Uncertain Modal Analysis of Unmanned Aircraft Composite Landing Gear

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Abstract: Through taking uncertain mechanical parameters of composites into consideration, this paper carries out uncertain modal analysis for an unmanned aircraft landing gear. By describing correlated multi-dimensional mechanical parameters as a convex polyhedral model, the modal analysis problem of a composite landing gear is transferred into a linear fractional programming (LFR) eigenvalue solution problem. As a consequent, the extreme-point algorithm is proposed to estimate lower and upper bounds of eigenvalues, namely the exact results of eigenvalues can be easily obtained at the extreme-point locations of the convex polyhedral model. The simulation results show that the proposed model and algorithm can play an important role in the eigenvalue solution problem and possess valuable engineering significance. It will be a powerful and effective tool for further vibration analysis for the landing gear.

Key words: landing gear; composite; uncertain; convex polyhedral model; modal analysis

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0 Introduction

As an important part of aircraft, the landing gear is a special device for taking off, landing, taxiing and parking. Therefore, it plays an important role in aircraft structural design[1]. The landing gear should not only meet certain static strength design standards and requirements, but also meet the dynamic quality requirements under a series of working conditions such as take-off, landing and taxiing, etc.[2]. Especially, under the action of periodic dynamic load, the main component of the landing gears may have strong resonance due to unreasonable structural design or coupling between the natural frequencies of the system and the aircraft body, which will seriously affect the reliability and stability of the landing gear. As a result, in the process of dynamic design of landing gear structure, as the key parameter of structural vibration performance, it has very obvious engineering significance to accurately predict the eigenvale, natural frequency and modal shape of the landing gear structure[3-4].

On the other hand, owing to the advantages of high specific strength, high specific stiffness and good design-ability, advanced composite materials have been widely applied in the field of aerospace and aviation. And it is gradually moving towards the structural design direction of the landing gear[5-6]. Generally speaking, the landing gear accounts for 3%—6% of the takeoff weight of an aircraft. Therefore, it is of great significance to apply advanced composite materials into the structural design of landing gear. However, it should be noted that there exists a large amount of uncertainties in the design and service process of composite structures, mainly due to material properties, geometric dimensions and working loads, etc. It will seriously affect the safety and reliability of composite structures[7]. Therefore, in order to make full use of the superior performance and continuously tap its material potential, it is necessary to quantify these uncertain factors accurately in structural mechanical response analysis and structural optimization design[8], includ-

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ing probabilistic analysis approaches and non-probabilistic ones[9-10]. Especially, when the available uncertain information is limited or insufficient, the probabilistic methods will no longer be applicable. On the contrary, the non-probabilistic methods will show great advantages. Generally speaking, these non-probabilistic methods consist of interval models (or hyper-rectangles), ellipsoid models (or hyper-ellipsoid models) and convex polyhedral models. For example, Qiu et al.[10] treated the uncertain parameters of composite materials as uncertain-but-bounded variables, and then made use of the ellipsoid methods and interval methods to study the uncertain buckling problem of composite structures. Wang et al.[7] dealt with the performance parameters of composite materials as hyper-ellipsoid models or hyper-rectangle models by using the methods of minimum hyper-ellipsoid and minimum hypercube, and then studied the vibration and buckling problem of composite shells. Of course, the convex polyhedral model can also be an effective method to describe the uncertain-but-bounded mechanical parameters of composite materials, which have two aspects of advantages and will be very important and meaningful for engineering problems with uncertainties. One is that it can deal with dependent uncertainties and avoid the over-conservatism phenomenon compared to interval models or hyper-rectangle models. The other is that it has the advantage of low computational loss because of the linear structural form compared with the ellipsoidal[11] or hyper-ellipsoidal models[12-13]. Therefore, in the aspects of uncertain response analysis and eigenvalue prediction for engineering structures, how to take good advantages of the convex polyhedral model is still a hot issue worthy of discussion and research.

This paper carries out the modal analysis of the composite landing gear structure for an unmanned aircraft vehicle, where uncertainties of the composite mechanical properties are taken into consideration. As a consequence, a more credible natural frequency range can be obtained and it can provide necessary data support for further structural design and modal test verification. Here, the uncertain mechanical parameters can be represented as a convex polyhedral model, which is described by some linear inequality constraint equations. The upper and lower bounds of the natural frequency interval can be obtained by using the proposed extreme-point solution algorithm. The results show that the proposed model and method are feasible and of great significance.

1 Establishment of FEM for Unmanned Composite Landing Gear

The tail landing gear of an unmanned aircraft is mainly composed of fuselage mounting plates, composite buffer beam, wheel fork, wheel axle, wheel and tire. The landing gear mainly absorbs energy through the deformation of tire and buffer beam, with simple structure, high reliability and good maintainability. The landing gear structure is shown in Fig.1.

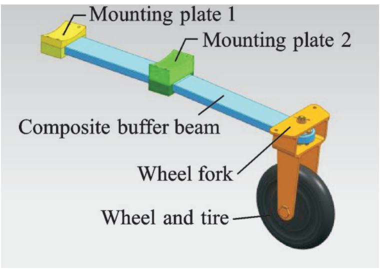


Fig.1 Sketch map of unmanned aircraft composite landing gear

The buffer beam structure of the landing gear is the fiber-reinforced composite structure with the sandwich materials. The core material is a foam structure, and the carbon fiber layer is reinforced on the upper and lower surfaces of the core body. The corresponding layer sequence (1-1—1-16) is shown in Fig.2.

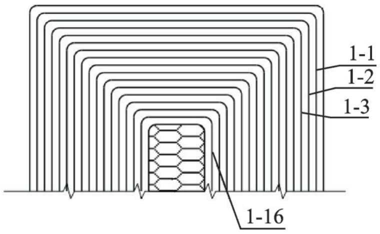


Fig.2 Sketch map of composite layer for buffer beam

Here, the composite material is the T300/QY8911 carbon fiber-reinforced unidirectional tape and the density is 1.62 g/cm^3. The laminate information of the composite structure is $[45/-45/0/0]_{2S}$ with 16 plies and the ply thickness is 0.125 mm. The other parts are 7050 aluminum alloy.

Four elastic mechanical parameters of composites are considered including the longitudinal tension elastic modulus E_1, the transverse tension elastic modulus E_2, the Poisson's ratio υ_{12} and the shear modulus G_{12}.

These mechanical parameters can be measured by tension test or shear test of composite materials, which shows that they are actually dispersive or uncertain. In the process of structural mechanical response analysis and reliability evaluation, it is necessary to consider the influence of uncertain mechanical parameters on the results.

2 Mathematical Characterization of Composite Uncertain Mechanical Parameters

Owing to their special structural form and superior mechanical properties, fiber-reinforced composite laminates have been widely applied in the fields of aerospace and successfully used in the structural design of the landing gear. In fact, the composite laminated structure is a series of laminas, which is composed of the reinforced phase (fibers) and the matrix phase (resin matrix). However, due to the complex manufacturing process and the inherent dispersion of component materials, the mechanical properties of composite laminates usually show great uncertainty and intrinsic correlation[14]. Furthermore, based on grey mathematical theories, these uncertain mechanical parameters can be dealt with as interval models or convex polyhedral models. They can be expressed as

$$\begin{cases} x_i \leqslant x_i^c + 3 \cdot s_i \\ -x_i \leqslant 3 \cdot s_i - x_i^c \end{cases} \quad i = 1,2,\cdots,m \tag{1}$$

$$\begin{cases} x_i \cdot \cos\theta_k + x_j \cdot \sin\theta_k \leqslant 7.5 \cdot \dfrac{\Delta_{\max}^{ij}}{N} + \\ \quad x_i^c \cdot \cos\theta_k + x_j^c \cdot \sin\theta_k \\ -x_i \cdot \cos\theta_k - x_j \cdot \sin\theta_k \leqslant 7.5 \cdot \dfrac{\Delta_{\max}^{ij}}{N} - \\ \quad x_i^c \cdot \cos\theta_k - x_j^c \cdot \sin\theta_k \end{cases} \\ i = 1,2,\cdots,m; \quad j = i+1,\cdots,m \tag{2}$$

where m is the number of uncertain mechanical parameters, x_i^c the mean value of uncertain mechanical parameters, s_i the corresponding standard variance, and θ_k the polar angle of any two mechanical parameters.

$$x_i^c = \sum_{j=1}^{N} x_{ij} \quad i = 1,2,\cdots,m \tag{3}$$

$$s_i = 2.5 \cdot \frac{\Delta_{\max}^{i}}{N} \quad i = 1,2,\cdots,m \tag{4}$$

where x_{ij} is the available experimental sample point, which has already been validated and sorted by the ascending order. N is the number of valid experimental sample points, and $\Delta_{\max}^i$ the maximum difference between the mean-value accumulated sequence and the accumulated data sequence, namely

$$\Delta_{\max}^{i} = \max\left(\left| k \cdot x_i^c - \sum_{j=1}^{k} x_{ij} \right|\right) \quad k = 1,2,\cdots,N \tag{5}$$

It can be seen that Eqs.(1), (2) are actually a series of linear inequality constrained equations. They can be written as

$$\begin{gathered} \boldsymbol{Ax} \leqslant \boldsymbol{b} \\ \boldsymbol{A} \in \boldsymbol{R}^{M \times m}, \boldsymbol{x} \in \boldsymbol{R}^{m}, \boldsymbol{b} \in \boldsymbol{R}^{M} \end{gathered} \tag{6}$$

where $\boldsymbol{A} = (a_{ij})_{M \times m}$ is the $M \times m$ dimensional coefficient matrix, and M the number of linear inequalities constraints. Furthermore, the feasible region $\boldsymbol{S}$ of uncertain mechanical parameters can be denoted as

$$\boldsymbol{S} = \{ \boldsymbol{x} \in \boldsymbol{R}^m \mid \boldsymbol{Ax} \leqslant \boldsymbol{b} \} \tag{7}$$

It is a closed and bounded convex polyhedral model. Then according to the Krein-Milman theorem, in the uncertainty region, every point $\boldsymbol{x}$ of a closed, bounded and not-null convex polyhedral model can be represented as

$$\boldsymbol{x} = \sum_{i=1}^{L} \alpha_i y_i \quad \sum_{i=1}^{L} \alpha_i = 1; \alpha_i \geqslant 0; i = 1,2,\cdots,L \tag{8}$$

where y_i are the extreme points of the convex polyhedral model, L is the number of all extreme points, and $\alpha_i \geqslant 0$ is the coefficient.

3 Convex Polyhedral Model of Uncertain Eigenvalue Problem

Considering that the eigenvalue problem without damping effects can be expressed as the following form for composite landing gear with uncertain mechanical parameters

$$\boldsymbol{K}(\boldsymbol{x})\boldsymbol{u}=\lambda\boldsymbol{M}(\boldsymbol{x})\boldsymbol{u} \tag{9}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are the structural stiffness matrix and mass matrix, respectively. $\boldsymbol{u}$ is the eigenvector or the mode shape for structural vibrations, and λ the corresponding eigenvalue or the square of natural frequency. Furthermore, one can take good advantages of optimization theories or numerical algorithm to determine upper and lower bounds, namely, the maximum and minimum values. According to the knowledge of the convex polyhedral model, the structural stiffness matrix $\boldsymbol{K}(\boldsymbol{x})$ and mass matrix $\boldsymbol{M}(\boldsymbol{x})$ can be expressed as the following convex combination form.

$$\begin{aligned}&\boldsymbol{K}(\boldsymbol{x})=\boldsymbol{K}(\boldsymbol{\alpha})=\sum_{i=1}^{L}\alpha_i\boldsymbol{K}_i\\&\boldsymbol{M}(\boldsymbol{x})=\boldsymbol{M}(\boldsymbol{\alpha})=\sum_{i=1}^{L}\alpha_i\boldsymbol{M}_i\\&\sum_{i=1}^{L}\alpha_i=1;\alpha_i\geqslant 0;i=1,2,\cdots,L\end{aligned} \tag{10}$$

where $\boldsymbol{K}_i$, $\boldsymbol{M}_i$ are the extreme-point of the stiffness matrix and mass matrix in the uncertain region, respectively. $\boldsymbol{\alpha}=(\alpha_i)$ is the corresponding coefficient vector. Then Eq.(9) can be rewritten as

$$\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}=\lambda\boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{u} \tag{11}$$

When the structural stiffness matrix $\boldsymbol{K}$ and mass matrix $\boldsymbol{M}$ vary in uncertain ranges, the changing region of structural eigenvalues can be expressed as

$$\boldsymbol{\Lambda}=\{\lambda:\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}=\lambda\boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{u}\,|\,\boldsymbol{K},\boldsymbol{M}\in\boldsymbol{\Phi}\} \tag{12}$$

where $\boldsymbol{\Phi}$ is a closed and bounded convex polyhedral model, namely

$$\begin{aligned}&\boldsymbol{\Phi}=\left\{(\boldsymbol{K},\boldsymbol{M}):\boldsymbol{K}(\boldsymbol{x})=\sum_{i=1}^{L}\alpha_i\boldsymbol{K}_i,\boldsymbol{M}(\boldsymbol{x})=\sum_{i=1}^{L}\alpha_i\boldsymbol{M}_i\right\}\\&\sum_{i=1}^{L}\alpha_i=1;\alpha_i\geqslant 0;i=1,2,\cdots,L\end{aligned} \tag{13}$$

Then the maximum and minimum values of structural eigenvalues can be written as

$$\lambda_{\min}\leqslant\underline{\lambda}\leqslant\boldsymbol{\Lambda}\leqslant\overline{\lambda}\leqslant\lambda_{\max} \tag{14}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum values of the eigenvalues, respectively. Furthermore, they can be expressed as

$$\begin{aligned}&\lambda_{\min}=\min\left\{\frac{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}}{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{u}}\right\}\\&\lambda_{\max}=\max\left\{\frac{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}}{\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{u}}\right\}\end{aligned} \tag{15}$$

By substituting Eq.(10) into Eq.(15), the following equation can be obtained as

$$\begin{aligned}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}(\boldsymbol{\alpha})\boldsymbol{u}=&\boldsymbol{u}^{\mathrm{T}}(\alpha_1\boldsymbol{K}_1+\alpha_2\boldsymbol{K}_2+\cdots+\alpha_L\boldsymbol{K}_L)\boldsymbol{u}=\\&\alpha_1\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}_1\boldsymbol{u}+\alpha_2\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}_2\boldsymbol{u}+\cdots+\alpha_L\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}_L=\\&\alpha_1p_1+\alpha_2p_2+\cdots+\alpha_Lp_L=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\alpha}\end{aligned} \tag{16}$$

$$\begin{aligned}\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\alpha})\boldsymbol{u}=&\boldsymbol{u}^{\mathrm{T}}(\alpha_1\boldsymbol{M}_1+\alpha_2\boldsymbol{M}_2+\cdots+\alpha_L\boldsymbol{M}_L)\boldsymbol{u}=\\&\alpha_1\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}_1\boldsymbol{u}+\alpha_2\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}_2\boldsymbol{u}+\cdots+\alpha_L\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}_L\boldsymbol{u}=\\&\alpha_1q_1+\alpha_2q_2+\cdots+\alpha_Lq_L=\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\alpha}\end{aligned} \tag{17}$$

$$p_i=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{u},q_i=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{u}\quad i=1,2,\cdots,L \tag{18}$$

Therefore, Eq.(15) can be rewritten as

$$\begin{aligned}&\lambda_{\min}=\min\left\{\frac{\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\alpha}}{\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\alpha}}\right\}\quad\lambda_{\max}=\max\left\{\frac{\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\alpha}}{\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\alpha}}\right\}\\&\boldsymbol{\alpha}\in\boldsymbol{\Omega}=\left\{\boldsymbol{\alpha}\,\middle|\,\sum_{i=1}^{L}\alpha_i=1\right\}\quad\alpha_i\geqslant 0;i=1,2,\cdots,L\end{aligned} \tag{19}$$

Generally speaking, Eq.(19) can be viewed as a linear fractional programming (LFP) problem. Here, the mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$ have non-negative decompositions. That is to say, one can have

$$\begin{aligned}&p_i=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{K}_i\boldsymbol{u}>0,q_i=\boldsymbol{u}^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{u}>0\\&i=1,2,\cdots,L\end{aligned} \tag{20}$$

Then Eq.(19) can be called as the convex polyhedral model of structural eigenvalue problems with uncertain parameters.

4 Extreme-Point Solution Algorithm of Convex Polyhedral Model for Eigenvalue Problems

For the sake of predicting lower and upper bounds of structural eigenvalues efficiently and accurately, the extreme-point solution algorithm is proposed for the convex polyhedral model. Let

$$u(\boldsymbol{\alpha})=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\alpha},v(\boldsymbol{\alpha})=\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\alpha} \tag{21}$$

Suppose that

$$\frac{u(\boldsymbol{A}_l)}{v(\boldsymbol{A}_l)}=\frac{\boldsymbol{p}^{\mathrm{T}}\boldsymbol{A}_l}{\boldsymbol{q}^{\mathrm{T}}\boldsymbol{A}_l}=\min\left\{\frac{\boldsymbol{p}^{\mathrm{T}}\boldsymbol{A}_i}{\boldsymbol{q}^{\mathrm{T}}\boldsymbol{A}_i}\middle| i=1,2,\cdots,L\right\} \tag{22}$$

where $\boldsymbol{A}_i, i=1,2,\cdots,L$ is the extreme point in the uncertain-but-bounded region $\boldsymbol{\Omega}$ and

$$\boldsymbol{A}_i=(0,\cdots,0,1,0,\cdots,0)^{\mathrm{T}} \tag{23}$$

Only the i-th element is one when other elements are zero. Besides, every extreme point $\boldsymbol{A}_i$ is corresponded to the extreme point y_i for the feasible region $\boldsymbol{S}$ of dependent uncertain-but-bounded parameters. Then one can have

$$u(\boldsymbol{A}_l)v(\boldsymbol{A}_i)\leqslant u(\boldsymbol{A}_i)v(\boldsymbol{A}_l)\quad i=1,2,\cdots,L \tag{24}$$

Through multiplying $\alpha_i(\alpha_i\geqslant 0, i=1,2,\cdots,L)$ on both sides of Eq.(24) and accumulating all linear inequlities, the following formula can be obtained as

$$u(\boldsymbol{A}_l)\sum_{i=1}^{L}\alpha_i v(\boldsymbol{A}_i)\leqslant v(\boldsymbol{A}_l)\sum_{i=1}^{L}\alpha_i u(\boldsymbol{A}_i) \tag{25}$$

Because $u(\boldsymbol{A}_i)=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{A}_i, v(\boldsymbol{A}_i)=\boldsymbol{q}^{\mathrm{T}}\boldsymbol{A}_i, i=1,2,\cdots,L$, then

$$\sum_{i=1}^{L}\alpha_i u(\boldsymbol{A}_i)=\boldsymbol{p}^{\mathrm{T}}\left(\sum_{i=1}^{L}\alpha_i\boldsymbol{A}_i\right)=\boldsymbol{p}^{\mathrm{T}}\boldsymbol{\alpha}=u(\boldsymbol{\alpha}) \tag{26}$$

$$\sum_{i=1}^{L}\alpha_i v(\boldsymbol{A}_i)=\boldsymbol{q}^{\mathrm{T}}\left(\sum_{i=1}^{L}\alpha_i\boldsymbol{A}_i\right)=\boldsymbol{q}^{\mathrm{T}}\boldsymbol{\alpha}=v(\boldsymbol{\alpha}) \tag{27}$$

Substituting Eqs.(26),(27) into Eq.(25), one can obtain that

$$u(\boldsymbol{A}_l)v(\boldsymbol{\alpha})\leqslant v(\boldsymbol{A}_l)u(\boldsymbol{\alpha}) \tag{28}$$

Namely

$$\frac{u(\boldsymbol{A}_l)}{v(\boldsymbol{A}_l)}\leqslant\frac{u(\boldsymbol{\alpha})}{v(\boldsymbol{\alpha})}\quad \forall\boldsymbol{\alpha}\in\boldsymbol{\Omega} \tag{29}$$

That is to say, the eigenvalues can achieve the minimum values at the location of extreme points for convex polyhedral models. Similarly, the eigenvalues can also achieve the maximum values at the location of extreme points for convex polyhedral models. Furthermore, the eigenvalue bounds of the composite landing gear can be obtained as

$$\begin{aligned}\lambda_{\min}&=\min\left\{\frac{p_1}{q_1},\frac{p_2}{q_2},\cdots,\frac{p_L}{q_L}\right\}\\ \lambda_{\max}&=\max\left\{\frac{p_1}{q_1},\frac{p_2}{q_2},\cdots,\frac{p_L}{q_L}\right\}\end{aligned} \tag{30}$$

Then the proposed extreme-point solution algorithm for the convex polyhedral model can be efficiently and accurately applied in the eigenvalue estimation problem.

5 Calculation Results and Discussions

The large-scale finite element analysis software ANSYS is made use of to establish the finite element analysis model of composite landing gear as shown in Fig.3. The shell 181 element is used to simulate the composite buffer beam and the solid 45 or solid 95 element is used to simulate the other parts. The shell 181 element is a 4-node three-dimensional shell element, whose node has 6 degrees of freedom. More than 250 layers of material are allowed in the element and the layer information can be defined by the section command. The fixed support constraint is applied to the upper end surfaces of the fuselage mounting plate 1 and mounting plate 2. In addition, in order to consider the influence of the mass of wheel and tire on the structural mode, a mass point of 0.5 kg is coupled in the center of the wheel axle by the multi-point constraint. The whole finite element model of the landing gear structure contains about 40 000 nodes and 90 000 elements.

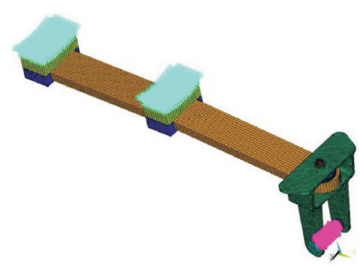


Fig.3 Finite element model of composite landing gear

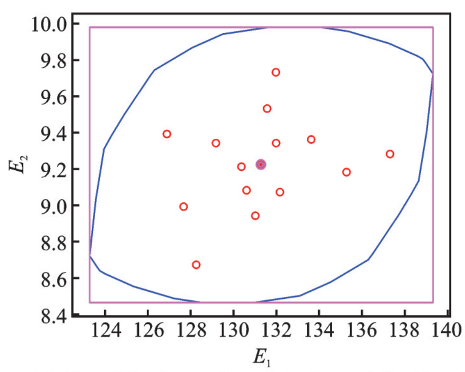
In order to consider the influence of uncertain mechanical parameters on the analysis results, the uncertainty analysis method is used to quantify the uncertainty of test data points[15]. Here, the test data points of elastic mechanical parameters of T300/QY8911 carbon fiber-reinforced unidirectional tape are shown in Table 1.

Through non-probabilistic analysis methods, the interval model and convex polyhedral model can be obtained for uncertain mechanical parameters of composite materials, as shown in Fig.4. Besides, the characteristic parameters of mechanical parame-

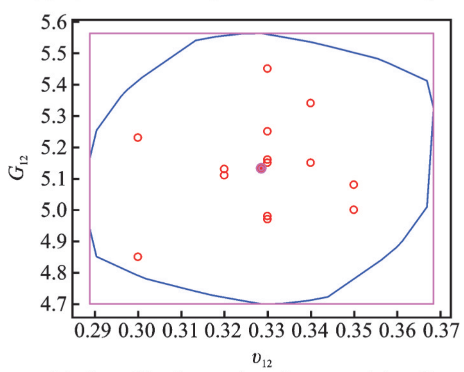
Table 1 Experimental points of elastic mechanical parameters for T300/QY8911

No.	E_1/GPa	E_2/GPa	υ_{12}	G_{12}/GPa
1	129.20	9.34	0.30	5.23
2	131.59	9.53	0.33	4.97
3	130.63	9.08	0.33	5.16
4	132.01	9.34	0.33	5.15
5	131.04	8.94	0.34	5.15
6	127.69	8.99	0.32	5.11
7	133.65	9.36	0.35	5.08
8	132.19	9.07	0.30	7.85
9	132.00	9.73	0.35	5.00
10	130.39	9.21	0.34	5.34
11	128.28	8.67	0.33	4.98
12	135.30	9.18	0.32	5.13
13	137.33	9.28	0.33	5.25
14	126.91	9.39	0.33	5.45

ters are shown in Table 2. Among them, the red hollow point is the test data sample point and the red solid point is the nominal value of uncertain mechanical parameters. The area enveloped by pink lines is the uncertain interval model of elastic parameters and the area enveloped by blue curves is the uncertain convex polyhedral model.



(a) Quantification region of elastic modulus E_1 and E_2



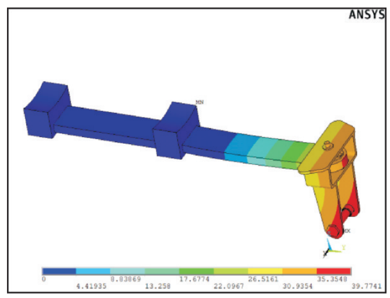
(b) Quantification region of gear modulus G_{12} and Poisson's ratio υ_{12}

Fig.4 Uncertain quantification regions of composite mechanical parameters

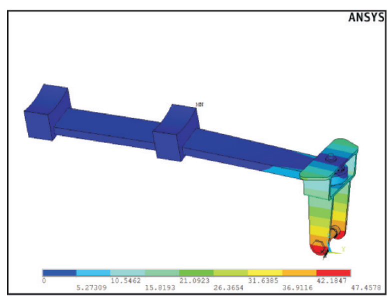
Table 2 Characteristics parameters for uncertain mechanical parameters

Parameter	E_1/GPa	E_2/GPa	υ_{12}	G_{12}/GPa
Nominal value	131.30	9.22	0.329	5.13
Standard variance	2.67	0.25	0.013	0.14
Lower bound	123.28	8.46	0.289	4.70
Upper bound	139.32	9.98	0.368	5.56

Furthermore, using ANSYS finite element analysis software, the first-order and second-order natural frequencies of composite landing gear structure are 51.04 Hz and 65.20 Hz, respectively. The modal shapes are shown in Fig.5, which are vertical bending and lateral waving, respectively.



(a) The first-order mode shape



(b) The second-order mode shape

Fig.5 Mode shapes of composite landing gear

Furthermore, the uncertain intervals including the lower bound (LB) and upper bound (UB) can be obtained for the natural frequencies of the landing gear structure by using the proposed extreme-point solution algorithm, as shown in Table 3.

Table 3 Natural frequencies of composite landing gear

Natural frequency	First-order		Second-order	
	LB	UB	LB	UB
Interval model	49.53	52.49	63.36	66.95
MCS for interval model	49.53	52.49	63.36	66.95
Convex polyhedral model	50.01	52.01	63.77	66.53
MCS for convex polyhedral model	50.01	52.01	63.77	66.53

Here, the interval model does not consider the correlations between uncertain mechanical parameters, while the convex polyhedral model can consider the correlations between mechanical parameters. Through comparisons of the results, the following conclusions can be found: (1) When considering the parameter correlations (convex polyhedral model), the estimation interval of natural frequencies is more compact than that without considering parameter correlations (interval model). In other words, a more precise and compact range of structural mechanical responses can be obtained when considering correlation of mechanical properties. (2) The lower and upper bounds of natural frequencies predicted by the proposed algorithm and Monte Carlo simulation (MCS) method are identical. Here, the number of sample points used in the proposed algorithm is $2^4=16$ (interval model) or 360 (convex polyhedral model), and the number of sample points in the MCS method is 10 000. It can be seen that when the amount of calculation is far less than that of MCS method ($360 \ll 10\ 000$), the calculation results by the extreme-point solution algorithm have the same accuracy with the MCS method.

In summary, the proposed extreme-point solution algorithm can be as an efficient and accurate method to solve natural frequencies, where the uncertain-but-bounded mechanical properties of composites can be represented as a convex polyhedral model.

6 Conclusions

This paper is aimed at taking good use of the non-probabilistic approach, namely, the convex polyhedral model to mathematically characterize the uncertain mechanical parameters of an unmanned aircraft composite landing gear. Furthermore, the proposed extreme-point solution algorithm of the convex polyhedral model is applied into carrying out uncertain modal analysis of composite landing gear structure, where uncertain mechanical parameters can be described as a series of linear inequality equations. Compared with the Monte Carlo simulation method, the proposed algorithm can greatly reduce the calculation cost while ensuring the calculation accuracy. The results show that the proposed method can play an important role in the uncertain eigenvalue problem of landing gear structure, and be a powerful calculation tool in further vibration analysis.

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无人机复合材料起落架不确定性模态分析

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摘 要:考虑复合材料力学参数的不确定性,对某无人机起落架进行模态分析研究。通过将相关多维力学参数描述为一个凸多面体模型,复合材料起落架的模态分析问题可转化为一个线性分式规划特征值求解问题,进而提出了特征值上下界的极点求解算法,即可在凸多面体的极点求得特征值的精确解。仿真结果表明,提出的模型和算法在工程结构特征值问题中具有真实的工程意义,在进一步起落架振动分析研究中可成为一种重要的强有力的计算工具。

关键词:起落架;复合材料;不确定性;凸多面体;模态分析