2}\boldsymbol{V}^{\mathrm{T}}\boldsymbol{E}_L$, and $\boldsymbol{GA}=\boldsymbol{E}_M^{\mathrm{T}}\boldsymbol{U\Sigma}^{1/2}$, then the eigenvalue and eigenvectors of the system matrix $\boldsymbol{A}$ are defined as $\boldsymbol{\Lambda}$ and $\boldsymbol{\psi}'$, respectively. So we have

$$\boldsymbol{\psi}^{-1}\boldsymbol{A\psi}=\boldsymbol{\Lambda} \tag{10}$$

It is well-known that exponential matrix function can be expressed as

$$\boldsymbol{A}_1=\boldsymbol{e}^{\boldsymbol{\psi\Lambda\psi}^{-1}\Delta t}=\boldsymbol{\psi e}^{\boldsymbol{\Lambda}\Delta t}\boldsymbol{\psi}^{-1},\boldsymbol{\psi}^{-1}\boldsymbol{A}_1\boldsymbol{\psi}=\boldsymbol{e}^{\boldsymbol{\Lambda}\Delta t}$$

So the eigenvectors of matrix $\boldsymbol{A}$ is as same as matrix $\boldsymbol{A}_1$ which eigenvalue matrix is

$$\boldsymbol{Z}=\boldsymbol{e}^{\boldsymbol{\Lambda}\Delta t}=\mathrm{diag}(z_1,z_2,\cdots,z_{2n}) \tag{11}$$

where the diagonal elements of matrix $\boldsymbol{Z}$ is $z_i=\boldsymbol{e}^{\lambda_i}\Delta t$, $i=1,2,\cdots,2n$, the eigenvector of matrix is $\boldsymbol{\Lambda}=\mathrm{diag}(\lambda_1,\lambda_2,\cdots,\lambda_{2n})$, and

$$\lambda_i=\frac{1}{\Delta t}\ln z_i \quad i=1,2,\cdots,2n \tag{12}$$

Therefore, the modal frequency, damping ratio and the modal sharp can be obtained as follows

Modal frequency $\omega_i=\frac{1}{2\pi}\sqrt{(\lambda_i\mathrm{Re})^2+(\lambda_i\mathrm{Im})^2}$

Damping ratio $\xi_i=\frac{\mathrm{Re}(\lambda_i)}{w_i}$

Modal sharp $\boldsymbol{\Phi}=\boldsymbol{G\psi}$

2 Method Based on Sliding Time Window Method and ERA

To solve the problem that the modal damping ratio is unstable, a new modal identification method is presented. The method based on sliding time window method and ERA combines the statistic way and average idea, and can improve the stability and veracity of the identification result. Fig.1 shows the strategy of the method. Firstly, the response signal is analyzed by power spectral density (PSD). Then the frequency bandwidth for modal identification can be chosen. Secondly, the signal is filtered by band-pass filter and intercepted by sliding rectangular. Thirdly, the modes of the sliding signal are identified, which include modal frequency, damping ratio and the modal sharp. Fourthly, the mode is filtered by the empirical value of modal damping ratio and the PSD result, where the empirical value of modal damping ratio is limited by the theoretical and experiential values. For example, the damping ratio of the metal is below 10%. At last, the statistical analysis is used to derive the final identification result.

As the sliding rectangular is used, the width of the rectangular is a key parameter. According to the experience, a longer time period is most likely needed to capture at least 6—8 cycles of the mode for mode identification, where the 8-cycle width of the lowest mode is adopted in this paper.

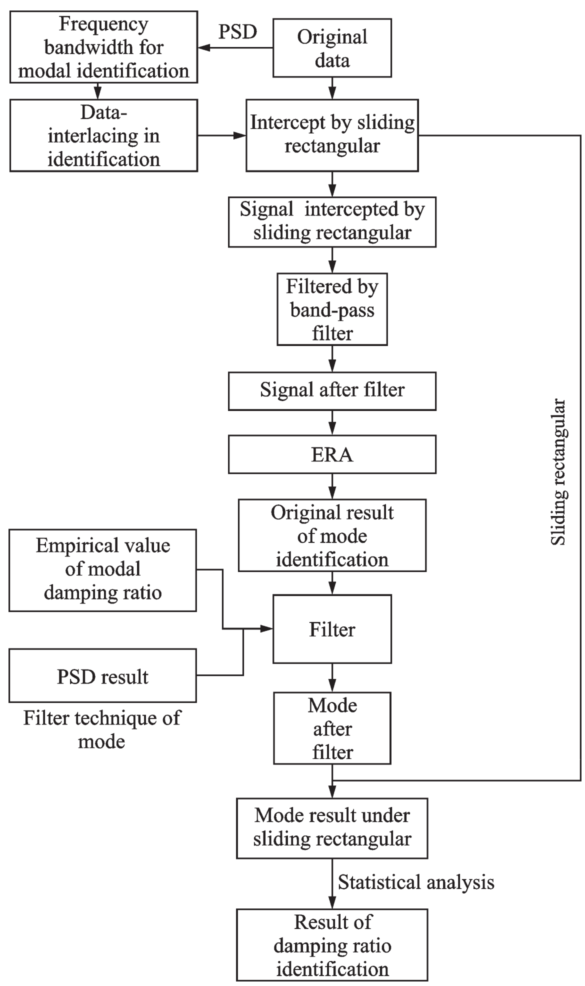

Fig.1 Strategy of the modal identification method

3 Modal Identification Method

3.1 Beam model

The cantilever beam is adopted in the paper for example. Gaussian white noise excitation is used as excitation at the end of the beam, and the acceleration response of the beam is taken for modal identification. Fig.2 shows the beam model, whose parameter is present in Table 1.

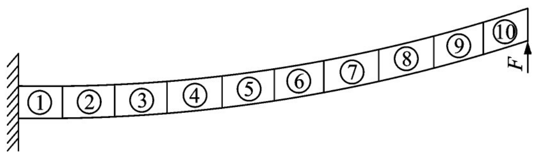

Fig.2 Beam model

Table 1 Beam's parameter

Item	Value
Length / m	1
Section / m×m	0.02(width)×0.025(high)
Density / (kg • m^{-3})	2 700
Elastic modulus / GPa	70
Number of element	10
Theoretical modal damping ratio / %	2

The Gaussian white noise excitation with 800 Hz band width is applied to the end of the beam, which is presented in Fig.3. And the acceleration response of the beam is calculated by the wilson-θ method, and the response of beam's end is shown in Fig. 4, whose PSD curve is shown in Fig.5.

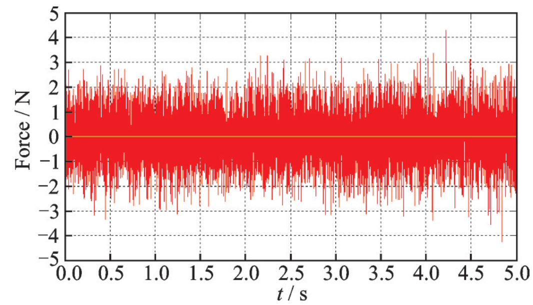

Fig.3 Gaussian white noise excitation

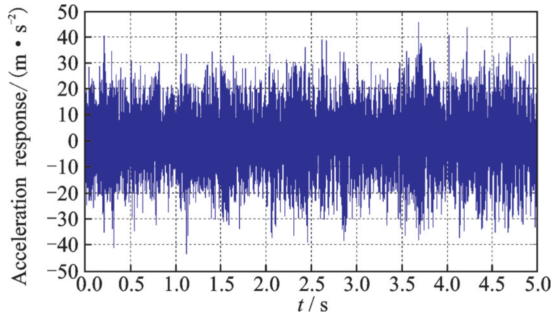

Fig.4 Response of beam's end

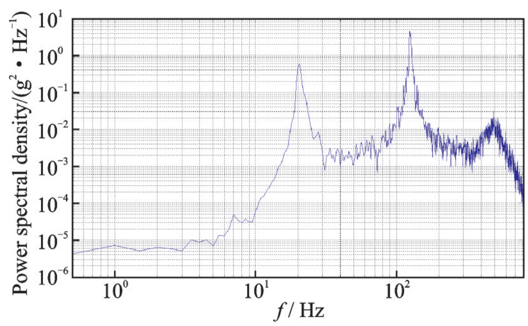

Fig.5 PSD curve of beam's end

3.2 Unstable problem of modal damping identification

The unstable problem of modal damping ratio identification is investigated firstly, which is variational with the selected signal. A selected signal with different length which is changing from 8-cycle to 25-cycle width of the lowest mode is analyzed and the mode is identified. The identification result is shown in Table 2. Form the table, it is derived that the modal frequency identification result of different signal length is the same, but the difference of the modal damping ratio result is obvious, where the biggest value is about four times the smallest one. Therefore, it is very significant to improve the robustness and stability of the damping identification.

Table2 Identification result with different signal lengths

Item	Parameter	Signal length				
		8	10	15	20	25
Identification of 1st order mode	Damping ratio	0.020 49	0.035 43	0.033 01	0.050 99	0.010 50
	Modal frequency / Hz	22.74	22.25	23.25	23.49	23.47
Identification of 2nd order mode	Damping ratio	0.039 62	0.021 94	0.009 90	0.021 14	0.035 07
	Modal frequency / Hz	120.78	122.17	121.39	119.84	121.47

3.3 Influence of data-interlacing on identification

The data-interlacing is a way to improve the utilization rate of the data and stability of the identification. So the influence of data-interlacing width on modal identification is needed to investigate. Table 3 presents the identification results of different data-interlacing width, where the width is changed from 3-cycle width of the lowest mode to 7-cycle one. Figs.6—7 present the identification of first two-order modal shapes, and Fig.8 presents the identification and statistical results of 6-cycle width case.

Table 3 Identification results of different data-interlacing widths

Item	Parameter	Signal length					Theoretical mode
		7.0	6.0	5.5	4.0	3.0	
Identification of 1st order mode	Damping ratio	0.020 17	0.018 70	0.017 60	0.022 81	0.031 60	0.02
	Modal frequency/Hz	22.05	21.90	22.00	21.94	21.30	20.56
	MAC	0.992	0.986	0.984	0.988	0.979	
Identification of 2nd order mode	Damping ratio	0.024 53	0.029 80	0.035 00	0.029 79	0.036 00	0.02
	Modal frequency/Hz	120.95	121.26	121.79	121.96	121.99	128.87
	MAC	0.993	0.987	0.979	0.975	0.969	

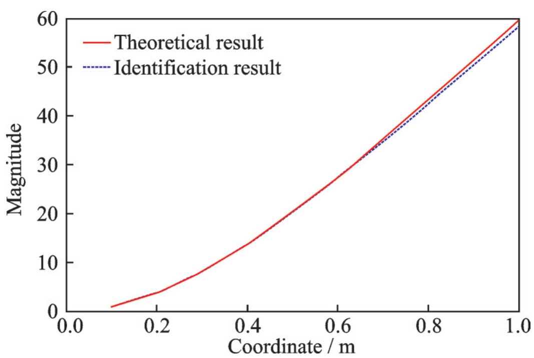

Fig.6 Identification of the first order modal shape

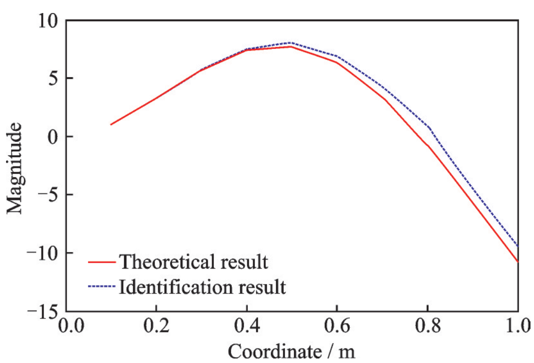

Fig.7 Identification of the second order modal shape

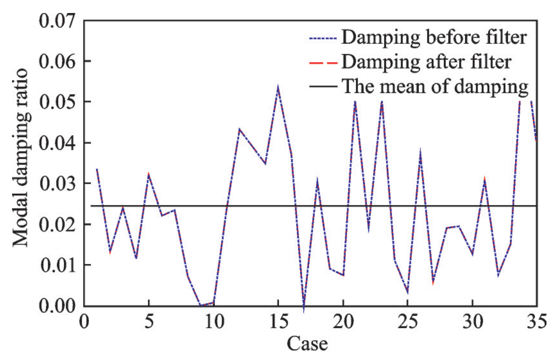

Fig.8 Modal damping ratio identification and statistical result of 6-cycle width case

From the result, it is derived the conclusion as follows:

(1) The consistency of the modal frequency identification result under different signal length is good, and the modal sharp is as same as the theoretical mode.

(2) The consistency of the modal damping ratio identification result under different signal length is poorer compared with the modal frequency, and the result under longer data-interlacing is better which is more closed to the theoretical mode.

(3) The difference of the modal damping ratio results decreases, where the biggest value is about one and a half times the theoretical one.

3.4 Robustness to signal-to-noise ratio (SNR)

According to the research above, the robustness of the method to SNR is investigated, where we consider 7-cycle data-interlacing. The identification of four cases is shown in Table 4, where the measurement noise is changed from 0% to 50% of the signal's magnitude.

From the result above, we can obtain the following conclusions.

(1) The consistency of the modal frequency identification result under different signal length is

Table 4 Identification of four noise cases

Item	Parameter	Measurement noise/%				Theoretical mode
		0	15	30	50	
Identification of 1st order mode	Damping ratio	0.020 17	0.025 06	0.018 29	0.024 21	0.02
	Modal frequency/Hz	22.05	21.69	21.85	22.04	20.56
	MAC	0.991	0.986	0.992	0.986	
Identification of 2nd order mode	Damping ratio	0.024 53	0.026 87	0.016 60	0.022 35	0.02
	Modal frequency/Hz	120.95	122.20	120.36	121.90	128.87
	MAC	0.992	0.985	0.979	0.985	

good, and the modal sharp is as same as the theoretical mode.

(2) When the measurement noise is applied, the modal damping ratio identification result has warps compared with the theoretical one, but the biggest warp is about 20%.

(3) As the accretion of the measurement noise, the consistency of the modal damping ratio identification result is good, which proves the method is robust.

3.5 Robustness to information quantity

According to the research above, the robustness of the method to information quantity of the response is investigated, where we also consider 7-cycles data-interlacing. Four cases with different combinations of position response are presented as follows.

Case 1 Translational response of element is from 1 to 10.

Case 2 Translational response of element is 2, 4, 6, 8, 10.

Case 3 Translational response of element is 1, 2, 3, 4, 5.

Case 4 Translational response of element is 6, 7, 8, 9, 10.

Case 1 represents all nodes' translational response of the beam, where the information quantity is comprehensive. Case 2 represents the information quantity is the uniformity and sparse. Case 3 and Case 4 represent the information quantity is closed to the constrained end and free end of the beam. The identification of four cases is shown in Table 5.

Table 5 Identification of four information quantity cases

Item	Parameter	Case				Theoretical mode
		1	2	3	4	
Identification of 1st order mode	Damping ratio	0.020 17	0.017 16	0.019 56	0.029 39	0.02
	Modal frequency/Hz	22.05	22.10	21.91	22.11	20.56
	MAC	0.991	0.989	0.995	0.986	
Identification of 2nd order mode	Damping ratio	0.024 53	0.019 37	0.018 31	0.017 36	0.02
	Modal frequency/Hz	120.95	121.21	120.98	122.12	128.87
	MAC	0.987	0.978	0.981	0.988	

From the result above, we can obtain the follow conclusions.

(1) The consistency of the modal frequency identification result under different cases is good, and the modal sharp is as same as the theoretical mode.

(2) The modal damping ratio identification result has warps compared with the theoretical one under different combinations of position response, but the biggest warp is about 10%, which proves the method is robust.

4 Conclusions

In this paper, we investigate the method to improve the precision and stability of the identified result of the damping ratio which is eurychoric in operational mode identification under ambient excitation. The operational mode identification method based on sliding time window method and ERA is investigated to improve the identification accuracy and stability. It is found that the mode can be realized by using the proposed method, and the method has high robustness to SNR and signal size.

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Acknowledgement This work was supported in part by the National Basic Research Program of China (No. JCKY2016203B032).

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Author contributions Dr. **WANG Liang** given out idea, complied the models and wrote the paper. Ms. **ZHANG Yan** designed calculation and conducted the analysis. Mr. **CAI Yipeng** analyzed sequencing data. Dr. **NANGONG Zijun** guided the work. All authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

(Production Editor: Zhang Huangqun)