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Probabilistic Tolerance Method for Omitting Small Fatigue Loads

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Abstract: The simplification of fatigue load spectrum, which can effectively reduce experimental cost, is of great importance for structural fatigue tests. By introducing random variables, the probabilistic tolerance method of removing small amplitude cycles proposed in this paper takes into account the randomness of both load and fatigue limit. The probability of the damage occurrence caused by the removed small loads is calculated to ensure that it cannot exceed the given probabilistic tolerance. Accordingly, the omission level is obtained and the truncated spectrum is formed. The unnotched aluminum sheet specimens are used to perform the fatigue test on the original fatigue spectrum and truncated fatigue spectrum of a transporter. The test results show that there is no statistical difference between the test life of the truncated spectrum and that of the original spectrum, which demonstrates the validity of the small-load-omitting method that considers randomness.

Key words: load spectrum truncation; P-S-N curve; exceedance frequency curve of load; probabilistic tolerance value

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0 Introduction

The load-time histories of aircraft structures often contain a large percentage of small amplitude cycles[1-5]. These numerous but nearly "non-damaging" small loads take up a large amount of test time. Removing these small cycles is of great significance for saving time and cost of fatigue test.

The process of fatigue failure contains crack initiation and crack propagation, and the load of the crack initiation stage accounts for a large proportion. A lot of research on the omission of small fatigue loads at this stage has been carried out. Heuler and Seeger[6] regarded 50% of the endurance limit (107 cycles) as the omission level. Jonge et al.[7] conducted fatigue tests on 2024-T3 notched sheet specimens under the transport wing standard (TWIST) and Mini-TWIST load sequences, finding out that the omission of the lowest gust load cycles (81% of the fatigue limit) resulted in a crack initiation life increase by a factor of 2.4. Yan et al.[8] carried out the fatigue test of 45 steel notched specimens under three kinds of block spectrums and came to the conclusion that small loads below fatigue limit had no effect on fatigue life and could be removed directly. Bao et al.[9] studied the low-load omission level of crack propagation of aluminum alloys 2324-T39 and 7050-T7451 under transport spectrum and indicated that an omission level of 11.72% or 13.98% was reasonable. Through the fatigue tests of LY12CZ and 30CrMnSiNi2A specimens under fighter spectra, Zhang et al.[10] demonstrated that the omission of low amplitude cycles with an overload ratio ($\sigma_{\max,\max}/\sigma_{\max}$) $r\geqslant 2.5$ had no distinct influence on

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crack initiation life. Schubbe studied the crack behavior of Al-Li alloy that an omission level of 30% will increase fatigue life by 15%[11].

In the above-mentioned omission methods which give a direct cut to the small loads, the dispersive characteristic of fatigue loads is out of consideration. In this paper, a cluster of load-frequency curve which reflects the load spectrum of a fleet with the same transporter composition is studied. Fig.1[12] demonstrates the dispersion of a fleet spectrum.

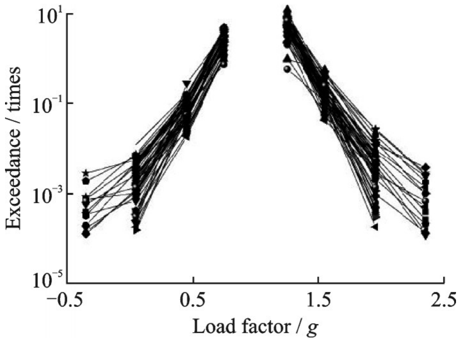
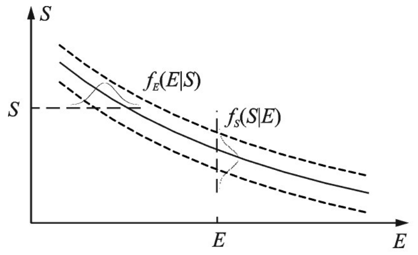


Fig.1 Fleet load spectrum of Fokker 27[12]

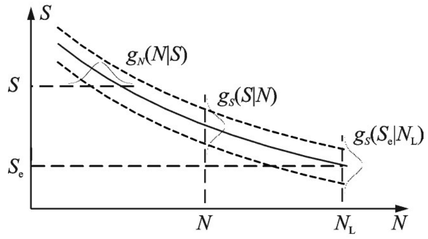
Due to dispersion, the corresponding fatigue load under a certain load frequency is a random variable. Some of these loads are comparatively large, which could cause fatigue damage, and some are small, causing no damage. In this paper, random variables are used to describe the fatigue load and fatigue limit. The probability of damage occurrence caused by the corresponding load at a fixed frequency is calculated to guarantee that it cannot exceed the given probabilistic tolerance value. Accordingly, the omission level is obtained and the truncated spectrum is formed. This method takes into account both the external dispersion of fatigue loads and the inherent dispersion of material fatigue limit. Under the condition of a given probabilistic tolerance value, the original fatigue load spectrum was truncated. Fatigue tests were performed and the rationality of this method was verified.

1 Probability Tolerance Method for Small Load Omission

The load-frequency curve is recorded as S-E curve, where S is the stress (maximum stress or stress amplitude), E is the exceedance frequency corresponding to stress S, and the fatigue curve of the material is S-N curve, where N is the fatigue life. The fatigue limit of the material is assumed to be S_e. Due to the randomness of load and material properties, the stress S and fatigue limit S_e are both random variables, as shown in Fig.2.



(a) Exceedance frequency curve of loads



(b) Fatigue curve of material

Fig.2 Randomness of variables S and S_e

According to the fatigue theory, when the random variable S is greater than S_e, fatigue damage occurs, and when the random variable S is less than S_e, no fatigue damage occurs. The occurrence of damage under a certain stress S is assumed to be event A. When the load group (S, S_e) is located in the shadow region in Fig.3, event A occurs, that is, $A = \{(S, S_e)\,|S > S_e\}$. The $f_S(S|E)$ and $g_{Se}(S_e|N_L)$ in Fig.3 are the probability density function of load S corresponding to a fixed exceedance frequency E and the probability density function corresponding to fatigue limit S_e, respectively. In gen-

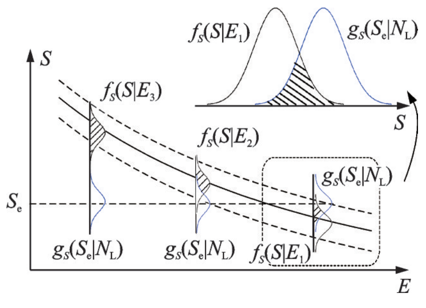


Fig.3 Schematic diagram of the occurrence of event A

eral, both are considered to obey normal distribution law. From the area of the shadow region in Fig.3, it can be seen that the probability of damage occurrence gradually increases with the external load S. When load S is small enough, the probability of damage approaches zero. When load S is developed to a certain value (load corresponding to surpassing frequency E_3), the probability of damage occurrence can be infinitely close to 1.

Let the occurring probability of event A be $P(A)$, then the calculating formula for $P(A)$ is

$$P(A)=\iint_{\Omega} g_{S_e}(y)f_S(x)\mathrm{d}x\mathrm{d}y=$$

$$\int_0^{\infty}\left[\int_0^x g_{Se}(y|N_{\mathrm{L}})dy\right]f_S(x|E)\mathrm{d}x \tag{1}$$

where $P(A)$ is the function of load S, and the probability of damage occurrence can be different with different load values. The integral region Ω in Eq. (1) is shown in Fig.4.

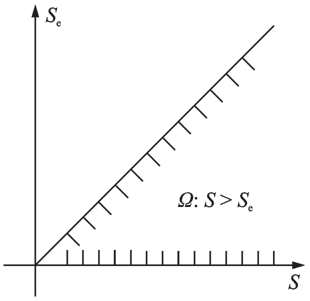


Fig.4 Integral region

When the omission level of load is S_{omit}, the equivalent probability δ is defined to describe the average probability of damage produced by loads within S_{omit} range.

$$\delta=\frac{1}{S_{\mathrm{omit}}}\int_0^{S_{\mathrm{omit}}}P(A)\mathrm{d}S \tag{2}$$

The effect of omitted small loads on fatigue life is considered acceptable if the equivalent probability is limited below the probability tolerance value δ_{CR}. The omission criterion for small fatigue loads is

$$\delta\leqslant\delta_{\mathrm{CR}} \tag{3}$$

2 Examples and Test Verification

2.1 Measurement of material fatigue properties

Fatigue tests were conducted using unnotched sheet specimens made of LC4CS (7A04T6) aluminum alloy which is widely used in aircraft structures. The configuration of the specimen is shown in Fig.5.

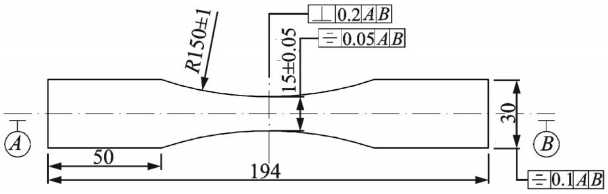


Fig.5 Configuration of unnotched sheet specimen

It is assumed that the S-N curve with different reliability can be expressed by a three-parameter power function formula

$$N_{cp}(S-S_{0p})^{H_p}=C_p \tag{4}$$

After logarithmic operations on both sides

$$\lg N_{cp}=-H_p\lg\left(S-S_{0p}\right)+\lg C_p \tag{5}$$

After the test, the double weighted least square method[13] was used for fitting the S-N curves under different reliabilities. The curve parameters are shown in Table 1. The curves with different reliabilities are obtained and shown in Fig.6.

It can be concluded from the above that the mean value of fatigue limit is 40.6 MPa, and the

Table 1 P-S-N curve parameters with different reliabilities

Reliablity p/%	H_p	$\lg C_p$	S_{0p}/MPa
99.87	1.394 8	7.257 7	38.5
97.72	1.381 1	7.282 8	39.2
84.1	1.366 0	7.305 2	39.9
50	1.349 3	7.324 7	40.6
15.9	1.330 7	7.341 0	41.3
2.28	1.310 2	7.353 9	42.0
0.13	1.287 6	7.363 0	42.7

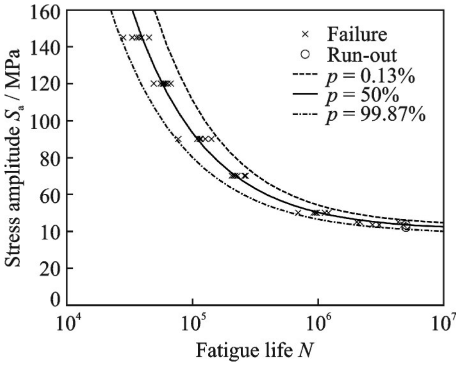


Fig.6 P-S-N curve of LC4CS unnotched specimen standard deviation is 0.7 MPa.

2.2 Truncation of load spectrum and test verification

The original load spectrum is derived from the actual center-of-gravity overload spectrum of a transport aircraft. Each spectrum block represents 2 000 flights and the number of load cycles is 431 681. The stress at the flight state of 1-g is 210 MPa. Since the nominal stress and the center-of-gravity overload have a linear relationship, the center-of-gravity overload spectrum is converted into stress spectrum, and the exceedance curve of the original spectrum is

$$S_0(E) = -37.15\lg E + 209.37 \tag{6}$$

In addition, the ground-air-ground cycles are also included. The valley stress is $S_g = 0$ MPa, and the peak stress, which occurs once in each flight, is determined as 296.7 MPa. The ground-air-ground cycle remains unchanged during the truncation of load spectrum. It's worth noting that the exceedance curve mentioned above only considers the average of a fleet usage, and the spectrum of fleet cannot be acquired, so the dispersion of load spectrum is not considered in the verification.

Different omission levels S_{omit} are taken to calculate the corresponding equivalent probabilities, and the calculation results are shown in Fig.7. The equivalent probability is a monotonically increasing function of the omission level S_{omit}, and the desired omission level can be obtained by finding the S_{omit} value corresponding to the probability tolerance δ_{CR} on their relationship curve. The probability tolerance, which is selected according to specific requirements in engineering application, is the maximum equivalent probability that the omitted small loads can cause damage.

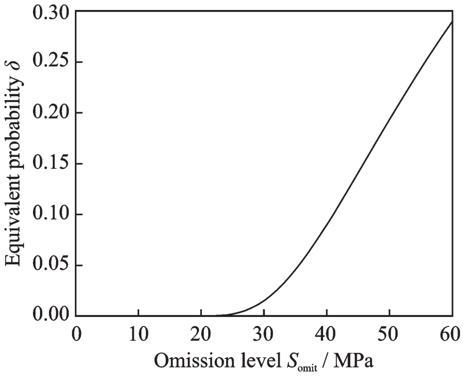


Fig.7 Change of tolerance with omission level

The small-load-omission criterion was verified by experiment. Based on the TWIST method[14-15], the truncated spectrum with omission levels 22, 31, 41, 52 and 64 MPa were compiled to be flight-by-flight testing spectrum, which were recorded as $L1$, $L2$, $L3$, $L4$ and $L5$, respectively. The detailed information of the five truncated spectrum and the original spectrum $L0$ are given in Table 2. The five truncated spectrum were tested in fatigue experiment and the results are shown in Table 3. It can be seen that the fatigue life did not change much after the omission level was reduced to 41 MPa, thus the test life of the $L1$ spectrum can be used as the fatigue life of the original spectrum.

T-test method was used to test the mean value of crack formation life under different load spectra, judging whether the logarithmic life expectancy of spectra $L2$—$L5$ had a large difference with that of spectrum $L1$. The significance level $\alpha = 0.05$ was taken, and the calculated and critical values of the statistic parameter T are shown in Table 4. It can be seen that the value of T satisfies $T_{\alpha/2} < T < T_{1-\alpha/2}$ for spectra $L2$—$L3$, and the mean logarithmic lives of spectrum $L2$ and spectrum $L3$ exhibit no significant difference with that of spectrum $L1$.

Table 2 Spectra *L0*—*L5*

Spectrum	$L0$	$L1$	$L2$	$L3$	$L4$	$L5$
Omission level S_{omit}/Mpa	0	22	31	41	52	64
Cycle number	431 681	99 266	62 918	34 135	17 218	8 159
Percentage of remainder cycles/%	100	23	15	8	4	2
Equivalent probability δ	0	3.05×10^{-4}	0.02	0.10	0.21	0.32

Table 3 Fatigue test results of spectra $L1$—$L4$

Spectrum	Crack initiation life for 33 specimes (spectrum block, 1 block=2 000 flights)							Average value	Standard deviation
$L1$	5.07	4.62	6.10	5.27	4.59	5.16	5.60	5.20	0.53
$L2$	4.86	4.74	4.22	4.42	4.47	4.18	5.37	4.61	0.42
$L3$	4.84	6.00	5.42	5.22	6.34	5.84	5.77	5.63	0.55
$L4$	7.53	7.31	8.11	8.10	6.74	7.84		7.61	0.53
$L5$	9.17	8.40	10.83	10.85	10.49	9.54		9.88	1.00

Table 4 Test results of mean logarithmic fatigue life under different load spectra

Spectrum	T	$T_{\alpha/2}$	$T_{1-\alpha/2}$
$L2$	2.12	−2.45	2.45
$L3$	1.21	−2.45	2.45
$L4$	15.7	−2.57	2.57
$L5$	14.9	−2.57	2.57

The F-test method was used to test the variance of the crack formation life under different load spectra, judging whether the log-normalized standard deviation of spectra $L2$—$L5$ was significantly different from that of spectrum $L1$. The calculated and critical values of the statistic parameter F are listed in Table 5. It can be concluded that the value of F satisfies $F_{\alpha/2} < F < F_{1-\alpha/2}$ for all load spectra $L2$—$L5$, so the logarithmic life standard deviation of the truncated spectra has no significant difference from that of spectrum $L1$.

Table 5 Test results of logarithmic standard deviation of fatigue life under different load spectra

Spectrum	F	$F_{\alpha/2}$	$F_{1-\alpha/2}$
$L2$	0.76	0.17	5.88
$L3$	0.81	0.17	5.88
$L4$	0.51	0.14	6.98
$L5$	1.08	0.14	6.98

According to the results of T-test and F-test, it can be concluded that the fatigue life distribution of spectrum $L2$ and spectrum $L3$ is the same as that of spectrum $L1$, which means that the omission level set to 41 MPa is reasonable, and the corresponding probability tolerance δ_{CR} is found to be 0.10 in Fig.7.

3 Conclusions

(1) A probabilistic tolerance omission method for fatigue loads is proposed. When the equivalent probability is equal to the probabilistic tolerance, the omission level can be obtained, and the truncated spectrum is further formed.

(2) The proposed omission method takes into account the effect of load dispersion and fatigue limit dispersion. If the structural P-S-N curve, the original load spectrum, and the probabilistic tolerance value are given, the omission level as well as the corresponding truncated spectrum can be achieved.

(3) The test results demonstrate the validity of the proposed small-load-omission method in engineering applications, which is useful for saving both fatigue test time and cost.

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Author contributions Dr. **LIN Hanyu** designed the study, complied the models, conducted the analysis, interpreted the results and wrote the manuscript. Dr. **XU Lipu** contributed to data and conducted the test. Prof. **YAO Weixing** contributed to the design and discussion of the study. Prof. **HUANG Jie** contributed to the discussion and background of the study. All authors commented on the manuscript draft and approved the submission.

Competing interests The authors declare no competing interests.

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疲劳小载荷删除的概率容限值方法

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摘要：疲劳载荷谱的浓缩可以大大减少试验费用，对于结构的疲劳试验非常重要。本文提出的载荷谱删除的概率容限值方法考虑了载荷及疲劳极限的随机性，采用随机变量描述载荷和疲劳极限。通过计算删除的小载荷能够产生损伤的概率，使其不能超过设定的概率容限值，从而得到删除水平，形成删除谱。采用铝合金光滑试验件对某运输机疲劳载荷的原始谱和删除谱进行疲劳试验，试验结果表明删除谱的试验寿命与原始谱试验寿命在统计上没有差别，说明本文提出的考虑随机性的载荷谱删除方法的有效性。

关键词：载荷谱删除；*P-S-N*曲线；载荷超越频次曲线；概率容限值